Forty Years of "Unnatural" Natural Deduction and Quantification: A History of First-order Systems of Natural Deduction, From Gentzen to Copi

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Abstract. Systems of natural deduction as a method of theorem proving were first developed in detail by Gentzen and Jáskowski. The complex nature of the rules for quantification in these early systems of natural deduction, and in particular of the parallel rules of Universal Generalization (UG) and Existential Instantiation (EI) led to an active research program in the period from the late 1950's to the late 1960's in an effort by Quine, Suppes, Leblanc, and Copi to develop simplified and correct rules of inferences for the quantified formulae of first-order functional logic. We explore the history of these efforts and examine the difficulties found in the "unnatural" natural deductive systems that were developed for first-order logic. Our survey covers roughly forty years, from 1929 to 1971, which includes the early work of Hertz on which Gentzen's work was based through the development of the quantifation rules in Copi's system of natural deduction as presented in the third edition of his textbook *Symbolic logic*. Also considered is the extent to which O.H. Mitchell made a start at developing a system of natural deduction in his 1883 paper *On a new algebra*.

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§0. Introduction. One of the principal aims of Gentzen's famous [1934] paper *Untersuchungen über das logische Schließen* was to develop a "natural calculus" in which it would be possible to bring purely logical proofs into normal form in which everything required for the proof would appear in one way or another in the conclusion. Indeed, the development of quantification theory as a family of formal first-order systems was undertaken, I have argued [Anellis 1991], from questions raised by the Löwenheim-Skolem theorem about the nature of *being a proof.* One aspect of this work was to develop proof procedures which are more "natural" than the axiomatic methods to be found in Hilbert's, Frege's and Whitehead and Russell's axiomatic systems.

In *The development of logic* [1962, p. 539], William and Martha Kneale state that "Gentzen has in fact presented logic in a fashion more natural than that of Frege, Whitehead, and Russell." Others, such as Dziob [1972], Fang [1984], and even Szabo [1969a, pp. 4-5], have argued that Gentzen's method of natural deduction, as well as corrective attempts, such as Copi's, to render Gentzen's system more natural, are not entirely natural or elegant and simple. Fang [1979, p. 210] and (especially) [1984, p. 14], for example complains that Gentzen's system of natural deduction is unnatural because it is too complicated and contains too many inference rules. [Fang 1984, p. 14] also claims that Gentzen himself was "perfectly aware of the 'unnatural' aspect in some artificial rules of inference," and he points in particular to Gentzen's [1934, p. 186] rules (3a) and (3b), the laws of addition: $P \rightarrow (P \vee Q)$ and $Q \rightarrow (P \vee Q)$, which he goes so far as to call "artificial or even nonsensical" [Fang 1979, p. 210]. Nevertheless, as [Szabo 1969a, p. 4] reminds us, the development of **NK** was intended specifically as "Gentzen's attempt to find a more 'natural' approach to formal reasoning."

Despite the comparatively large number of inference rules required for Gentzen's system **NK** of natural deduction and for the more recent successors to **NK**, such as Copi's system of natural deduction, there is the more serious problem of requiring restrictions on quantifier rules in **NK** and its successors in order to avoid some rather embarrassing consequences. In this study, our primary concern will be precisely the elucidation of difficulties with the rules for quantification in systems of natural deduction and the history of the various attempts to deal with these difficulties.

§1. The early history of natural deduction, from Mitchell to Gentzen. In the manuscript *On the Algebra of Logic: Part II* which was unpublished during his lifetime, Charles Peirce noted [1884] that Oscar H. Mitchell (1851-1889) [1883] had developed a system of logic which differed from his own work insofar as the only logical connectives which it used were negation and disjunction (logical summation) or conjunction (logical product), and that its only inference rules were elimination (simplification) and amplification. Indeed, in Mitchell's logic, as described by [Peirce 1884], the methods of adding and dropping parts of assertions (as opposed to having implication as the foundation of logical passage) is reminiscent of Gentzen's [1934] *Untersuchungen über das logische Schließen*, rules for natural sequences, in particular

$$\frac{G,D}{G \wedge D} \quad \frac{G \wedge D}{G} \quad \frac{G \wedge D}{D} \quad \frac{G}{G \vee D} \quad \frac{D}{G \vee D} \quad \frac{G \vee D,[G]F,[D]F}{F}$$

This invites a question of a possible historical connection between Mitchell's work and Gentzen's.

One might conjecture that Gentzen knew of Mitchell's work. Van Heijenoort [1976, p. 29], however, follows Bernays [1965, pp. 3, 5] in tracing the origin of Gentzen's natural sequents calculus to the "Satzsystem" of the Göttingen logician Paul Hertz (1881-1940) in his series of papers *Über Axiomensysteme für beliebige Satzsysteme*, *Über Axiomensysteme beliebiger Satzsysteme*, and *Über Axiomensysteme von Satzsysteme* [1922-1923; 1929; 1929a; 1929b]. Szabo [1969a, p. 2] does not even raise the question of whether Gentzen knew of the work of Peirce or Mitchell, but traces the origin of his work on natural deduction to Hertz. There are in fact no indications in either Gentzen's famous [1934] paper that he read either Peirce or Mitchell, or in his [1932-33] paper *Über die Existenz unabhängiger Axiomensysteme zu unendlichen Satzsystemen*, which served as the bridge between Hertz's work and in own [1934]. The absence of references to Peirce or to Mitchell does not, of itself, of course, show that Gentzen was unaware of their work. Similarly, the presence of two references to Schröder's *Vorlesungen über die Algebra der Logik* by Hertz in [192bb, p. 465] does not by itself constitute evidence that Hertz was familiar with the work of Mitchell. Still less do Hertz's references to Schröder prove that he knew of Peirce's unpublished work. Moreover, a close examination of Mitchell's published paper shows that Mitchell's system cannot be understood as a precursor of the method of natural deduction. The elimination and addition rules which Mitchell presented were known to Boole. The familiar rules, suggested by Mitchell did not provide an early version of natural deduction or stimulate the development of systems of natural deduction. It is clear, in particular, that Mitchell's system is not a deductive zero order system; and as is well known, a system is a natural deductive system if it is a deductive zero order system, that is a system all of whose rules are substitution-instance rules, none of which specify axioms. Instead, what is significant about Mitchell's suggestion is that it could have become an early version of natural deduction or towards stimulating the development of systems of natural deduction if it had been followed up. There is no evidence that Gentzen knew of or was influenced by the work of Peirce or of Mitchell.

On 2 February 1932, Gentzen submitted his first paper for publication to *Mathematische Annalen* (see [Szabo 1969a, p. 1]). This was his [1932-33] paper *Über die Existenz unabhängiger Axiomensysteme zu unendlichen Satzsystemen*, which began with the presentation of the theory of sentence systems which Hertz had developed. In the paper, Gentzen constructed a counterexample of a sentence system that did not have an independent axiom system (although so-called *linear* sentence systems do have independent axiomatizations). Gentzen's proof of this result were carried out with Gentzen's simplification of Hertz's inference rules. More importantly, a generalization of Hertz's *syllogism* allowed Gentzen to transform Hertz's syllogisms into the famous *Schnitt* (cut). Hertz's *sentences* are thereby transformed into Gentzen's *sequents* (*sequentzen*) and the components of the sequences are Hertz's *antecedents* and *succedents.* As Szabo [1969a, p. 2] has noted, 'the notion of 'logical consequence' as used by Gentzen is...largely inspired by Hertz." We carefully distinguish between *sequents* and *sequences* by noting that for Gentzen, a *sequent* is an array of two finite sequences of formulae related by '→'.

While Gentzen was developing **NK**, Stanislaw Jaśkowski (1906-1965) in Warsaw was developing his own system of natural deduction. In common with Gentzen's system of natural deduction and with all subsequent systems of natural deduction, such as that presented by Fitch [1952], Quine, and others, Jaśkowski's system was a deductive zero order system, based upon rules of substitution and without any rules specifying axioms. Proofs are begun with assumptions and the consequences of these assumptions are obtained by discharging the assumptions by conditionalization.

Jaśkowski [1934, p. 1] noted that an alternative to the axiomatic approach to logic was suggested by his teacher Jan Łukasiewicz (1878-1956), as early as 1926, during a seminar which Łukasiewicz conducted at the University of Warsaw. As reported by [Wolenski 1989, p. 110], the specific question posed follows from the fact that mathematical proofs do not refer to logical theses, but to assumptions and inference rules, or the rules of reasoning. (As stated by [Wolenski 1989, p. 111], in Łukasiewicz's sense, *theses* are theorems; in Jaśkowski's wider sense, *theses* can be any formulae of a proof, i.e. may be suppositions as well as their consequences. Jaśkowski [1934, p. 8] explicitly states that he adopts the use of *thesis* and *system* in Leśniewski's [1929] and [1930] sense.) The questions are (a) whether proofs can be contained in a system of structural rules and (b) if their relation to theorems of an axiomatic sentential calculus can be studied. The aim was to develop a system in which inferences were drawn from assumptions only according to the principles of informal reasoning. This was in effect a call to develop a method of natural deduction. During the course of the seminar, Jaśkowski first worked out some of the ideas for a system of natural deduction. These were presented in the following year at the First Polish Mathematical Congress in Lwów in 1927, and published in [1929] the Congress Proceedings. In fact, however, the first proposal to found proofs on the rules of intuitive (or informal) logic was published by Łukasiewicz in [1925] in a paper *On a certain way of conceiving the theory of deduction,* along with discussions by Kuratowski, Leśniewski, and Tarski. Jaśkowski developed the system of natural deduction in [1934], at the same time that Gentzen presented his [1934] system. Jaśkowski began by providing examples explaining the intuitive sense of the method based upon assumptions, using the Polish notation developed by Łukasiewicz ([1929]; see also [Łukasiewicz & Tarski 1930]). If the symbol 'S' represents the assumption operator 'it is supposed that...', then the proof of the formula *CpCCpqq* can be encoded (see [Jaśkowski 1934, pp. 6-7]; see also [Wolenski 1989, p. 111]) by the sequence

1. *Sp,*
1.1. *SCpq,*
1.1. *q,*
1. *CCpqq,*
CpCCpqq

where we have informal proof of *CpCCpqq*

1. *Assume that p.*
2. *Assume that Cpq.*
3. *From 1 and 2 it follows that q.*
4. *Since q is a consequence of the supposition Cpq, we obtain the implication Cpqq.*
5. *Given the supposition p, we obtain the formula CpCCpqq.*

In the encoded proof devised by Jaśkowski, each supposition has a numerical prefix. Prefixes of principal suppositions are indicated by one number and a dot; subsidiary or additional suppositions have prefixes consisting of several numbers and dots. If a prefix has an initial number (or number segment) which is identical with the initial number (or number segment) of a preeceding supposition, then the new supposition is in the "range" of that preceeding supposition. In the example given, therefore, the supposition 1.1. *SCpq* is in the "range" of the supposition 1. *Sp*. Formulae which are prefixed but which do not contain the S-operator, are direct consequences of the supposition having the same prefix. Thus, in the example given, 1.1. *q* is a direct consequence of 1.1. *SCpq*.

In Jaśkowski's system, if a thesis *T* of the system has a number *n*, then all of the theses with the number *n* in their initial segment of their prefix, together with *T*, belong to the "domain" of *T*. In the example given, 1. *Sp*, 1.1. *SCpq*, and 1.1. *q* are all in the "domain" of *T*. An *absolute domain* of Jaśkowski's system is the set of all theses of the system which have been presented. The absolute domain of a system prior to the presentation of the first thesis is the empty set. An absolute domain increases as the system is expanded. In this respect, Jaśkowski borrows Leśniewski's [1929] and [1931] conception of a formal system, rather than follow the prevalent conception. The four expansion rules of Jaśkowski's system (see [Jaśkowski 1934, pp. 10-11]; see also [Wolenski 1989, p. 112]) are:

(I) *To any domain D may be added a formula consisting of (a) a prefix which differs from the initial segment of the prefix of any element of D, (b) a dot, (c) the symbol S, (d) a sentence.*

(II) *If in a domain D of a supposition x a sentnece y is true, then the sentence Cxy may be added to the domain of which D is a direct subdomain. For t wo domains D and* D_1*, where D the domain of the x and* D_1 *is the absolute domain or the domain of a supposition* x_1 *whose prefix is identical with the initial segment of the prefix of the supposition x, then D is a subdomain of* D_1 *and D is a direct subdomain of* D_1 *if and only if D is not a sub-domain of any subdomain of D.*

(III) *If in a given domain D the sentences Cxy and x are true, then the sentence y may be added to D.*

(IV) *In a domain D of the suppoisition Nx, if the sentences y and Ny are true, then the sentence x may be added to the domain of which D is a direct subdomain.*

We should note that rule (III) is clearly the rule of detachment applied to suppositional proofs.

With these four expansion rules, Jáskowski was able to construct his system. He gives fifty-nine theses, obtained intuitively [1934, pp. 12-13]. The first twenty theses (along with the numbers of the rules and theses used in their proofs), as given by [Wolenski 1989, p. 112-113], are:

T1	1.	*Sp*	I
T2	1.1	*SCpq*	I
T3	1.1.	*q*	III, T2, T1
T4	1.	*CCpqq*	II, T2, T3
T5		*CpCCpqq*	II, T1, T4
T6	2.	*SCNpNq*	I
T7	2.1.	*Sq*	I
T8	2.1.1.	*SNp*	I
T9	2.1.1.	*Nq*	III, T6, T8
T10	2.1.	*p*	IV, T8, T7, T9
T11	2.	*Cpq*	II, T7, T10
T12		*CCNpNqCqp*	II, T6, T11
T13	1.2.	*Sq*	I
T14	1.	*Cqp*	II, T13, T1
T15		*CpCqp*	II, T1, T14
T16	1.3.	*SNp*	I
T17	1.3.1.	*SNq*	I
T18	1.3.	*q*	IV, T17, T1, T16
T19	1.	*CNpq*	II, T16, T18
T20		*CpCNpq*	II, T1, T19

Jaśkowski's suppositional system is therefore clearly restricted to suppositions and rules, without any axioms. The logical theses are end-formulae which end subdomains of absolute domains, and the entire system is comprised successive subsystems, denoted by different numerical prefixes. In this system, theorems are those formulae which have no numerical prefixes. In a metalogical theorem, Jaśkowski also proved that his system is equivalent to the axiomatic system – namely by showing that every thesis of the axiomatic system is also included among the theses of his system (without a prefix) and that every thesis of his system without a prefix is included among the theses of the axiomatic system.

Jaśkowski's system, at the quantificational level, appealing to the principle of economy, made use only of universal generalization (**UG**) and universal instantiation (**UI**) (although of course the negation " ~(x)~ " of the universal quantifier in Jaśkowski's system can be interpreted as the existential quantifier). Thus, the economy of Jaśkowski's first-order rules did not allow for the possibility of dealing with an empty universe. Moreover, it could be argued that the economy of his system was obtained at the cost of the naturalness of deduction in his system. Because it was deemed preferable by most logicians of the day to preserve the autonomy of existential quantification and thereby insuring a more natural method of deduction, Gentzen's [1934] system was the one which gained the wider acceptance. (The only recent well-known Jaśkowski-type system appearing to date in elementary textbook form is [Kalish & Montague 1964].)

As is well known, the Gentzen **N**-sequents are typically of the form

$$\text{(D)}\quad A_1, A_2, \ldots, A_n \rightarrow B_1, B_2, \ldots, B_m$$

where every line in the proof of a deduction of this form is an axiom or can be derived from previous lines in the proof by certain rules. As we have already noted, such a sequent is an array of two finite sequences of formulae related by '→'. (These rules, as we shall see, are similar to, but more limited than Copi's. For example, Gentzen's **UI** rule can be stated as:

$$\frac{K \rightarrow (x)A \quad , \quad K, A' \rightarrow B}{\therefore\ K \rightarrow B}$$

where K is an N-sequent of the type (D) and A' is like A except for exhibiting free occurrences of some individual variable x' (not necessarily distinct from x) where ever A exhibits free occurences of x (see [Leblanc 1966a, p. 161]).

As is readily seen, Gentzen's sequent

$$A_1 \,\&\, A_2 \,\&\, \ldots \,\&\, A_n \rightarrow (x)A$$

is equivalent to Copi's sequence

$$A_1 \cdot A_2 \cdot \ldots \cdot A_n \supset (x)\, A$$

provided that each A_i-component of K of assumptions is conjoined so that it is equivalent to the conjunction of the assumptions.

For Gentzen, (a) A' is considered to be an additional premise, and (b), all of K, i.e. *each* assumption, is to be repeated in every step (that is, each line) of the deduction, and therefore appears in the conclusion.

These two points are of considerable importance. It can be seen from them that Gentzen's derivations, although straightforward, are often long and tedious.

§2. Quine's interpretation of Gentzen's system. A search was undertaken for a simplification of Gentzen's system almost from the outset. In the late 1930s, Quine taught his version of Gentzen's system. He undertook the revision of Gentzen's quantification rules **UG** and **EI** because (as he stated at [Quine 1950, p. 96]), Gentzen's rules, and in particular the rule in Gentzen's system that does the work of **EI**, were "more complicated to state and less convenient to use." This new system of natural deduction was presented in textbook form by Quine in [1940]. In this system, Quine ([1940, p. 88]) adopts five "metatheorems" as axioms of quantification, namely

*100. If ϕ is tautologous, $\vdash \phi$
*101. $\vdash \ulcorner(\alpha)(\beta)\phi \supset.(\beta)(\alpha)\phi\urcorner$
*102. $\vdash \ulcorner(\alpha)(\phi \supset \psi) \supset.(\alpha)\psi\urcorner$
*103. If α is not free in ϕ, $\vdash \ulcorner\phi \supset (\alpha)\phi\urcorner$
*104. If ϕ' is like ϕ except for containing free occurrences of α' wherever f contains free occurrences of α, then $\vdash \ulcorner(\alpha)\phi \supset \phi'\urcorner$
*105. If $\ulcorner\phi \supset \psi\urcorner$ and ϕ are theorems, then so is ψ.

These rules bore some resemblance to rules presented in [Cooley 1942, pp. 126-140], although the restrictions in Cooley's rules were not formulated carefully.

Quine's presentation was challenged by Berry [1941] and Wang [1947]. In particular, they questioned Quine's interpretation of Gentzen's quantification rules. Berry [1941, p. 23] argued that *101 could be eliminated by "means of an otherwise trivial shift in the meanings of *100, *102, *103, and *104." Specifically, Berry [1941, p. 24], adopting a suggestion of Fitch, suggested that Quine's conception of closure be revised by replacing Quine's "Convention A", according to which the *closure of p* (written " $\vdash$ p ") is "p itself is a theorem" and is $\ulcorner(\alpha_1)(\alpha_2)(\alpha_n)p\urcorner$ where the α_i are the only variables free in *p* and each α_{i-1} is alphabetically prior to the corresponding α_i. By this convention, there is an alphabetically first variable, but no alphabetically last variable. Berry's "Convention B" proposes that there be an alphabetically last variable, but no alphabetically first variable.

Wang [1947, p. 130] accepts Berry's anti-alphabetical ordering of variables, and in addition and in view of the change in the definition of *closure* resulting from Berry's alterations, proposes that a new set of quantification rules:

Qp1. If ϕ is tautologous and α is not free in ϕ,
$\vdash \ulcorner(\alpha)\phi\urcorner$

Qp2. If α is not free in ϕ, $\vdash \ulcorner(\alpha)(\phi \supset \psi) \supset. \phi \supset (\alpha)\psi\urcorner$

Qp3. If ϕ' is like ϕ except for containing free occurrences of α' wherever ϕ contains free occurrences of α, then $\vdash \ulcorner(\alpha)\phi \supset \phi'\urcorner$

Qp4. If $\vdash \ulcorner\phi \supset \psi\urcorner$ and $\vdash \phi$, then $\vdash \psi$

replace Quine's metatheorems *100–*105. In the remainder of his paper, he shows how *100–*105 may be proved on the basis of his new rules.

As a result of these criticisms of Berry and Wang, Quine distributed mimeographed notes on the *Theory of Deduction* [1948]. In the [1948] version, Quine's restrictions on **UG** vary from Gentzen's, and the new existential instantiation rule (**EI**) is also quite different from Gentzen's. The set of restrictions which Quine placed on these rules in his mimeographed notes of [1946] for *A short course in logic* proved to be "insufficient" (see [Quine 1950, p. 96]). In the [1948] notes, the restrictions on **UG** were shown by J.W. Oliver to be too stringent, being "restrictive beyond necessity and convenience" (see [Quine 1950, p. 96]). The restric-tions on the rules were eased and presented in polished form in Quine's [1950] paper *On natural deduction*. Quine ([1950, p. 96]) viewed his [1950] system to be "superior both practically and aesthetically to that of *Theory of deduction*." The system of natural deduction developed by Rosser since 1940 and presented by him in mimeographed lecture notes from 1946-47 was reputedly very close to Quine's [1950] system.[1] Indeed,

[1] Both Prawitz [1965, p. 103] and Gupta [1968, p. 97] show that Rosser's [1953] quantifcation rules indeed are the same as those presented in [Quine 1950a]. [Gupta 1968, p. 97] and [Prawitz 1965, p. 104] also show that Suppes' [1957] rules are closely related to Quine's and Rosser's. Both Gupta [1968, p. 97] and Prawitz [1965, p. 104] show that Suppes' device of treating certain individual variables as "ambiguous names" as a means of dealing with **EI** is identical with that used by [Borkowski and Słupecki 1958]. Lemmon's [1961] is a simplification of Suppes' system. Gupta [1968,

Quine's [1950] system was influenced by "information that Rosser's **UG** and **EI** were symmetrical to each other" (see [Quine 1950, pp. 96-97]). In Rosser's [1953, pp. 103-107] discussion of the "Generalization Principle" for the restricted predicate calculus, that for a statement P and a variable x, if P is proved, then $(x)P$ can be inferred, it is crucial to note the restriction that "if any of our assumptions depend on P, ...then they put a restriction on P which prevents use of the generalization principle [while] if none of our assumptions depends on P, ...then our deduction is completely unrestricted as far as P is concerned, and use of the generalization principle is legitimate" ([Rosser 1953, p. 106]). Rosser's analogue for **EI** is his rule C, the "formal analogue of an act of choice" [Rosser 1953, pp. 127-133], which requires the restriction on the variable y that it should not occur free in any P precedes the closure sign, and if the rule C has already been used somewhere in the proof to go from $(\exists v)Gv$ to Gw, then y must not have occurred free in Gw.

Berry's modifications were accepted by Quine in the revised (1951) edition of [1940]. Berry's modification of the original definition of closure [Quine 1940, p. 79] is acknowledged to "yield a reduction in the axioms of quantification" [Quine 1940; 1951 rev. ed., p. ix]. The modification was mentioned, on p. 89, in the second printing of the first edition, and fully incorporated into the revised edition. Moreover, Quine's [1950] system was completely incorporated into his new [1950a] textbook *Methods of logic.*

In [1950, p. 94], Quine began by presenting the sample deduction

(1)	$(\exists x)(y)(Fy \supset Gxy)$	premiss
(2)	$(x)(y)(Gxy \supset Hyy)$	premiss
(3)	$(y)(Fy \supset Gwy)$	(1), w
(4)	$Fz \supset Gwz$	(3)

p. 97] also notes that both Suppes and Quine employ flagging as a device for treating **EI**. These relationships and similarities perhaps explain how Dziob [1972, p. 5] was led to erroneously assert that the criticisms which Lemmon [1961] and [1965] directed specifically at Suppes' [1957], as well as Schagrin's [1965] criticisms of [Lemmon 1961], are in fact directed at Quine.

(5)	(y)(Gyw ⊃ Hyy)	(2)
(6)	Gwz ⊃ Hzz	(5)
(7)	Fz	conditional proof
(8)	Hzz	(4), (6), (7)
(9)	(∃y)Hzy	(8)
(10)	Fz ⊃ (∃y)Hzy	(9)
(11)	(x)(Fx ⊃ (∃y)Hxy)	(10), z

and then listing the new rules for quantification required for the deduction. These rules, given at [1950, p. 96], are:

Rule of universal instantiation (UI): from $\ulcorner(\alpha)\phi\urcorner$ we may infer ϕ^{β}_{α}. This rule accounts for lines (4)–(6).

Rule of existential generalization (EG): from ϕ^{β}_{α} we may infer $\ulcorner(\exists\alpha)\phi\urcorner$. This rule accounts for line (9).

Rule of universal generalization (UG): from ϕ^{β}_{α} we may $\ulcorner(\alpha)\phi\urcorner$. (Alphabetical order: w, x, y, z, w′, x′,y, z′, w″, x″, y″, z″, etc.) This rule accounts for line (11).

Rule of existential instantiation (EI): from $\ulcorner(\exists\alpha)\phi\urcorner$ we may infer ϕ^{β}_{α} if β is alphabetically later than all free variables of $\ulcorner(\exists\alpha)\phi\urcorner$. This rule accounts for line (3).

Flagging: Off to the right of any line inferred by UG or EI, the variable β must be written. Cf. lines (3) and (11).

Restriction: Neither UG nor EI is permissible if β has previously been flagged.

> Also, we must impose this restriction upon what are to be regarded as finished deductions. No flagged variable is o be free in the last line of a finished deduction, nor in any premiss of the last line.

By way of preparatory explanation, Quine [1950, p. 95] noted that:

> Where α and β are any variables (*x*, *y*, etc), and ϕ is any schema in which α is free, the schema which is like ϕ except for containing free occurrences of β in place of all free occurrences of α will be called ϕ^{β}_{α}. In particular β may be free also in ϕ; and in particular β may be α, in which case ϕ^{β}_{α} is ϕ.

Thus Quine adopts Berry's anti-alphabetical conditions for **EI** and **UG**. In particular, Quine incorporateded both the anti-alphabetical and flagging restrictions in his [1950a] textbook *Methods of logic.* As evidence of the necessity of these restrictions, Quine [1950, p. 97] provided six examples of wrong results which are obviated by introduction of these restrictions. In summary, for Quine [1950a], (a) **EI** and **UG** can obtain only if β is alphabetically later than all free variables of (∃α)ϕ and (α)ϕ respectively (and has not been previously flagged), and (b) no flagged variable can occur free in the conclusion. This will become an issue in our consideration of Copi's quantification rules.

§3. The history of the tribulations of Copi's quantification rules for natural deduction. Many logicians still found Quine's new [1950a] system still too intractible to be "natural". Among those to attempt to provide a simplified alternative were Gumin and Hermes [1954-1956], who relax Quine's conditions on the ordering of the flagged variables.

The variant of **EI** given by Gumin and Hermes [1954-1956, p. 392] is

$$\frac{\exists x H}{\Theta}\ , \text{ falls Umbf } Hx\Theta y \text{ und Umbf } \Theta yxH$$

(where "Umbf HxyΘ" means "*H geht durch eine freie Umbenennung von x in y durch* Θ"). The flagging conditions added are that no variable can be flagged more than once and that "*Man kann die markierten Variablen so zu einer Folge ordnen, daß keine markierte Variable von einer in bezug auf dies Ordnung später markierten Variablen abhängt.*" Although this makes it easier to construct deductions, the conditions on **EI** still remain complicated and unnatural.

At the same time, the next step was taken by Irving M. Copi (b. Copilowitsch), who also found Quine's new [1950a] system to be too intractible to be "natural". Thus Copi formulated his own alternative system of natural deduction. Copi's system first appeared in Copi's [1954] textbook *Symbolic logic*. But [Dziob 1972, p. 3] has written that [Copi 1954] was "based to an embarrassing extent upon Quine's text."[2] In this system, Copi replaced some of Quine's conditions on flagging with some new restrictions on **EI** and **UG**. But these restrictions proved to be unnecessarily strong, too restrictive, and to render deductions to be much longer than they need to have been.

Very soon after Copi's text was published, Donald Kalish privately criticized Copi for the "undesirably unnatural" rule of **UG**. This criticism was taken up by Copi in his published reply to Kalish. In his paper *Another variant of natural deduction*, completed in mid-June 1955 and appearing in the March 1956 issue of the *Journal of Symbolic Logic*, Copi [1956, p. 52] thus wrote that "it has been pointed out to me by Professor Donald Kalish of U.C.L.A. that the restriction placed upon Universal Generalization (UG) and Existential Instantiation (EI) in [Copi 1954, p.

2 [Dziob 1972] is extremely critical of Copi, and readers are forewarned that her criticisms sometimes comes close to being *ad hominem*.

139] force one to construct a less natural proof than seems desirable for such arguments as

$$(x)(Ex \supset Ax) \;\therefore\; (x)[(\exists y)(Ey \cdot Hxy) \supset (\exists y)Ay \cdot Hxy)].\text{"}$$

The statement of **EI** at [Copi 1954, p. 104] is

$$\frac{(\exists\mu)\Phi\mu}{\therefore\ \Phi\nu}$$, provided that ν is a variable which occurs free in no earlier step ,

where ([Copi 1954, p. 100]) "the expression 'Φμ' will denote any propositional function in which there is at least one free occurrence of the variable denoted by 'μ' [and] the expression 'Φν' will denote the result of replacing all free occurrences of μ in Φμ by ν". Copi [1956, p. 52], admits that this is "relatively unrestricted," even "with the added proviso [(at [Copi 1954, p. 100])] that when ν is a variable it must occur free in Φν at all places at which μ occurs free in Φμ." The purpose of [Copi 1956], as stated there (p. 52) is "to formulate an alternative restriction on **UG** which will permit a more natural proof for such arguments, and to prove the consistency of the altered rule." The original rule **UG** [Copi 1954, p. 106] was

$$\frac{\Phi\mu}{\therefore\ (\nu)\Phi\nu}$$, provided that μ is a variable which has had no free occurrence in any propositional function inferred by **EI** .

The new rule **UG** [Copi 1956, p. 52] is

$$\frac{\Phi\mu}{\therefore\ (\nu)\Phi\nu}$$, provided that Φμ contains no free variable introduced by **EI**, and that μ is a

> variable which does not occur free in
> any assumption within whose scope $\Phi\mu$ lies.

The suggestion in [Copi 1956, p. 52] of a comparison of [Copi 1954, p. 139] with Quine's [1950a, pp. 175f] is an implicit admission that Quine's version of **UG** is, after all, the correct one. Copi [1956, p. 52] argues that the new rules permit "more natural proofs in some cases" than are found in [Copi 1954], and is an improvement over the Quine's statements in [1950] and [1950a] of requiring no "cluttering [of] our rules with an alphabetical stipulation, as if alphabetical had anything to do with logic" ([Copi, 1956, p. 52]; quoting [Quine 1950a, p. 161]).

The remainder of [Copi 1956] is devoted to a proof of the consistency of the new version of **UG** with the remainder of the system of natural deduction presented in [Copi 1954].

The new **UG** rule was incorporated into the second (1965) edition of Copi's *Symbolic logic*, which was submitted for publication in 1964. It was explained in the preface (p. viii) of the new edition that:

> The new quantification rules presented in Chapter 4 are both easier to understand and easier to apply in the second edition. They permit simpler proofs of validity to be given for some arguments... .

The appearance of the second edition of Copi's *Symbolic logic* opened the way to new criticisms of Copi's rules. It was agreed by Copi's critics that his new quantification rules were, if anything, too palatable. Although Copi's textbook was becoming a "best-seller," its author was attaining some degree of notoriety for what was being called his system of unnatural deduction. In the wake of the appearance of the second edition, at least eight articles appeared, in *Logique et Analyse*, the *Journal of Symbolic Logic*, the *Notre Dame Journal of Formal Logic*, and *Mind*, criticizing Chapter 4 of Copi's second edition. As we shall see from an examination of some of these criticisms, it became necessary for Copi to revise his quantification rules yet again, and thus to issue a third edition of his textbook.

In the September 1965 issue of *Logique et Analyse*, Hugues Leblanc argued that Copi's rule **UG**, as found in the second edition of [Copi 1954], "is clearly unsound" [Leblanc 1965, p. 210]. Leblanc selected one derivation using Copi's revised **UG** to show that the rule is incorrect. Leblanc [1965, p. 210] suggests that we "consider...the following eight lines, in which '*Vxy*' is short for '*x* voted for *y*' and '*m*' is short for '*Moe*':

1	(x)Vxm	
→ 2	(x)Vxy	
3	Vxy	2, UI
4	(y)Vyy	3, UG
5	(x)Vxy ⊃ (y) Vyy	2 - 4, C. P.
6	(y)((x)Vxy ⊃ (y)Vyy	5, UG
7	(x)Vxm ⊃ (y)Vyy	6, UI
8	(y)Vyy	1, 7, M.P.

> The inference abides by Copi's truth-functional and quantificational rules. Suppose though, that ' *m* ' and ' *y* ' are susceptible for the occasion of at least one extra value besides Moe, say Lefty, and that every one did vote for Moe. Then Lefty cannot have voted for himself. Hence 1 may be true and 8 be false. Hence 1 does not imply 8. Hence the inference is invalid. But C.P. (Conditional Proof), M.P. (Modus Ponens) and Copi's UI are sound. Hence Copi's UG isn't.

Leblanc [1966b] discusses these difficulties raised by the rules **UG** and **EI** and proposes some alternatives. He shows in particular that there is something deeper at stake than the ease and elegance of the rules. What is at stake is their correctness.

In [Leblanc 1965, p. 210], it is claimed that a "correction" is being offered to Copi's rules. But as we have seen, Leblanc has merely offered an example to show that Copi's rules are incorrect. This point was made by John Slater [1966], who reiterates Leblanc's example but then provides a new rule for **UG**, namely

$$\frac{\Phi\mu}{\therefore\ (\nu)\Phi\nu}$$

, provided that μ is a variable ccurring free in Φμ at all and only those places where ν occurs free in Φν, and that Φμ contains no free variable introduced by **EI**, and that μ is a variable which does not occur free in any assumption within whose scope Φμ lies,

which will "rule out this class of invalid arguments **UG** in Copi's system.... The 'all and only' requirement renders the [illegitimate] use of **UG**... invalid."

The difficulties which Copi faced and which had been pointed out by Leblanc and Slater were compounded by William Tuthill Parry in a [1965] paper responding to Copi's [1956]. In particular, Parry [1965, p. 119]

> ....shows that the system of natural deduction proposed by Copi in this JOURNAL [of Symbolic Logic] (1956), made by varying one restriction on Universal Generalization (**UG**) of the system of his ***Symbolic logic*** (1954), is incorrect. The original ***Symbolic logic*** system, also incorrect, was corrected in the third printing (1958) by modification of another restriction on **UG**; but combining this modification with that of this JOURNAL article does not give a correct system.

[Parry 1965] showed that Copi's restrictions on the **UG** rule were inadequate *precisely because* Copi had failed to make use of Quine's far from superfluous device of flagging the instantial variable in an inference by **EI** or **UG**.

It is true, says Parry, that Copi's third printing of the first edition *Symbolic logic* version of **UG** is an improvement over the original printing of that edition, where it was possible to obtain "$(x)Fxx$ " from "$(\exists x)(y)Fxy$ " – clearly false in a universe with just two elements. But Copi's revised restrictions on **UG**, which now took into account Quine's anti-alphabetical stipulation, necessary though they were, were nevertheless still insufficient. Parry proved that Copi's new restrictions were insufficient by choosing a derivation that [Quine 1950, p. 98] had used to specifically show the need for the flagging variables.

In Quine's proof as presented by [Parry 1965, p. 120],

1.	$(x)(\exists z)(Fzx \cdot (Gz \supset Gx)$	/ $\therefore$ $(x)(\exists z)(Fxz \cdot (Gz \supset (y)Gy)$
2.	$(\exists z)(Fzx \cdot (Gz \supset Gx))$	1, **UI**
3.	$Fxz \cdot (Gz \supset Gx)$	2, **EI**, z (x)
4.	$Gz \supset Gx$	3, Simp.
5.	Gz	
6.	Gx	4,5, M.P.
7.	$(y)Gy$	6, **UG** (x)
8.	$Gz \supset (y)Gy$	5–7, C.P.
9.	Fxz	3, Simp.
10.	Fxz	9,8, Conj.
11.	$(\exists z)(Fxz \cdot (Gz \supset (y)Gy))$	10, **EG**
12.	$(x)(\exists z)(Fxz \cdot (Gz \supset (y)Gy))$	11, **UG** ,

although Copi's restriction that x not be introduced by **EI** is satisfied, nevertheless a false conclusion is obtained, invalidly for Quine (see [Parry 1965, p. 121]). That is, as [Parry 1965, p. 121] notes, "the above argument-form, Quine points out, has a true premiss and a false concludion if 'F ' is identity, 'G' redness, and the universe contains some red and some non-red things. Line 12 violates Quine's rule against flagging the same variable twice." Parry's moral is that Copi should have borrowed *all*, and not merely a part, of Quine's rule of **UG**. Several similar examples are given, including an example showing that the original version of Copi's rules (in [Copi 1954]) were also incorrect.

An added difficulty for Copi came from criticisms by Lemmon [1961] that Suppes' [1957] formulation of **EI**, and by Leblanc [1966] that Quine's formulation, and hence ultimately Copi's as well, were incorrect.

Lemmon [1961, p. 594] argued that Suppes' [1957] rule of **EI** is too restrictive, making it impossible, for example, to derive $(\exists x)(Fxy$ & $(\exists z)Gxz)$ from $(\exists x)(Fxy$ & $(\exists y)Gxy)$ by applying **EI**. Thus, Lemmon [1961, p. 596, n. 1] declares that these difficulties "forced [him] reluctantly to conclude that a natural deduction formulation of the predicate calculus which employs a rule like **ES** [i.e. **EI**] involves hazards which are not straightforwardly overcome." As it happened, Copi's rule of **EI** did not include some of the restrictions which Quine had imposed on his version of that rule. Thus, Copi fortuitously escaped unscathed from at least this criticism. But then one might ask whether this particular lack in Copi's version of **EI** of the Quinean restrictions *always* make for valid arguments in Copi's system. The unfortunate negative replies were given by Leblanc, by Prawitz, and by Gupta.

Leblanc's [1966] textbook *Techniques of deductive inference* includes a new version of **EI**, similar to Gentzen's, but less unwieldy than Gentzen's rule (see [Leblanc 1966, pp. 102-109]). Instead of flagging variables, as Quine did, Leblanc treated each instance of **EI** as an additional assumption. This assumption may not be validly discharged until a certain conclusion "*C*" (acting as a constant obtained by **EI** and with "*C*" carrying certain restrictions, that is, as a "subsidiary derivation") has been obtained. In particular, Leblanc's **EI** (called **E∃**) states [1966, p. 102] that, where "B' is like B except for exhibiting free occurrences of some individual X' wherever B exhibits free occurrences of some individual variable X,"

> If from a set $\{A_1, A_2, ..., A_n\}$ of premisses one may derive the conclusion $(\exists x')B'$ and from the superset $\{A_1, A_2, ..., A_n, B\}$ of the original set $\{A_1, A_2, ..., A_n\}$ derive a conclusion C, then from the original set $\{A_1, A_2, ..., A_n\}$ one may derive C as a conclusion, so long as $X'$and X, should

they occur free in ***B***, do not occur free in any one of A_1, A_2, ..., A_n , and *C*.

These restrictions on **E∃** require Leblanc [1966, p. 104] to distinguish between "deëxistentialization" and "quasi-deëxistentialization," according to which, if one were to

> suppose that, having obtained $(\exists X')\boldsymbol{B}'$ in the course of a derivation, one assumes ***B*** as a provisional premiss. If *X′* is the same as *X*, we say that *X′* (if it occurs free in ***B***) is deëxistentialized upon in the said derivation; if *X′* is distinct from *X*, we say that *X′* (if it occurs free in ***B***) is deëxistentialized upon and that *X* (if it occurs free in ***B***) is quasi-deëxistentialized upon in the derivation.

Leblanc [1966, pp. 104-105] adds that

> the restrictions appended to **E∃**, ...have their point. Suppose indeed that an individual variable deëxistentialized or quasi-deëxistentialized upon in a derivation could occur free in one or more of the premisses of that derivation. Then '$(\exists x)(f(x)$ & $\sim f(x))$' , though not implied by $\{f(x), \sim f(y)\}$, would nonetheless be derivable from it [two examples given].
>
> Or suppose that an individual variable deëxistentialized or quasi-deëxistentialized upon in a derivation could occur free in any conclusion on the strength of **E∃** under the premisses of that derivation. Then '$f(x)$' and '$f(y)$', though not implied by $\{(\exists x) f(x)\}$, would nonetheless be derivable from it [two examples given].

Next, Leblanc [1966, p. 107, n. 1] refers to Copi's **EI** rule, calling it **E∃*** (and to Quine's [1950a] version), that is, "a variant of **E∃** that does without the subsidiary derivation that **E∃** calls for." **E∃***, says Leblanc ([1966, pp. 107-108]) "unfortunately has a shortcoming which in our opinion far outweighs its undeniable merits." The "shortcoming" – soon to be understood as devastating for Copi's **EI** rule – is that , in Copi's rule, a conclusion obtained by **EI** from a set of premisses need not be implied by these premisses.

Leblanc, aware that Copi's rule of **EI** would soon come under attack due to criticisms of Quine's and Suppes' **EI** rules and Slater's modification of Copi's **UG** rule, wrote to Copi. He mentioned that, although Copi's rule of **EI** took effective care of the deëxistentialized individual variables, it did not allow for those which are quasi-deëxistentialized. Leblanc suggested, however, that a minor change of wording could account for the former.

More importantly, Leblanc argued that Copi's **EI** rule violated the hallowed dictum that held that, in natural deduction, each line of a proof is itself a valid formula. But for each use of the **EI** rule according to Copi, that the line of proof which appeals to **EI** for its justification is not a valid formula. More specifically, we recall that, for Gentzen, because the lines of a proof are sequences, each assumption is repeated in each line of the deduction. Thus, each line is a valid formula. For Quine, too, because of his flagging restriction, no flagged variable can appear free in a premiss of the last line, each formula which has been existentially instantiated is regarded as an additional premiss. Therefore, in these cases as well, each line is a valid formula. But Copi's second (1965) edition of *Symbolic logic* would allow, for example, the inference

$$\begin{array}{ll} & A_1 \\ & A_2 \\ & \cdot \\ & \cdot \\ & \cdot \\ \text{i} & (\exists x)(Gx \lor By) \\ \text{i+1} & Gz \lor By \end{array}$$

which, rewritten in the form of N-sequences, is equivalent to

$$\{A_1, A_2, \ldots, A_n\},\ (\exists x)(Gx \vee By) \rightarrow (Gz \vee By)\ ,$$

which cannot be said to be a valid formula. Leblanc then suggested that, should Copi be considering a revision of his textbook, he might perhaps wish to inspect Leblanc's new rule for **EI**, which respects this law of implication in derivations.[3] The [1965] paper *Minding one's X's and Y's* was Leblanc's public critique of the version of **EI** appearing in the second (1965) edition of Copi's *Symbolic logic*. It must have been written upon the book's first appearance, since Leblanc quoted the rules of quantification directly from the end-papers of the book (see [Leblanc 1965, p. 209]).

Other published criticisms of the versions of Copi's **EI** rules in the first two editions of his textbook appeared several years after the first printing of the second edition. Among the chief critics was Dag Prawitz.

In his paper A note on existential instantiation, completed before mid-March 1966 and appearing in the March 1967 issue of the *Journal of Symbolic Logic*, Prawitz noted the "difficulties" in the (first-edition, [1954], version) of Copi's rule for **EI**. Prawitz may not yet have have seen [Slater 1966] while he wrote his [1967], nor does he refer to it; he does, however, refer specifically to [Parry 1965]. Hence, his argument is

3 There appear to have been rumors concerning Copi's reply to Leblanc's letter, begun in the late 1960s by Leblanc's students at Bryn Mawr, and which had by early 1972 spread to the universities in Pittsburgh (see, e.g., [Dziob 1972, p. 7]), which claimed that Copi arrogantly and offhandedly dismissed Leblanc, suggesting that each man retain his own rules, and reducing the claims of the correctness of their respective versions of **EI** to a contest decided by the commercial success of their respective textbooks.

Anne Marie Dziob was a philosophy graduate student at Duquesne University in Pittsburgh at the time she made these claims. I must caution the reader that I do not know the source of her information or the origin of the rumors which she reports, nor have I been able to confirm her allegations. The justification for inclusion of these rumors in the present account stems from my recollection of them, as part of the intellectual milieu of my graduate student days, as associated with the folklore of history of the development of the method of natural deduction generally and specifically of the evolution of Copi's book.

vitrually equi-valent to the one that asserts that Copi's difficulties are of the sort that result when converting to Slater's [1966] rule for **UG** from Copi's incorrect **UG** rule. Prawitz's primary concern here ([1967, p. 82]) is primarily that "from a deduction of *A* from *G*...it is to be required among others that no variable introduced by **EI** occurs in *A* or in some formula of Γ." He explains [Prawitz 1967, p. 81] that

> The presence of a rule for existential instantiation (**EI**) in a system of natural deduction often causes some difficulties, in particular, when it comes to formulate necessary restrictions on the rule for universal generalization (**UG**). A system containing rules for **EI** and **UG** that avoided Quine's rather cumbersome restrictions on these rules was formulated by Copi [1954], but the system was found to be inconveniently restrictive. A less restrictive system was therefore suggested by Copi [1954].
>
> Also that system forces some deductions to be unnecessarily long as is shown in Prawitz [1965, Appendix C, p. 104], where a way to liberalize Copi's restriction on **UG** is suggested [Prawitz, p. 105]. However, the system suggested by Copi [1954] is also incorrect (i.e. unsound) as has recently been shown by Parry [1965].

Prawitz [1967, p. 82] then reminds his readers that Parry's [1965] system has (in [Prawitz 1965]) been shown to be correct. This is done by transforming deductions of Parry's system into corresponding deductions in Gentzen's system of natural deduction, although Gentzen's handling of inferences of existential formulae, seems to Prawitz [1965, Appendix C, §3] to be "much more transparent."

Prawitz's criticisms of Copi were comparatively mild in comparison with the implications which Gupta's criticsims of Suppes' version **ES** of **EI** held not only for Suppes but for Copi. In his paper *On the rule of existential specification in systems of natural deduction*, Gupta [1968], having Suppes' [1957] rules specifically in mind, brought to a forceful conclusion the point that Leblanc had raised in his letter to Copi.

Gupta's interpretation of **EI** not only seeks to overcome Lemmon's qualms respecting the "hazards" of **EI**, and reassesses Quine's and Suppes' **ES** rule, but in effect dispenses entirely with **EI**. That is, Gupta treats **EI** much as Gentzen, Quine, and Leblanc treated it – as providing an additional premiss for a proof, rather than as an inference rule. Specifically, Gupta [1968, pp. 96-97] wrote that

> ..much uneasiness and some amount of mystery surrounds the rule of Existential Specification [i.e. **EI**, Suppes' E.S.]. In this paper I present my view on the matter, my main contention being that this rule is not a rule of inference in the sense that the rules of Universal Specification, Universal Generalization and Existential Genenalization are, but that it is a proof-strategical in the sense that its use shortens an otherwise long proof in which this rule is not used. More specifically, I will show that if the other rules have been chosen suitably, then any proof using the E.S. rule corresponds to a proof (often longer) involving no applications of the E.S. rule. ...In this way we shall regard the E.S. rule as a rule of strategy, as a rule of introducing additional auxiliary premises.
>
> ...Each of the rules U.S., U.G., and E.G. when applied with the appropriate restrictions leads to a formula to which is a logical consequence of the formula to which it is applied. But this is no longer so if the rule E.S. has been applied.

Gupta [1968, p. 98] regards the hazards of which [Lemmon 1965] spoke as "occasioned solely by one's acceptance of the rule E.S. as a rule of inference on [a] par with the other rules of inference," and suggests, therefore, that "if the rule of E.S. is regarded as a rule of introducing aditional auxiliary premises for purposes of proof-strategy, then the hazards disappear... ." Suppes' device of treating certain individual variables as "ambiguous names" as a means of dealing with **EI** (a device also used by Kalish and Montague in their [1964] Jáskowski-style system) made it possible to treat formula obtained by E.S. as such additional **auxiliary premisses or "quasi-derivations."**

To develop his "hazard-free" system *L*, Gupta presents the basic inference rules for propositional logic, along with the rules of quantifier negation, U.G., E.G., and U.I.. Finally, [Gupta 1968, pp. 101-102] introduced E.S. as a rule which governs so-called "quasi-derivations." A *quasi-derivation* in *L* is a sequence $\psi_1, \psi_2, ..., \psi_n$ in which **E.S.** has not been applied and which are not derivations of ψ_n in *L* but serve as tokens for a longer derivation of ψ_n in *L*. In *L*,

> ...every line would be a logical consequence of the premises on which that line depends. ...Within this system we may for purposes of proof-strategy permit the following rule of introducing additional premises
>
> E.S.: $\dfrac{(\exists x)\phi(x)}{\phi^{x}_{\alpha}}$ provided α has not occurred earlier.

To summarize, Gupta shows that **EI** need not at all be a quantification rule, that it may, and commonly is, applied within a proof as a short-cut, as a quasi-derivation, to make deduction easier. Its use, as such, meets the requirement (set forth by Leblanc) that in natural deduction each line of a proof must itself be a valid formula. The sub-proof or quasi-derivation which makes use of **EI** is not formally considered to be a part of the proof. Much later, Michael Scanlan, in an unpublished [1987] manu-script, presented his full formalization of the deductive system CKP which "was presented by Copi, corrected by Kalish and revised by Fine to permit use of arbitrary names" (see [Scanlan 1987, p.8]).

Even before the appearance of Gupta's [1968], Copi acknowledged, as we have seen, that the **EI** rule of his second (1965) edition of *Symbolic logic* left much to be desired. In his second edition, Copi's **EI** rule stated (p. 110) that

$$\frac{(\exists\mu)\Phi\mu}{\Phi\nu}$$, provided that ν is a variable that occurs free in no earlier line.

From the criticisms of **EI** which we have considered, it is evident that Copi's rule now needed to be both implemented and "justified" as applicable only in a quasi-derivation. Indeed, as a result of these criticisms to the formulations of **UG** and **EI** in the earlier editions of his *Symbolic logic*, Copi provided one more reformulation of the rules for the third (1967) edition. Thus Copi once again wrote to Leblanc, this time acknowledging that Leblanc had been correct from the outset. The upshot of this round of correspondence was that Copi became "indebted" to Leblanc, as he admits in a footnote (n. 10, p. 111) of the third (1967) edition of his *Symbolic logic*.

For example, Copi now revised the rule for **UG** by adding the restriction that ν is a variable that does not occur free "*either in* (μ)Φμ *or* in any assumption within whose scope Φν lies" ([Copi 1954; 3rd ed., 1967, p. 114], my italics). "The third edition of *Symbolic logic* differs from the second in...one major respect," says Copi [1954; 3rd ed., 1967, pp. iii-iv]. "The major change"

> is due to Professor William Tuthill Parry, who discovered that the quantification rules set forth in Chapter 4 of the second edition were incorrect [references to [Parry 1965], [Leblanc 1965], and [Slater 1966]. Those rules were originally published in [Copi 1965], and appeareed to offer some advantages in simplicity and ease of application over the quantification rules presented in the first edition of this book. When no objections to them had come to my attention by the end of 1964, I decided to include them in the second edition, which was published in the Spring of 1965. Its appearance was followed shortly by the publication of Parry's article.
>
> In the third edition new quantification rules replace the unsatisfactory old ones. The changes are in Existential Instan-

> tiation (EI) and Universal Generalization (UG). They now represent a closer approximation to Gentzen's original version of these rules, though modified to conform to the general approach of the present volume.

Among those to whom Copi expressed some indebtedness were Dag Prawitz, who corresponded with the Copi about the rules and offered his advice, Parry, Kalish, and Leblanc (see [Copi 1954, 3rd ed. 1967, p. iv]). Much is owed in particular to Leblanc. The new rules which Copi presents in the third edition are essentially those of Leblanc. In particular, as [Copi 1954, 3rd ed. 1967, p. iv] notes,

> the new phrasing of **EI** is similar to, and the new phrasing of **UG** is the same as, rephrasings of Gentzen's rules suggested by Professor Hugues Leblanc. [Copi is] indebted to Professor Leblanc for permission to borrow his new formulations, which he plans to substitute in the second edition of his *Techniques of Deductive Inference* [Leblanc 1966], for the rules now employed there.

In fact, the new rule of **EI** which Copi included in the third edition was provided by (or as Copi says, "borrowed" from) Leblanc himself.[4] It is exactly like Leblanc's rule except that it uses Copi's terminology and Copi's arrow of conditional proof. Although Copi nowhere admits – as had, for example, Gupta and Leblanc – that the rule of **EI** is not essential to a system of natural deduction, Copi does point out ([Copi 1954, 3rd ed., p. 111]) that it is "useful"

> to establish the logical truth of equivalences of the form

[4] The rumors mentioned in the previous note add at this point that Leblanc actually wrote the new EI rule for Copi, and that Copi paid Leblanc $300 for his work. The reader is again warned against accepting these unsubstantiated rumors, reported by Dziob, without additional verification.

(E) $(\nu)\,[\Phi\nu \supset p] \equiv (\exists\mu)(\Phi\mu \supset p]$

where ν occurs free in $\Phi\nu$ at all and only those places that μ occurs free in $\Phi\mu$, and where p contains no free occurrence of the variable ν.

And noting this time around ([Copi 1954, 3rd ed., p. 111]) that "we want to permit going from $(\exists x)Fx$ to Fx or Fy only under very stringent restrictions" and that "we never end a proof with a propositional function containing a free variable," Copi fully implements his new rule of **EI.**

Finally, then, Copi's schematization ([Copi 1954, 3rd ed., p. 112]) of a proof using the version of **EI** given to him by Leblanc, is a formalization of the thesis offered in [Gupta 1968]. Thus, given the required restrictions, we have the proof

i.	$(\exists\mu)\Phi\mu$	
	.	
	.	
	.	
→ j.	$\Phi\nu$	
	.	
	.	
	.	
k.	p	
k+1.	$\Phi\nu \supset p$	j-k, C.P.
k+2.	$(\nu)(\Phi\nu \supset p)$	k+1, **UG**
k+3.	$(\exists\mu)\Phi\mu \supset p$	k+2, Equivalence (E)
k+4	p	k+3, i, M.P.

This proof "can be regarded," says Copi [1954, 3rd ed., p. 113], "as providing an informal justification for the rule of Existential Instantiation," which is now stated ([Copi 1954, 3rd ed., p. 113]) as

EI: $(\exists\mu)\Phi\mu$

.
.
.

$\Phi\nu$

.
.
.

p

$\therefore$ p

"provided that ν is a variable that does not occur free either in *p* or in any line preceeding $\Phi\nu$" [Copi's emphasis].

Copi's new third-edition rule, "borrowed" from Leblanc, in effect restricts existential instantiations to quasi-derivations, or subproofs of natural deduction proofs in Copi's system. Copi is thereby able to respect the requirement that every line of a proof in natural deduction must be a valid formula.

§4, Post-script. In the period immediately following the appearance of the third edition of Copi's Symbolic logic and the 1970 printing of that edition, there were no new published criticisms of the new quantification rules. There were, however, criticisms of the completeness proof which Copi presented both in the second edition of his book and in the third edition.

Gerald Massey [1963] showed that the proof of the completeness which Copi gave of his system for propositional logic in the second edition was incorrect, and John Thomas Canty [1963] provided a completeness proof. Both articles are briefly summarized by Kalish [1965]. John A. Winnie [1970] provided a simplification of the proof which Canty had used. In effect, Winnie proves the completeness of both Copi's system and the system based upon Copi's which Canty devised to carry out his proof of the completeness of Copi's system. Bradley [1971] notes that Copi adpots Canty's proof for the third edition of his *Symbolic logic*, and then suggests a simplification of the proof. Wilcox [1971] points out that the last two of the three definitions which Copi gave of *completeness* in the third edition of his *Symbolic logic* (pp. 188-189), and which are to be found unchanged from all of the previous editions, are not equivlaent, although Copi had claimed that they were equivalent. Wilcox states, but does not attempt to prove, that the non-equivalence of Copi's second and third definitions of completeness follows from the provable completeness in Copi's third sense of the propositional calculus of the *Principia mathematica*.

In our consideration of some of the difficulties involved in designing sound quantification rules for systems of natural deduction, we have explored some of the more salient historical trends, given an account of the search to render quantified deduction "natural", and in partcular focused attention on the arguments against Quine's, and especially of Copi's, rules of **EI** and **UG**, and on Copi's "comeback" in the third edition of his popular textbook.

It is clear that the difficulties involved in developing a set of quantification rules for systems of natural deduction which are at once both simple, i.e. "natural", and sound required a significant effort of roughly half a century, by many of the better logicians and textbook writers. The task was not quite as easy as one might have initially supposed. The difficulties involved in a correct formulation in particular of the **EI** rule for systems of natural deduction were indeed of sufficient significance, both pedagogically and theoretically, to warrant a separate consideration of alternative treatments by Prawitz [1965, §3, pp. 103-105].

One important proof-theoretic method which grew out of Gentzen's original work in natural deduction, and in particular in the calculus of N-sequences has not been considered here. The tree method, growing out of Beth tableaux in the work of Hintikka, Smullyan, and van Heijenoort, avoided some of the difficulties which other systems of natural deduction encountered with their quantification rules. This fortuitous situation arose in part because the tree method attained its mature form during the period in which the difficulties with the quantifiers rules for systems of natural deduction of Quine, Suppes, and Copi were being discussed and solved, and immediately afterward. It is also due in large measure to the straightforward and uncomplicated nature of the tree method. Indeed, the tree method is perhaps the most intuitive, and hence "natural" or all deductive systems arising out of Gentzen's calculus of **N**-sequences. Thus, the tree method requires only two quantification rules, namely **EI** and **UI**. As we know, there were no difficulties encountered with **UI** in any natural deduction system; and the simple condition on **EI** in the tree method that *any instantiation of an existentially bound variable is restricted to cases in which the instantiating term is **new** to the path in which it occurs* has therefore been proven to be adequate. There is no clear evidence, however, that the tree method was designed specifically and explicitly to avoid the the difficulties which arose with the quantification rules of other systems of natural deduction. (For a more detailed consideration of the history of the tree method, see [Anellis 1990]).

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