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Research Article

Stability of the *n*-Dimensional Mixed-Type Additive and Quadratic Functional Equation in Non-Archimedean Normed Spaces

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We will prove the stability of the functional equation $2f(\sum_{i=1}^{n} x_i) + \sum_{1\le i,j\le n, i\ne j} f(x_i - x_j) = (n+1)\sum_{i=1}^{n} f(x_i) + (n-1)\sum_{i=1}^{n} f(-x_i)$ in non-Archimedean normed spaces.

1. Introduction

A classical question in the theory of functional equations is "when is it true that a function, which approximately satisfies a functional equation, must be somehow close to an exact solution of the equation?" Such a problem, called *a stability problem* of the functional equation, was formulated by Ulam in 1940 (see [1]). In the following year, Hyers [2] gave a partial solution of Ulam's problem for the case of approximate additive functions. Subsequently, his result was generalized by Aoki [3] for additive functions and by Rassias [4] for linear functions. Indeed, they considered the stability problem for unbounded Cauchy differences. During the last decades, the stability problems of functional equations have been extensively investigated by a number of mathematicians (see [5–23]).

A non-Archimedean field is a field $\mathbb{K}$ equipped with a function (valuation) $|\cdot| : \mathbb{K} \to [0,\infty)$ such that

(F_1) $|r| = 0$ if and only if $r = 0$;

(F_2) $|rs| = |r||s|$;

(F_3) $|r + s| \leq \max\{|r|, |s|\}$ for all $r, s \in \mathbb{K}$.

Clearly, it holds that $|1| = |-1| = 1$ and $|n| \leq 1$ for all $n \in \mathbb{N}$.

Let X be a vector space over a scalar field $\mathbb{K}$ with a non-Archimedean and nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \to \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

(N_1) $\|x\| = 0$ if and only if $x = 0$;

(N_2) $\|rx\| = |r|\|x\|$ for all $r \in \mathbb{K}$ and $x \in X$;

(N_3) $\|x + y\| \leq \max\{\|x\|, \|y\|\}$ for all $x, y \in X$.

Then $(X, \|\cdot\|)$ is called a non-Archimedean space. Due to the fact that

$$\|x_n - x_m\| \leq \max_{m \leq i < n} \|x_{i+1} - x_i\| \quad (n > m), \tag{1.1}$$

a sequence $\{x_n\}$ is Cauchy if and only if $\{x_{n+1} - x_n\}$ converges to zero in a non-Archimedean space. A complete non-Archimedean space is a non-Archimedean space in which every Cauchy sequence is convergent.

Recently, Moslehian and Rassias [24] proved the Hyers-Ulam stability of the Cauchy functional equation

$$f(x + y) = f(x) + f(y), \tag{1.2}$$

and the quadratic functional equation

$$f(x + y) + f(x - y) = 2f(x) + 2f(y) \tag{1.3}$$

in non-Archimedean normed spaces.

We now consider the *n-dimensional mixed-type quadratic and additive functional equation*

$$2f\left(\sum_{i=1}^{n} x_i\right) + \sum_{1 \leq i,j \leq n, i \neq j} f(x_i - x_j) = (n+1)\sum_{i=1}^{n} f(x_i) + (n-1)\sum_{i=1}^{n} f(-x_i), \tag{1.4}$$

whose solution is called *a quadratic-additive function*.

In 2009, Towanlong and Nakmahachalasint [25] obtained a stability result for the functional equation (1.4), in which they constructed a quadratic-additive function F by composing an additive function A and a quadratic function Q, where A and Q approximate the odd part and the even part of the given function f, respectively.

In this paper, we investigate a general stability problem for the n-dimensional mixed-type quadratic and additive functional equation (1.4) in non-Archimedean normed spaces.

2. Solutions of (1.4)

In this section, we prove the generalized Hyers-Ulam stability of the n-dimensional mixed-type quadratic and additive functional equation (1.4). Assume that H is an additive group and X is a complete non-Archimedean space.

For a given function $f : H \to X$, we use the abbreviations

$$\begin{aligned} f_e(x) &:= \frac{f(x)+f(-x)}{2}, \\ f_o(x) &:= \frac{f(x)-f(-x)}{2}, \\ Af(x,y) &:= f(x+y)-f(x)-f(y), \\ Qf(x,y) &:= f(x+y)+f(x-y)-2f(x)-2f(y), \\ D_n f(x_1,x_2,\ldots,x_n) &:= 2f\left(\sum_{i=1}^{n} x_i\right)+\sum_{1\le i,j\le n, i\neq j} f(x_i-x_j) \\ &\quad - (n+1)\sum_{i=1}^{n} f(x_i)-(n-1)\sum_{i=1}^{n} f(-x_i) \end{aligned} \tag{2.1}$$

for all $x, y, x_1, x_2, \ldots, x_n \in H$ and for an arbitrarily fixed $n \in \mathbb{N}$.

Theorem 2.1. *Assume that $n \ge 2$ is an integer. Let H and X be an additive group and a complete non-Archimedean space, respectively. A function $f : H \to X$ is a solution of* (1.4) *if and only if f_e is quadratic, f_o is additive, and $f_e(0) = 0$.*

Proof. If a function $f : H \to X$ is a solution of (1.4), then we have $f_e(0) = 0$,

$$\begin{aligned} Qf_e(x,y) &= f_e(x+y)+f_e(x-y)-2f_e(x)-2f_e(y) \\ &= \frac{1}{2}D_n f_e(x,y,0,\ldots,0)+\frac{1}{2}(n-2)(n+3)f_e(0) \\ &= 0, \\ Af_o(x,y) &= f_o(x+y)-f_o(x)-f_o(y)=\frac{1}{2}D_n f_o(x,y,0,\ldots,0)=0 \end{aligned} \tag{2.2}$$

for all $x, y \in H$, that is, f_e is quadratic and f_o is additive.

Conversely, assume that f_e is quadratic, f_o is additive, and $f_e(0) = 0$. We apply an induction on j to prove $D_n f_e(x_1, x_2, \ldots, x_n) = 0$ for all $x_1, x_2, \ldots, x_n \in H$. For $j = 2$, we have

$$\begin{aligned} &D_n f_e(x_1,x_2,0,\ldots,0) \\ &\quad = 2f_e(x_1+x_2)+2f_e(x_1-x_2)-4f_e(x_1)-4f_e(x_2)-(n-2)(n+3)f_e(0) \\ &\quad = 0. \end{aligned} \tag{2.3}$$

If $n > 2$ and $D_n f_e(x_1, x_2, \dots, x_j, 0, \dots, 0) = 0$ for some integer j $(2 \le j < n)$ and for all $x_1, x_2, \dots, x_j \in H$, then a routine calculation yields

$$\begin{aligned} &D_n f_e(x_1, x_2, \dots, x_{j+1}, 0, \dots, 0) \\ &\quad = Qf_e(x_1 + \cdots + x_j, x_{j+1} - x_j) + \frac{1}{2} D_n f_e(x_1, \dots, x_{j-1}, 2x_j, 0, \dots, 0) \\ &\qquad + \frac{1}{2} D_n f_e(x_1, \dots, x_{j-1}, 2x_{j+1}, 0, \dots, 0) - \sum_{k=1}^{j-1} (Qf_e(x_k, x_j) + Qf_e(x_k, x_{j+1})) \\ &\qquad - \frac{j}{2} Qf_e(x_{j+1}, x_{j+1}) - \frac{j}{2} Qf_e(x_j, x_j) \\ &\quad = 0 \end{aligned} \tag{2.4}$$

for all $x_1, x_2, \dots, x_{j+1} \in H$. Hence, we conclude that

$$D_n f_e(x_1, x_2, \dots, x_n) = 0 \tag{2.5}$$

for all $x_1, x_2, \dots, x_n \in H$.

Since f_o is additive, a long calculation yields

$$\begin{aligned} &D_n f_o(x_1, x_2, \dots, x_n) \\ &\quad = \sum_{1 \le i, j \le n, i \ne j} Af_o(x_i, -x_j) + 2 \sum_{i=1}^{n-1} Af_o\left(\sum_{j=1}^{i} x_j, x_{i+1} \right) \\ &\quad = 0. \end{aligned} \tag{2.6}$$

Hence, it follows from (2.5) and (2.6) that

$$D_n f(x_1, x_2, \dots, x_n) = D_n f_e(x_1, x_2, \dots, x_n) + D_n f_o(x_1, x_2, \dots, x_n) = 0 \tag{2.7}$$

for all $x_1, x_2, \dots, x_n \in H$; that is, f is a solution of (1.4). □

3. Generalized Hyers-Ulam Stability of (1.4)

In the following theorem, we will investigate the stability problem of the functional equation (1.4).

Theorem 3.1. *Assume that $n \ge 2$ is an integer. Let H and X be an additive group and a complete non-Archimedean space, respectively. Assume that $\varphi : H^n \to [0, \infty)$ is a function such that*

$$\lim_{m \to \infty} \frac{\varphi(n^m x_1, n^m x_2, \dots, n^m x_n)}{|n|^{2m}} = 0 \tag{3.1}$$

for all $x_1, x_2, \dots, x_n \in H$*. Moreover, assume that the limit*

$$\tilde{\varphi}(x) := \lim_{m\to\infty} \max_{0\le i<m} \left\{ \frac{\varphi(n^i x, \dots, n^i x)}{|4||n|^{2i+2}}, \frac{\varphi(-n^i x, \dots, -n^i x)}{|4||n|^{2i+2}} \right\} \tag{3.2}$$

exists for each $x \in H$*. If a function* $f : H \to X$ *satisfies the inequality*

$$\left\| D_n f(x_1, x_2, \dots, x_n) \right\| \le \varphi(x_1, x_2, \dots, x_n) \tag{3.3}$$

for any $x_1, x_2, \dots, x_n \in H$*, then there exists a unique quadratic-additive function* $T : H \to X$ *such that*

$$\left\| f(x) - T(x) \right\| \le \tilde{\varphi}(x) \tag{3.4}$$

for each $x \in H$*. In particular,* T *is given by*

$$T(x) = \lim_{m\to\infty} \left(\frac{f(n^m x) + f(-n^m x)}{2n^{2m}} + \frac{f(n^m x) - f(-n^m x)}{2n^m} \right) \tag{3.5}$$

for all $x \in H$*.*

Proof. If we replace x_i in (3.1) with 0 for each $i \in \{1, 2, \dots, n\}$, then we have

$$\lim_{m\to\infty} \frac{\varphi(0, 0, \dots, 0)}{|n|^{2m}} = 0. \tag{3.6}$$

Since $|n| \le 1$, it holds that $\varphi(0, 0, \dots, 0) = 0$ and

$$\left\| (n-1)(n+2) f(0) \right\| = \left\| D_n f(0, 0, \dots, 0) \right\| \le \varphi(0, 0, \dots, 0) = 0. \tag{3.7}$$

Hence, we conclude that $f(0) = 0$.

Let $J_m f : H \to Y$ be a function defined by

$$J_m f(x) = \frac{f(n^m x) + f(-n^m x)}{2n^{2m}} + \frac{f(n^m x) - f(-n^m x)}{2n^m} \tag{3.8}$$

for all $x \in H$ and $m \in \{0,1,2,\ldots\}$. A tedious calculation, together with (F_2), (N_3), and (3.3), yields

$$\begin{aligned}
\left\| J_i f(x) - J_{i+1} f(x) \right\| &= \left\| -\frac{D_n f(n^i x, \ldots, n^i x)}{4n^{2i+2}} - \frac{D_n f(-n^i x, \ldots, -n^i x)}{4n^{2i+2}} \right. \\
&\qquad \left. - \frac{D_n f(n^i x, \ldots, n^i x)}{4n^{i+1}} + \frac{D_n f(-n^i x, \ldots, -n^i x)}{4n^{i+1}} \right\| \\
&\le \max\left\{ \frac{\|D_n f(n^i x, \ldots, n^i x)\|}{|4||n|^{2i+2}}, \frac{\|D_n f(-n^i x, \ldots, -n^i x)\|}{|4||n|^{2i+2}}, \right. \\
&\qquad \left. \frac{\|D_n f(n^i x, \ldots, n^i x)\|}{|4||n|^{i+1}}, \frac{\|D_n f(-n^i x, \ldots, -n^i x)\|}{|4||n|^{i+1}} \right\} \\
&\le \max\left\{ \frac{\varphi(n^i x, \ldots, n^i x)}{|4||n|^{2i+2}}, \frac{\varphi(-n^i x, \ldots, -n^i x)}{|4||n|^{2i+2}} \right\}
\end{aligned} \tag{3.9}$$

for all $x \in H$ and $i \in \{0,1,2,\ldots\}$. It follows from (3.1) and (3.9) that the sequence $\{J_m f(x)\}$ is Cauchy. Since X is complete, we conclude that $\{J_m f(x)\}$ is convergent.

Let us define

$$T(x) := \lim_{m\to\infty} J_m f(x) \tag{3.10}$$

for any $x \in H$. It follows from (N_3) and (3.9) that

$$\begin{aligned}
\| f(x) - J_m f(x) \| &= \left\| \sum_{i=0}^{m-1} \left(J_i f(x) - J_{i+1} f(x) \right) \right\| \\
&\le \max_{0\le i<m} \left\| J_i f(x) - J_{i+1} f(x) \right\| \\
&\le \max_{0\le i<m} \left\{ \frac{\varphi(n^i x, \ldots, n^i x)}{|4||n|^{2i+2}}, \frac{\varphi(-n^i x, \ldots, -n^i x)}{|4||n|^{2i+2}} \right\}
\end{aligned} \tag{3.11}$$

for all $m \in \{0,1,2,\ldots\}$ and $x \in H$. In view of (3.2), if we let $m \to \infty$ in (3.11), then we obtain the inequality (3.4).

Replacing x_i in (3.3) with $n^m x_i$ for $i \in \{1,2,\ldots,n\}$ and considering (F_2) and (N_3), we get

$$\begin{aligned}
\| D_n J_m f(x_1, x_2, \ldots, x_n) \| &= \left\| \frac{D_n f(n^m x_1, \ldots, n^m x_n) - D_n f(-n^m x_1, \ldots, -n^m x_n)}{2n^m} \right. \\
&\qquad \left. + \frac{D_n f(n^m x_1, \ldots, n^m x_n) + D_n f(-n^m x_1, \ldots, -n^m x_n)}{2n^{2m}} \right\|
\end{aligned}$$

$$\leq \max\left\{\frac{\varphi(n^m x_1,\ldots,n^m x_n)}{|2||n|^m}, \frac{\varphi(-n^m x_1,\ldots,-n^m x_n)}{|2||n|^m}, \frac{\varphi(n^m x_1,\ldots,n^m x_n)}{|2||n|^{2m}}, \frac{\varphi(-2^m x_1,\ldots,-2^m x_n)}{|2||n|^{2m}}\right\} \tag{3.12}$$

for all $m \in \{0,1,2,\ldots\}$ and $x_1, x_2, \ldots, x_n \in H$. If we let $m \to \infty$ in the last inequality, then it follows from the condition (3.1) that $D_nT(x_1, x_2, \ldots, x_n) = 0$ for all $x_1, x_2, \ldots, x_n \in H$; that is, T is a quadratic-additive function.

Assume that $T' : H \to X$ is another quadratic-additive function satisfying (3.4). By the definition of D_n, a routine calculation yields

$$\begin{aligned}
&-\frac{D_nT'(n^i x,\ldots,n^i x)}{4n^{2i+2}} - \frac{D_nT'(-n^i x,\ldots,-n^i x)}{4n^{2i+2}} - \frac{D_nT'(n^i x,\ldots,n^i x)}{4n^{i+1}} + \frac{D_nT'(-n^i x,\ldots,-n^i x)}{4n^{i+1}} \\
&\quad = -\frac{1}{2n^{2(i+1)}}\left(T'\left(n^{i+1}x\right) + T'\left(-n^{i+1}x\right)\right) + \frac{1}{2n^{2i}}\left(T'\left(n^i x\right) + T'\left(-n^i x\right)\right) \\
&\qquad - \frac{1}{2n^{i+1}}\left(T'\left(n^{i+1}x\right) - T'\left(-n^{i+1}x\right)\right) + \frac{1}{2n^i}\left(T'\left(n^i x\right) - T'\left(-n^i x\right)\right)
\end{aligned} \tag{3.13}$$

for each $i \in \{0,1,2,\ldots\}$ and $x \in H$. Hence, it follows from (3.8) that

$$\begin{aligned}
&\sum_{i=0}^{k-1}\left(-\frac{D_nT'(n^i x,\ldots,n^i x)}{4n^{2i+2}} - \frac{D_nT'(-n^i x,\ldots,-n^i x)}{4n^{2i+2}}\right. \\
&\qquad \left. -\frac{D_nT'(n^i x,\ldots,n^i x)}{4n^{i+1}} + \frac{D_nT'(-n^i x,\ldots,-n^i x)}{4n^{i+1}}\right) = T'(x) - J_kT'(x)
\end{aligned} \tag{3.14}$$

for any $k \in \mathbb{N}$ and $x \in H$. Since T' is a solution of (1.4), it follows from the last equality that

$$T'(x) = J_kT'(x) \tag{3.15}$$

for any $k \in \mathbb{N}$ and $x \in H$. Obviously, this equality also holds for T.

Consequently, by considering that $|n| \leq 1$, it follows from (N_3), (3.1), (3.4), and (3.8) that

$$\begin{aligned}
&\left\|T(x) - T'(x)\right\| \\
&\quad = \lim_{k\to\infty}\left\|J_kT(x) - J_kT'(x)\right\| \\
&\quad \leq \lim_{k\to\infty}\max\left\{\left\|J_kT(x) - J_kf(x)\right\|, \left\|J_kf(x) - J_kT'(x)\right\|\right\}
\end{aligned}$$

$$\begin{aligned}
&\leq \lim_{k\to\infty} |2|^{-1}|n|^{-2k} \max\Big\{ \left\|T\left(n^k x\right) - f\left(n^k x\right)\right\|, \left\|T\left(-n^k x\right) - f\left(-n^k x\right)\right\|, \\
&\qquad\qquad \left\|f\left(n^k x\right) - T'\left(n^k x\right)\right\|, \left\|f\left(-n^k x\right) - T'\left(-n^k x\right)\right\| \Big\} \\
&\leq \lim_{k\to\infty} \lim_{m\to\infty} \max_{k\leq i<m+k} \left\{ \frac{\varphi(n^i x,\ldots,n^i x)}{|8||n|^{2i+2}}, \frac{\varphi(-n^i x,\ldots,-n^i x)}{|8||n|^{2i+2}} \right\} \\
&= 0
\end{aligned} \tag{3.16}$$

for all $x \in H$. Therefore, $T = T'$, which proves the uniqueness of T. □

Corollary 3.2. *Let X and Y be non-Archimedean normed spaces over $\mathbb{K}$ with $|n| < 1$. If Y is complete and $f : X \to Y$ satisfies the inequality*

$$\left\|Df(x_1, x_2, \ldots, x_n)\right\| \leq \theta \sum_{i=1}^{n} \|x_i\|^r \tag{3.17}$$

for all $x_1, x_2, \ldots, x_n \in X$ and for some $r > 2$, then there exists a unique quadratic-additive function $T : X \to Y$ such that

$$\left\|f(x) - T(x)\right\| \leq \frac{n\theta}{|4||n|^2} \|x\|^r \tag{3.18}$$

for all $x \in X$.

Proof. Let $\varphi(x_1, x_2, \ldots, x_n) = \theta \sum_{i=1}^{n} \|x_i\|^r$. Since $|n| < 1$ and $r - 2 > 0$, we get

$$\lim_{m\to\infty} |n|^{-2m} \varphi(n^m x_1, n^m x_2, \ldots, n^m x_n) = \lim_{m\to\infty} |n|^{m(r-2)} \varphi(x_1, x_2, \ldots, x_n) = 0 \tag{3.19}$$

for all $x_1, x_2, \ldots, x_n \in X$. Therefore, the conditions of Theorem 3.1 are satisfied. Indeed, it is easy to see that $\tilde{\varphi}(x) = n\theta(|4|^{-1}|n|^{-2})\|x\|^r$. By Theorem 3.1, there exists a unique quadratic-additive function $T : X \to Y$ such that (3.18) holds. □

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