The Review of Modern Logic
Volume 10 Numbers 3 & 4 (March 2005–May 2007) [Issue 32], pp. 183–193.

Vincent F. Hendricks and Jacek Malinowski, eds.
Trends in Logic: 50 Years of Studia Logica
Dordrecht, Boston and London: Kluwer Academic Publishers, 2003
viii + 384 pp. ISBN 1402016018

REVIEW

DAVID DEVIDI

If, like me, you reckon that *if you are doing your philosophy right, sooner or later you end up doing algebra*, you'll really enjoy this volume. It was published to commemorate the 50th Anniversary of the Polish logic journal *Studia Logica*, which began publication in 1953. The volume begins with two papers of a historical or retrospective sort. The first is a review by Ryszard Wójcicki and Jan Zygmunt of the evolution of Polish logic in the past five decades and a description of the fascinating history of the journal. The second is a valiant attempt by Johan van Bentham to describe a broader context in which to consider the contributions later in the volume by delineating the most significant changes and the most significant constants of the past 50 years of logical investigations, all in a mere 20 pages. The rest of the volume consists of nine review articles. Each of these is devoted to an important area of logical investigation that has seen important developments in the pages of *Studia Logica*. They therefore focus on what one might call logic in the heavily algebraic Polish Style.

The title, *Trends in Logic*, is suitably ambiguous in suggesting, on the one hand, a historical review and, on the other, informed speculation about what to expect in coming years, since in their selection of topics the editors picked topics whose evolution can be followed in the back issues of the journal, but also ones which one might call, risking the pun, trendy. Assuming that the editors have chose wisely, then, there are presumably some lessons about where logic is now and where it is heading to be learned by considering the patterns that can be seen across the various review articles, in addition to those to be learned about specific areas of research within each of them. So in addition to describing some of what is in the various articles, I will offer some thoughts about what those lessons might be.

This volume begins with Rijzard Wójcicki and Jan Zygmunt's chapter "Polish Logic in Postwar Poland," which includes a fascinating sketch of the prehistory and history of *Studia Logica*. The story begins, unsurprisingly, with the pre-War "Polish School of Logic" and its overlap with the "Lvov-Warsaw School of Philosophy," thus with the achievements of Tarski, Łukasiewecz, Leśnieski, Jaśkowski and others in the period between the two World Wars. This hey day of Polish Logic saw many prominent researchers housed in Philosophy Departments (*e.g.*, Ajdukiewicz, Kotarbiński and Łukasiewicz) and many others in Mathematics Departments (*e.g.*, Leśniewski and Tarski), with productive interchange between them.[1]

World War II, of course, resulted in the death of many Polish logicians, several of whom were murdered by the Nazis, and the escape of many others abroad. It also smashed the research infrastructure built up in the period between the wars. In the immediate Post-War period, the surviving logicians still living in Poland worked to re-establish the research capacity of the Polish logic community in both philosophy and mathematics departments. However, research in formal logic proved to be insufficiently dialectical for those charged with ensuring the ideological purity of Polish sciences. In the wake of the First Congress of Polish Sciences in 1953, which involved a public attack on the leaders of the Lvov-Warsaw School, measures were taken to ensure that such thinkers as Kazimierz Ajdukiewicz, Tadeusz Kotarbiński, Maria Ossawska and Stanislaw Ossowski no longer be able to teach philosophy, humanities or social science; however, rather than being purged from the academy, these scholars were shuffled to less ideologically sensitive posts.

Most significantly for our story, Ajdukiewicz was able to found the new Section of Logic of the Polish Academy of Sciences. It was in this ideologically charged atmosphere that, from his position in the Academy of Sciences, he launched *Studia Logica*, with the goal of building bridges between philosophical and mathematical logic. From its very early days it attracted contributions from both mathematicians and philosophers, and from foreign contributors.

In 1976, the Editorial Board managed to gain approval for an International Editorial Board—in spite of requirements that all contacts with foreigners be cleared by the Security Police—and established English as the sole language of publication. This was, no doubt, a pivotal

[1] A longer, highly readable discussion of this period in Polish logic is contained in [1].

year in the process by which *Studia Logica* established itself as one of the world's preeminent logic journals.

Wójcicki and Zygmunt's chapter includes many fascinating historical nuggets. It was pressure to suppress investigations of formal logic in favour of more suitable studies of the "dialectical logic" supposedly lying, waiting for discovery, in the writings of Hegel and the classic texts of Marxism that led researchers in mathematics departments to opt for the name "foundations of mathematics" to avoid unwanted attention from the ideological police; the Marxist philosopher Leszek Kołakowski was an original member of the *Studia Logica* editorial board not because he had a great interest in formal logic, but to meet a requirement that at least one person on the Board be a member of the Party (p. 22). While he never used the word "dialectical" in his papers, Jaśkowski's discussive logic was an attempt to describe a logical system in which contradictions were present for discussion, and so one of several attempts, unfortunately vane at the time, by logicians to show that the methods of formal logic could be used to clarify and analyze the often obscure pronouncements of those working on dialectics (p. 21). All this makes especially impressive the vision and courage the founders of the journal showed by creating it in the politically charged times in which they did so.

Turning now to the review articles, as one might expect in a volume dedicated to one of the world's top logic journals, the editors have been able to recruit an all-star cast of authors. They also seem to have sensibly adopted a policy of giving their impressive cast the freedom to write the articles as they saw fit. The result is that the articles vary considerably in approach from fairly comprehensive introductions to contributions that are much more focused; some authors presuppose more familiarity with related fields or more mathematical sophistication, others less; and the notation varies widely, according to the various authors' tastes. This means that not all the articles will serve the same purpose but, perhaps counter-intuitively, this increases the usefulness of the volume.

Consider Melvin Fitting's paper, "Intensional Logic — Beyond First Order." Fitting essentially welds together two utterly familiar features of the logical landscape: (1) that the early 20th Century paradoxes in the foundations of mathematics could be circumvented either by moving to a set theorerical foundation, or by adopting a higher order system involving types; moreover, these can be regarded as "extensions" of first-order logic of quite different types, but each provides a foundation for mathematics; (2) first order modal logic is a frequently studied and philosophically important field of study. One might conjecture that there should be two useful and distinct ways of extending modal logic that parallel the ways of extending non-modal logic, and Fitting's paper is devoted to showing that this is indeed true. The paper is basically self-contained, and is extremely easy reading, by logic-paper standards. This is because Fitting includes all and only what is required to make his point, his point can be made with additions to the very familiar that he explains clearly enough to make seem obvious, and he never feels the need to pause and show off his technical virtuousity. By contrast, Vincenzo Mara and Daniele Mundici's paper "Łukasiewicz Logic and Chang's MV Algebras in Action" is technically quite demanding. The paper is an introduction to the research program they describe as "the generalization of sets, partitions, and functions to a many valued setting"; among other things, this involves them in the conceptually prior introduction of topics such as non-Boolean partitions and finite and infinite multi-sets so they can give a proof of an MV-algebraic Stone representation theorem for infinite multi-sets.

Clearly, Fitting's paper is not intended to be nor is it suited to provide the relevant background in the same was as is Mara and Mundici's. But an article that did so would be redundant in a way that Mara and Mundici's is not: there are dozens of sources that will fill in the background to Fitting's argument for readers not already familiar with it, and including it here would have both bored a large percentage of the likely readership and clouded his message. Mara and Mundici, on the other hand, are on less familiar terrain, so a map is in order; many readers will have only a passing familiarity with Łukasiewicz logic and may never have heard of MV algebras. There is much more of a technical nature to be learned in their paper, so one would not expect an argument of the sort that gives Fitting's paper its structure.

Each of the technical chapters will teach something useful and interesting to all readers except those already most expert in the particular field the chapter covers. As one would expect from the talented authors the editors succeeded in recruiting to the project, all of the chapters display impressive mastery of the material covered, and the authors of the technical chapters have devoted considerable effort to organization and pedagogy, evidently having grasped the differences between an expository article and a research paper and between the likely readers of each. Being a non-expert in all the fields covered, I learned interesting things from every article. But this same lack of deep expertise in any of various fields means I feel unqualified to rank the articles as better or worse, since I suspect such estimates are likely to be too much influenced by how close the topics are to things I have personally investigated in the past. I will simply note that all struck me as at least very good at the task the authors set themselves.

I hinted above that the articles are mostly examples of logic done in the Polish Style. Of course, Poles have made important contributions in most every field that could be regarded at all plausibly as *logic*, so talk of a Polish style will involve all the inaccuracies and oversimplifications of any such description. My impression is that one is on fairly solid ground, though, in suggesting that when people speak of logic in the Polish style, they have in mind an approach in which algebra and algebraic methods play a particularly central role. This impression might be mere autobiography—when it became clear that I hoped to do a thesis in logic, my PhD supervisor pressed a copy of Rasiowa and Sikorski's [4] into my hand and told me to start reading. But the impression is confirmed by two lists in the volume. The editorial introduction lists the topics that "have become the quint-essential trademarks of the journal" (p. 4). At the top of their list they put algebraic logics and consequence operations, which they characterize as "investigating, by means of algebraic methods, the consequence operations and deductive systems defined by means of classes of abstract algebras or more generally by classes of logical matrices" (p. 4). Another item on the list: *abstract* algebraic logic. But even some of the other topics on the list which needn't be thought of as particularly involving algebraic methods, such as modal logic, non-classical and many valued logics, or paraconsistency get a particularly algebraic treatment in *Studia Logica*. Wójcicki and Zygmunt's list of main areas of accomplishment by Polish logicians over 50 years, of course, overlaps the editors' list considerably, once again includes accomplishments in albegraic logic, the theory of logical matrices, and consequence operations. Again like the editors, when they list accomplishments in other areas of logic, they tend to mention algebraic approaches; for instance, in their section on Polish accomplishments in Model Theory they mention Rasiowa and Sikorski's book as "another important milestone, since it enables logicians to extend the field of model theory to non-classical logics."(p. 27)

By now, of course, the Polish style is hardly the special preserve of Poles, even if we include the Polish diaspora. Outstanding practitioners of this style of logical investigation are to be found around the world—for instance, the authors in the volume are an impressively international cast. Why this is so is probably an interesting and complicated story. An important part of the explanation is of course the influence of Tarski, including the influence transmitted by the absolutely amazing number of influential and productive PhD students he supervised, and others (*e.g.*, Dana Scott) with whom he worked as graduate students though in the end he did not supervise their dissertations (for a description of Tarski's remarkable influence, see [1]). There no doubt are also other important personal lines of influence. But there also is the manifest productivity of the approach, and the way the approach fit nicely with other developments in logic and foundations of mathematics at around the same time. For instance, the approach to Boolean and Heyting algebra valued semantics described in Rasiowa and Sikorski's important book arrived on the scene in the early 1960s, just in time to link up with the Scott-Solovay recasting of Cohen's methods of forcing in set theory in terms of Boolean Valued Models of Set Theory (see [3], which includes an interesting foreword by Scott). Nowadays, too, the Polish style more often is seen as a hybrid with other approaches, such as category theory, on which other nations such as the US and France, probably have a stronger claim, rather than in its purer form.

One lesson is obvious from the contents of the volume, if the view from *Studia Logica* is at all an accurate picture of the state of contemporary logic: non-classical logics of various sorts are where the action is. This is striking to me because, remarkably often, I hear reports from people I meet at conferences or from former students, of philosophers from big name departments, especially in the US, saying things like "Friends don't let friends be logical pluralists," or remarking that they have no patience for solutions to philosophical problems that involve "changing logic." Obviously one doesn't want to put too much weight on a perceived mismatch between the trends in logic as read off one collection and some philosophical hearsay, but I suspect that there may be a growing failure to communicate between logicians and some of the current heavyweights in Philosophy of Language, Metaphysics and Epistemology, and Philosophy of Mind. For instance, I sat through a keynote address at the Western Canadian Philosophical Association meetings some years back purporting to discuss where philosophy was headed in the then new century; in his address, a rather famous philosopher opined that logic was a spent force, and claimed that no philosophically significant result had been proved in logic in at least twenty years. A book review is not the place for a call for curricular reform in philosophy departments, of course, but it is very tempting to make one in hope of curing the logical ignorance, if not of the current generation, then perhaps of the next.

More specifically than merely suggesting that non-classical logics are where the action is, a large number of the chapters deal with systems in which, one way or another, the law of non-contradiction "fails to be valid." The scare quotes are in order here because not all of these systems are ones that will be of use to dialetheists, who hold that non-contradiction fails in the very strong sense that one and the same statement can be both true and false. Some of the authors would probably disavow any intention to give comfort to the dialetheists. But the prevalence of systems in which, to try to turn a suitably non-dialetheic phrase, contradiction is not catastrophy, is a trend that the broader philosophical community is not as aware of as it should be, I think. As one would expect, given *Studia Logica*'s goal of facilitating communication between philosophical and mathematical logicians, each of the fields to which a chapter is devoted originates with philosophical motivations, however mathematical the techniques involved in the investigation. The fact that so many philosophically motivated logicians are pursuing systems that dislodge non-contradiction from its status as the least open to question of any principle probably means something important, and it would be a worthwhile philosophical enterprise to try to figure out what it is.

The amount of space devoted to making clear the philosophical motivations and usefulness of the technical material varies considerably from chapter to chapter. For instance, Graham Priest devotes more than half his article to this purpose, describing the history and philosophical merits of his material, and defending his claims on its behalf against previously published criticisms. Perhaps this is no surprise, as his chapter is titled "Inconsistent Arithmetics." Few claims are more likely to draw an incredulous stare than the suggestion that arithmetic might be inconsistent. As George Boolos once said about an even stronger theory than arithmetic,

> [U]nlike ZF, analysis did not arise as a direct response to the set-theoretic antinomies, and the discovery of the inconsistency of analysis would be the most surprising mathematical result ever obtained, precipitating a crisis in the foundations of mathematics compared with which previous "crises" would seem utterly insignificant. [2, pp. 219–20]

Of course, Priest does not prove the inconsistency of arithmetic; instead, he describes a recipe for producing inconsistent *models* of arithmetic. The basic idea is that if one allows the underlying logic to be one on which particular sentences can be both true and false, then one can prove a "collapsing lemma". This lemma generates, for any model and any equivalence relation its domain that is a congruence for the basic functions of the model, a new model whose domain is the equivalence classes with respect to that relation. The collapsed model "in effect, identifies all members of an equivalence class to produce a composite individual which has properties of all its members. This may be inconsistent, even if its members are not" (p. 277). Since there are non-trivial equivalence relations on the natural numbers that are congruences with respect to addition, multipication and successor, it is easy enough to construct "collapsed models" of the standard model of the natural numbers. These are models in the sense that all sentences true in the standard model remain true in the collapsed model. On the other hand, assuming that the standard model is consistent, the collapsed model will declare some of statements that are true (but not false) in the standard model, for instance statements $m \neq n$ for those distinct m and n which are equivalent according to the relation, to be both true and false in the collapsed model.

Hioakira Ono's chapter, "Substructural Logics and Residuated Lattices," is a highly efficient and very readable introduction to a framework that allows us to see the commonalities among, and so to gain a clear picture of the differences between, many systems of logic. While there are many contenders for the title, the residuated lattice semantics Ono describes perhaps deserves to be regarded as the successor to the algebraic approach to logics presented in Rasiowa and Sikorski's 1963 book for the classical and intuitionistic cases, then generalized to a variety of other logics in Rasiowa's 1974 book. Restricting attention to propositional logic for simplicity, classical semantics involves two parts: truth value assignments, *i.e.*, functions from the set of atomic statements to the set $\{T, F\}$, and the truth tables, which supply a recipe for extending an assignment to all statements in the language. Algebraic semantics starts with the realization that classical semantics is precisely equivalent to ordering the set of truth values with $F < T$, and interpreting $\wedge$ as meet, $\vee$ as join, and $\neg$ as complement in the resulting lattice, which is a Boolean algebra. The semantics gets off the ground with the question "What's so special about the two-element Boolean algebra?" An algebraic valuation, then, is defined as a map from the set of atoms to the domain of some algebra (and not just to the two element Boolean algebra). Since the standard proof theoretic introduction and elimination rules for $\wedge$ and $\vee$ correspond exactly to meets and joins, the algebras in question are often lattices. What makes the approach productive is that restricting attention to maps into particular classes of algebras results in semantics suitable for different systems of logic;

for instance, assignments in Boolean algebras give rise to Boolean valued semantics for classical logic, while assignments in Heyting algebras yield a semantics for intuitionistic logic.

It is natural to suppose that any conditional operator in a logic with "and" in it should be related to it so as to make the deduction theorem and modus ponens both turn out valid, which in algebraic terms amounts to requiring that for all members of the algebra a, b, and c,

$$a \wedge b \leq c \text{ iff } a \leq b \rightarrow c,$$

as is the case in both Boolean and Heyting algebras. Unfortunately, this saddles the $\rightarrow$ operation with unfortunate features that are anathema to relevance logicians: $a \leq b \rightarrow a$, for instance. The way around this, while preserving the advantages of the algebraic method, is to introduce a binary operation, $*$, that corresponds to a possibly distinct sort of "and", and allowing the conditional $\rightarrow$ to be related to *that* notion just as the intuitionistic and classical conditionals are related to $\wedge$:

$$\text{(res)} \qquad a * b \leq c \text{ iff } a \leq b \rightarrow c$$

Assuming for the present that $*$ is symmetric, we say that $\rightarrow$ *residuates* $*$ when this condition is satisfied for all a, b, and c. A *residuated lattice* is a lattice that includes also a binary operation $*$ (which must also be monotone increasing, for technical reasons that needn't detain us and) that is residuated by some operator $\rightarrow$. Boolean and Heyting algebras are obviously residuated lattices, since we may set $* = \wedge$. But it turns out that many other philosophically interesting logics can be modeled using residuated lattices in this way, including relevance logics (including the First Degree Entailment system underlying Priest's inconsistent models of arithmetic), linear logics, the Łukasiewicz logics discussed in much detail by Marra and Mundici, and others. Since it is well known that Relevance logicians are hostile to the principle of explosion and non-contradiction is not valid in Łukasiewicz logic, it is no surprise that, in many applications, in these algebras negation operators are taken to be neither the complements of classical logic nor the psuedo-complements of intuitionistic logic, so that in general $a * \neg a \not\leq b$ and $a \wedge \neg a \not\leq b$ are possible (and so $a * \neg a$ and $a \wedge \neg a$ need not be 0).

Thus, residuated lattices can be seen to be a very general framework for interpreting logical systems. But it is more general still: in the case where $*$ need not be symmetric, one can have two conditional operators, $\rightarrow$ and $\leftarrow$, such that

$$b \leq c \leftarrow a \text{ iff } a * b \leq c \text{ iff } a \leq b \rightarrow c.$$

Such algebras can be used to model Categorical Grammars, the subject of Wojciech Buskowski's chapter "Type Logics in Grammar."

Two other chapters also deal with logics where non-contradiction need not be valid. "Inconsistency Tolerant Description Logics" by Serguei P. Odintsov and Heinrich Wansing, describes some *constructive* but *paraconsistent* (*i.e.*, non-explosive) logics that they suggest have advantages for *knowledge representation*. That is, they suggest these logics are well suited for attempts to represent an agent's state of information about some real state of the world, but as not suitable for representing states of the world themselves—precisely because of their tolerance of inconsistency. Here, clearly, are authors not willing to toe a dialetheist line. The material requires considerable subtlety to keep straight, but once again the authors do an admirable job of sticking to the basics of the program they describe and making the material clear and digestible—though, for me, the notation took some getting used to.

For many years now, Maria Luisa Dalla Chiara has been one of the central figures in the investigation of quantum logics. Her article in this volume, co-authored with Roberto Giuntini and Roberto Leporini, is a sort of departure from earlier work since it describes a quite different animal from orthodox quantum logics, "Quantum Computational Logics." In it the atoms are interpreted not as closed sub-spaces of a Hilbert space, but as "Qumixes", which are quantities of information of a sort described in quantum information theory, while the logical operators are interpreted as "gates." The resulting logic is strikingly different from orthodox quantum logic: non-contradiction fails to be valid; it includes peculiar operators such as "square root of negation," an operation such that

$$\sqrt{\neg}\sqrt{\neg}A \equiv \neg A;$$

$\wedge$ fails to be idempotent, *i.e.*, $A \not\models A \wedge A$; and, contrary to expectations,

$$(A \wedge B) \vee (A \wedge C) \models A \wedge (B \vee C)$$

but

$$A \wedge (B \vee C) \not\models (A \wedge B) \vee (A \wedge C).$$

Clearly we are very far from a lattice theoretic understanding of $\wedge$ and $\vee$ here. But the authors make a case that these operators nevertheless have some important connection to *and* and *or*, by showing that they correspond to *reversible* versions of the classical conjunction and disjunction operations (because they keep track, for instance, of *which* conjunct(s) falsify $A \wedge B$ in case that conjunction is false).

Of course, not every article involves some sort of paraconsistency. I have not mentioned the articles by Josep Maria Font, "Generalized Matrices in Abstract Algebraic Logic," and Robert Goldblatt, "Questions of Canonicity," simply for this reason. Given the identity of the authors, it will surprise nobody when I say that the articles are excellent; my failure to discuss them in any more detail is due only to my attempt to describe an overarching theme that unites the other articles. In conclusion, this is a volume that it is well worth having access to. Encourage your librarian to buy it if it's not already in your collection!

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University of Waterloo, Department of Philosophy, Waterloo, Ontario N3G 2L1, Canada

E-mail address: ddevidi@uwaterloo.ca