The Review of Modern Logic
Volume 10 Numbers 1 & 2 (September 2004–February 2005) [Issue 31], pp. 145–154.

E. Husserl
Logik, Vorlesung 1896 (*Materialien*, vol. 1)
Dordrecht: Kluwer, 2001
xviii + 328 pp. ISBN 0792369114
and
E. Husserl
Logik, Vorlesung 1902/03 (*Materialien*, vol. 2)
Dordrecht: Kluwer, 2001
xvi + 310 pp. ISBN 0792369122

REVIEW

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Logik, Vorlesung 1896 and *Logik, Vorlesung 1902/1903* are the first two volumes of *Materialien*, a new subseries of *Husserliana* now being published by the Husserl Archives of Leuven, Belgium to provide reliable transcriptions of essential and historically instructive manuscripts, lecture courses, research manuscripts and of drafts and compilations from Edmund Husserl's *Nachlass* worked out by his assistants. These two books come as a welcome complement to the existing *Edmund Husserl–Gesammelte Werke, Dokumente* and *Collected Writings* series, whose volumes include Husserl's *Vorlesungen über Bedeutungslehre* [13], his *Logik und allgemeine Wissenschaftstheorie* [15], and his *Einleitung in die Logik und Erkenntnistheorie* [12] (presently being translated by this reviewer). Of interest as well to modern logicians are *Alte und Neue Logik, Vorlesung 1908/09* [18], and *Allgemeine Erkenntnistheorie, Vorlesung 1902/03* [16], recently published in this subseries.

In close professional and personal contact for over four decades with many of the makers of modern logic and mathematics [7], Husserl was well versed in the pioneering work being done in those fields. *Logik, Vorlesung 1896* is a lecture course given at the University of Halle, where since 1886 he had enjoyed the support and friendship of Georg Cantor [1], then at the height of his creative powers. Husserl was actually one of the very first to tangle with the challenging questions raised by set theory. He himself characterized his 1891 *Philosophy of Arithmetic* as an initial attempt on his part "to obtain clarity regarding the original genuine meaning of the fundamental concepts of the theory of sets and cardinal numbers" (*Klarheit über den ursprungssechten Sinn der Grundbegriffe der Mengen- und Anzahlenlehre zu gewinnen*) ([10 §27a; §24 and note]). Between 1886 and 1893, he related in a 1901 letter, he had busied himself with the theory of geometry, formal arithmetic, and the theory of manifolds, at times exclusively devoting himself to this ([11, p. 396]). During that time Husserl's ideas changed considerably and durably as he forged the insights that went into the making of his *Logical Investigations*.

Most evident in the 1896 lecture course are Husserl's preoccupations as already known through his *Early Writings in the Philosophy of Logic and Mathematics* [14], *Philosophy of Arithmetic* [17] (both available in English translations by Dallas Willard), and *Studien zur Arithmetik und Geometrie* [11]. Among these may be named questions about extensions, sets, and classes, intensions, differences between equality and identity, numbers without reference, operating with meaningless symbols, the foundations of arithmetic, manifolds, "calculating" with concepts. The main text of this course is devoted to what Husserl calls the three "traditional" sections of logic: theory of concepts and their objects (pp. 54-132); theory of propositions and judgments (pp. 133-231); and theory of inference (pp. 232-64). The final sixty pages are taken from an 1895 lecture course entitled "On Recent Research into Deductive Logic." They concern logic as a theoretical discipline and contain critical discussions of the ideas of William Rowan Hamilton, Augustus De Morgan, and George Boole.

The section of the course devoted to concepts and their objects is a study of the various issues surrounding what Frege termed saturated and unsaturated expressions. It teems with interesting observations about definite articles and descriptions, properties and objects, properties and predicates, properties of properties, concepts of concepts, concepts, extensions and intensions, relations, wholes and parts, the one and the many, colors, states of affairs, relations, sets, classes, aggregates, manifolds, negation, identity, equality, equivalence, equipollency, the concepts of cardinal and ordinal numbers.

A recurring theme in the course is that of intensions and extensions. Whereas Frege had assumed it "known what the extension of a concept is" (ex. [4, §69, n. 2]), Husserl defined intensions and extensions for his students, discussed the different issues surrounding them, and critically examined the different relations that the one type of meaning might entertain with the other. He identified extensions (defined on p. 117 as the whole of the objects falling under the concept, *i.e.* those attributable to the predicates in question, or to each of them making up the concept, independently of whether such objects really exist or not) with sets, which he distinguished from classes. He closes a discussion of the various different types of intensional relationships obtaining between concepts by stressing that the extensional relationships of concepts and relationships equivalent to them prove of incomparably greater significance for logical investigation than intensional relationships (1896, pp. 70-76, 111-25, 129, 147-48, 271).

In §29, concepts are compared in terms of their extensions. There, Husserl teaches that two concepts can have extensions of equal size and a one to one correspondence obtain between their objects. As an illustration, he suggests the concepts cardinal number and ordinal number which, though they do not have the same extension—for no cardinal number is an ordinal number—, do have extensions of equal size for which a mutual one to one correspondence can be established and an infinite number of the type designated by ω by Cantor assigned. Husserl points out that it cannot be said of all concepts that they are of equal or unequal size, because not all concepts are in general comparable in this regard. It must only be noted that quantitative comparison must not stop for infinite multiplicities, and thus for infinitely large extensions of concepts, as demonstrated by the example of cardinal and ordinal numbers. He cites Kant's assertion that concepts can only be compared with respect to extension when they are mutually subordinate because one cannot otherwise know which of them contains more objects (1896, pp. 117-18).

In a way that invites comparison with Frege's and Quine's well known theories on the same subjects, Husserl tackles the issues surrounding intensions and extensions again in the section on propositions. In §43, he discusses extensional misinterpretations of 'S is P.' He especially stresses that it is fundamentally false to see constructions of the form 'S is P' in terms of a relationship of extensions S and P (1896, p. 149). For us, Husserl teaches, S is an object of the concept P, hence perhaps an individual, but in the formulas of traditional logic, S must also, and in all circumstances, designate a concept, and never a concept that is the object of the predicate concept. If we say: 'All or some humans are mortal,' then 'human' represents a concept and certainly not one that is the object of the concept 'mortal.' The mortals are not concepts, but humans, and nothing like the 'concept human' is running around under humans. So the formula 'All A are B' must correspond to 'S is P' in a completely different way in which A is absolutely not identified with S and B with P (1896, p. 150).

In §44, Husserl examines intensional interpretations of constructions of the form 'S is P.' His examples are 'Socrates is sick' and 'Red is a color.' His discussions should leave no doubt, if any persists, as to whether he heeded Frege's scathing attack ([3, pp. 197, 201-02]) on his failure to use extensions in Frege's way in the *Philosophy of Arithmetic.* Recall that before, during, and after his struggle with Russell's paradoxes, though not in his review of Husserl, Frege himself recognized his own use of extensions as the Achilles heel of the foundations he was laying for arithmetic ([9, pp. 67-108]). Again in contrast to Frege ([4 §§55-65]), in both lecture courses, Husserl finds several occasions to emphasize that equality is not identity and that translating subject predicate into statements expressing identities is an error (ex. 1896, pp. 44, 45, 111-15, 151-53). At one point, he condemns Lotze's rendering of 'Caesar crossed the Rubicon' as the tautology 'The crossing of the Rubicon of Caesar was the crossing of the Rubicon Caesar' as being intrinsically flawed (1896, pp. 152-53).

Logik, Vorlesung 1902/1903 is a lecture course given during Husserl's first years as a protégé of David Hilbert [5] at the University of Göttingen. Most interesting in this book are Husserl's teachings about the relationship between logic and mathematics, for these were years that found Husserl pursuing his interest in axiomatization and the theory of the manifolds that he saw as akin to Hilbert's axiomatic systems ([6] [8, pp. 179-98]). In both books under review, Husserl pointedly defends the view that he attributes to Frege's teacher Lotze ([19, pp. 34, 138f.]) that pure arithmetic is basically no more than a branch of logic that had undergone independent development and had developed very early through independent treatment (ex. 1902/03, pp. 19, 249; 1896, p. 241). He bids his students not to be "scared" ("*Ich bitte Sie nicht zu erschrecken!*") by this thought (1902/03, p. 34) and "to get accustomed to the initially strange view of Lotze that arithmetic is only a relatively independent, and from time immemorial, particularly highly developed piece of logic" (1896, pp. 271-72).

"All of arithmetic," Husserl considered, "is grounded in the arithmetical axioms. The unending profusion of wonderful theories that it develops (*entwickelt*) are already fixed, enfolded (*eingewickelt*) in the axioms, and theoretical-systematic deduction effects the unfolding (*Auseinanderwicklung*) of them" (1902/03, p. 33). As he saw it, the axioms of pure arithmetic were self-evident. On the basis of them, the theorems of the discipline were derived by pure deduction following methodical, simple procedures. The field branched out into more and more theories and partial disciplines, ever new problems surfaced and were finally solved by expending greatest mathematical acumen and following the most rigorous methods (1902/03, pp. 39).

All purely mathematical concepts like unit, multiplicity, cardinal number, order, ordinal number, and manifold, Husserl taught, are purely logical, for they plainly relate in the most general way to numbers in general and are only made possible out of the most general concept of object. Each and every thing can be counted as one, and to formulate the concept of number or any arbitrarily defined number, we need nothing more than the concept of something in general (1902/03, p. 32). Pure number theory, is a science that unfolds (*entfaltet*) the mere meaning of the idea of number. If we limit ourselves to the theory of cardinal numbers, then plainly each of the axioms is a proposition that unfolds (*auseinander legt*) the idea of number from some side or unfolds (*auseinander legt*) some of the ideas inseparably connected with the idea of cardinal number. So denying the truth of primitive number propositions like $a+b=b+a$ or 'For any two numbers there is a sum $a+b$' would be a contradiction; anyone who does so uses 'cardinal number' in some other way, does not know what the words mean (1902/03, p. 33).

Husserl considered that eminent thinkers like Lotze and Leibniz had correctly recognized number as specifically derived from the concept multiplicity (*Vielheit*) and multiplicity as representing the most universal logical concept combining objects in general (1896, pp. 241-42). With the extension of a concept, he reasoned, one thinks of the whole (*Gesamtheit*) of the objects falling under the concept as united. This whole is a multiplicity. If, as is unavoidable, the concept of multiplicity is included in logic, then the entire *a priori* set theory is also. And then, why should not numbers, which are no more than specific instances of sets, be included? Then, however, the whole of arithmetic belongs within the scope of a sufficiently broadly understood logic (1896, p. 271).

In the 1902/03 course, Husserl explains to his students that the formal discipline of propositions in general and of concepts in general is a mathematical discipline and as such has precisely the same character and the same methods as the mathematical disciplines known to them, for example arithmetic. What is mathematical in the way that arithmetical and geometrical disciplines proceed, he taught, is not dependent on our dealing with numbers in them. It was by chance historical development that pure mathematics was first constituted in the sphere of numbers and quantities and people thereby grew accustomed to identifying the mathematical and the quantitatively determinable. The essence of the mathematical is not, however, to be found in the quantitative, but in the establishment of a purely apodictic foundation for the truths of a field from apodictic basic principles. Handling inferences mathematically does not mean reducing them to mathematics in the ordinary sense, but rather signifies no more or no less than a rigorously scientific, *a priori* theory which builds from the bottom up and derives the manifold of possible inferences from the axiomatic foundations *a priori* in a rigorously deductive way (1902/03, p. 231).

Husserl finds nothing extraordinary about the idea of calculating with concepts and propositions. He considered that thanks to the theory of manifolds (*Mannigfaltigkeitslehre*) ([6] [8, pp. 179-98]) , which he calls a wonderful tool without which mathematics would have remained stuck in its early stages, modern mathematics had actually been able to develop the notion of an algebraic theory of concepts and states of affairs. In ten pages, he details his axioms, notation, rules of inference for so doing. It is to be noted in this regard that, like his contemporaries in Germany, he employs Peirce's symbols for the universal and existential quantifiers $\prod, \sum$ which, unlike Frege's, were widely used (1896, pp. 272-73; 1902/03, pp. 231, 239-49).

The discussion of pure mathematics that closes *Logik, Vorlesungen 1902/1903* is followed, and complemented, by a twenty page discussion of the theory of probability, *i.e.* "the principles by which, on the basis of alleged knowledge about certain states of affairs, we rightly surmise certain new states of affairs that are not to be inferred purely logically from them, and how from allegedly justified surmises acquire the right to make new surmises" (1902/03, p. 249).

By providing the material necessary for assuring that Husserl's ideas will one day have the impact on the history, philosophy, invention, and pedagogy of modern logic, set theory, and the foundations of mathematics that his ideas should have had from the beginning, the publication of his lecture courses on logic that has been underway for a number of years is finally affording modern logicians a fair chance to understand how exactly the father of phenomenology managed to exercise the profound philosophical impact he did on the times that gave birth to twentieth century philosophy of logic and mathematics. However, although these lecture courses are laced with pertinent lessons about logical matters that lived on to become the stuff of modern logic, to be honest, in spite of a wealth of interesting material presented in them, long stretches of these books will not long hold the interest of most modern logicians. Much of what Husserl taught was standard fare, or now principally of historical interest. As an example, one might cite his really thorough discussion of existence and predication in Descartes, Kant, and Hume or, say, the critical discussion of quantification in the work of William Rowan Hamilton. This is particularly true of *Logik, Vorlesung 1902/1903*, the stated intent of which to be an elementary presentation of the subject for beginners that will introduce them to the treasures of two thousand years of logic (1902/03, p. 3).

However, Husserl taught his students to take a critical attitude towards the subject matter. While he spoke to them of the essentially different character of most recent logical investigations, the number of truly significant recent works on logic affording dazzling proof of the intelligence and erudition of their authors and full of theories, multifarious fruitful observations, even felicitous suggestions that it would be ridiculous to call a waste, he confessed that he could not rid himself of the impression that logic was still in its early stages (*noch in den Kinderschuhen*) (Husserl 1896, p. 28). His attitude is, for example, evident in his critical discussions of the theories of classes and identity, equality, and equivalence in Hamilton, De Morgan, and Boole (1896, p. 276), where we find him criticizing Hamilton's attempt to reduce subject/predicate constructs to equation (1896, ex. pp. 290, 291), but using Boole's endeavors as a springboard for a defense of endeavors to develop formal logic as a discipline for calculating with concepts and states of affairs.

Much of what is taught in these volumes would primarily prove of interest to those interested in the evolution of Husserl's ideas about logic and mathematics. In the remainder of this review I shall look at a few of those themes. For example, in the first forty-two pages published from the introductory lectures for the 1896 course, Husserl carries out his hallmark fight against psychologism in defense of the objective side of logic in a most explicit way. Of interest in this regard is light that editor Elisabeth Schumann sheds on Husserl's comments, in introductions to the *Logical Investigations* and elsewhere, to the effect that in its essential content the *Prolegomena to Pure Logic* was simply the reworking of two complementary series of lectures given in Halle in the summer and autumn of 1896. She has discovered that no such lectures existed. Rather, significant portions of the *Prolegomena* coincide with material from the course that Husserl gave on logic and theory of knowledge in 1901/02 and left among notes from the 1896 logic course presumably used again for lectures in 1901/02 (1896, pp. X-XIII). The last thirty pages of *Logik, Vorlesungen 1902/1903* are taken from a 1901/02 lecture on logic and theory of knowledge.

In these volumes, we find Husserl laboring to liberate his students from damage wrought by psychologistic interpretations of Kant's logic and to introduce them to his own very Bolzanian alternative. For, even more than the *Logical Investigations*, these lectures bear the unmistakable imprint of Bolzano's *Wissenschaftslehre*; Husserl explicitly tells his students that more is to be learned about descriptively laying the foundations of formal logic from that book than from all the other past and recent logical works together (1896, p. 96). We find him complaining that "no psychological and logical term is laden with so many pernicious ambiguities as the term "*Vorstellung*" (1902/03, p. 82) and taking pains to spell out the differences between the subjective *Vorstellungen* (presentations) as psychological experiences and the objective logical *Vorstellungen*, which he regards as the "completely lost" distinction that Bolzano for the first time identified as a "cornerstone of all genuinely pure logic" (1902/03, p. 56). (Remember that in the *Philosophy of Arithmetic* Husserl had incurred Frege's wrath by using the word '*Vorstellung*,' which Frege had decided to use only to designate the subjective, psychological phenomena (ex. any object in so far as sensibly perceptible or spatial ([2 p. 59])), to designate both what Frege had decided was subjective and what he thought of as objective ([4 §27n.]).)

Unfortunately, most of what Husserl was trying to impress upon his students in this regard will be lost on modern logicians owing to his abundant, be it Bolzanian, use of the very ambiguous term '*Vorstellung*'. Except for this major terminological stumbling block, in ignorance of Bolzano's writings, modern logicians primed to adulate Frege could well regard much of what is taught in these lectures as Fregean in spirit, but I doubt whether many readers today will have the patience to sort through the various meanings of '*Vorstellung*' to ferret out Husserl's true meaning ([9]). As an illustration of the problem, one might take Husserl's explanation that when the same star has two names, like 'Arcturus' and 'Alpha Boötes,' then the name itself is a subjective, not objective, component of the presentation. The name, he teaches, is the means of calling forth the objective content of the presentation, thus producing a corresponding subjective presentation of it. But only what belongs to the meaning belongs to the objective presentation. Different proper names for a thing are not signs of different presentations. Arcturus and Alpha Boötes represent the same presentation (1896, p. 79). Or, take Husserl's explanation that, looking at the meaning of the expression 'the wisest Athenian,' we find that it refers to a certain Socrates. At first glance, this object may accordingly seem to be the objective reality which the expression means. However, we immediately see that this is not right. The objective reality that is expressed and remains identical is not the real object Socrates, but only the *Vorstellung* of him, the *Vorstellung* in the objective sense. From examples, we learn that the expressions 'Socrates,' 'the wisest Athenian,' 'the founder the theory of definition,' 'the teacher of Plato,' *etc.* all refer to identically the same object, but the *Vorstellungen* are different. Not simply the subjective *Vorstellungen* are different, but the meaning of the expressions is objectively different. The objective thought content changes if we exchange the expressions with one another, and that goes together with the fact that truths concerning the same object can give us completely different information depending on the concept imparted. Indeed, the knowledge of the identity of the object of different conceptual *Vorstellungen* of this kind can represent a meaningful extension of knowledge, and many will learn something from the fact that Socrates is the same person as the founder of the theory of definition. But anyone in this situation surely already understood both the expressions before learning the identity of the person, always was in possession of them as different *Vorstellungen.* And also after the knowledge is attained, the *Vorstellungen* will not be identical. It has only turned out that their objects are identical (1896, pp. 54-55).

To limn the true and ultimate structure of reality, Husserl taught, science needed metaphysical foundations. By metaphysics, he said, he did not mean some abstract conceptual mysticism, but the much more modest and fruitful, level-headed clarification and testing of those general presuppositions which the sciences of reality make about real being and the recuperation of the most mature, recent knowledge of real being, of its elemental principles, forms, and laws that the present state of the individual sciences permits (1896, p. 5). The realm of truth, he strives to impress upon students, is no disorderly hodgepodge. Truths are connected in systematic ways, are governed by consistent laws and theories, and so the inquiry into truth and its exposition must be systematic. The systematic representation of knowledge must to a certain degree reflect the systematic representation grounded in the things themselves (1896, p. 9). No blind omnipotent power has heaped together some pile of propositions P, Q, R, stringed them together with a proposition S, and then organized the human mind in a such a way that the knowledge of the truth of P unfailingly, or in certain normal circumstances, must entail knowledge of S. Not blind chance, but the reason and order of governing laws reigns in argumentation (1896, p. 13). All invention and discovery involves formal patterns without which there is no testing of given propositions and proofs, no methodical construction of new proofs, no methodical building of theories and whole systems (1896, pp. 16-17).

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