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Erratum: Group symmetry and covariance regularization

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This erratum concerns paper EJS723: "Group symmetry and covariance regularization", *Electronic Journal of Statistics*, Vol. 6 (2012) 1600–1640.

The paper contains a notational error in the statement of Schur's lemma and the paragraph immediately succeeding it. The corrected version is below. We would like to thank Ilya Soloveychik for pointing out the error to us.

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Lemma 2.1 (Schur's lemma [37]). *For a finite group $\mathfrak{G}$ there are only finitely many inequivalent irreducible representations (indexed by $\mathcal{I}$) $\vartheta_1, \ldots, \vartheta_{|\mathcal{I}|}$ of dimensions $m_1, \ldots, m_{|\mathcal{I}|}$. Every linear representation of $\mathfrak{G}$ has a canonical decomposition*

$$\rho = s_1\vartheta_1 \oplus \cdots \oplus s_{|\mathcal{I}|}\vartheta_{|\mathcal{I}|},$$

and s_i is the multiplicity of the i^{th} irreducible representation. Correspondingly, there is an isotypic *decomposition of $\mathbb{C}^N$ into invariant subspaces W_i:*

$$\mathbb{C}^N = W_1 \oplus \cdots \oplus W_{|\mathcal{I}|}$$

where each W_i is again a direct sum of isomorphic copies $W_i = W_{i1} \oplus \cdots \oplus W_{i,s_i}$.[1]

A basis of this decomposition (that depends only on the group) transforming with respect to the matrices $\vartheta(g)$ is called symmetry adapted, and can be explicitly computed algorithmically [20, 37]. This basis defines a change of coordinates by a unitary matrix $T \in \mathbb{C}^{N \times N}$. One of the main consequences of Schur's lemma is that the symmetry adapted basis *block diagonalizes the fixed point subspace*, i.e. every matrix in the fixed point subspace is commonly diagonalized by T. If $M \in \mathcal{W}_{\mathfrak{G}}$ is a matrix in the fixed point subspace, then changing coordinates with respect to T decomposes M into a block diagonal form as follows:

$$T^*MT = \begin{bmatrix} M_1 & & 0 \\ & \ddots & \\ 0 & & M_{|\mathcal{I}|} \end{bmatrix} \qquad M_i = \begin{bmatrix} B_i & & 0 \\ & \ddots & \\ 0 & & B_i \end{bmatrix}. \tag{3}$$

[1] Schur's lemma, as stated classically, provides a decomposition over the complex field $\mathbb{C}^N$. However, a real version can be adapted in a straightforward manner [37, pp. 106–109], [23] and the irreducible real representations are called *absolutely irreducible*.

In the above decomposition the diagonal blocks $M_i \in \mathbb{R}^{m_i s_i \times m_i s_i}$ can be further decomposed into m_i repeated diagonal copies $B_i \in \mathbb{R}^{s_i \times s_i}$ (recall that m_i are the dimensions of the irreducible representations and s_i are the multiplicities).[2] Thus, the symmetry restriction $M \in \mathcal{W}_{\mathfrak{G}}$ reduces the degrees of freedom in the problem of interest. This observation plays a central role in our paper.

[2]Note that in general the diagonal blocks in (3) are complex. In this paper, for the sake of simplicity, we will assume that the M_i are real; indeed this is the case in all the examples that we consider. The complex case can be handled in a straightforward manner by working with concentration bounds for complex Gaussian covariance matrices. Doing so leaves the overall behaviour of the sample complexity unchanged (up to constant factors).