PROPOSITION A. *Let V be a valuation ring having a proper prime ideal P which is not branched; then $P = \bigcup_{\lambda \in \Lambda} M_\lambda$, where $\{M_\lambda\}_{\lambda \in \Lambda}$ is the collection of prime ideals of V which are properly contained in P. In this case, $P \cdot V[[X]] = P[[X]]$ if and only if* (*) *given any countable subcollection $\{M_{\lambda_i}\}$ of $\{M_\lambda\}$, $\bigcup_{i=1}^\infty M_{\lambda_i} \subset P$.*

Proof. Assuming (*), let $f(X) = \sum_{i=0}^\infty f_i X^i \in P[[X]]$. For each i, $f_i \in M_{\bar\lambda_i}$ for some $\bar\lambda_i \in \Lambda$. Let $p \in P$, $p \notin \bigcup_{i=0}^\infty M_{\bar\lambda_i}$; since $p \notin M_{\bar\lambda_i}$, it follows that $f_i \in M_{\bar\lambda_i} \subseteqq (p)V$ for each i and $f(X) \in (p)V[[X]] \subseteqq P \cdot V[[X]]$.

Conversely, assuming that (*) fails, let $\{M_{\lambda_i}\}_{i=1}^\infty$ be a countable subcollection of $\{M_\lambda\}_{\lambda \in \Lambda}$ such that $\bigcup_{i=1}^\infty M_{\lambda_i} = P$. By extracting a subsequence of $\{M_{\lambda_i}\}$, we may assume that $M_{\lambda_i} \subset M_{\lambda_{i+1}}$ for each i. Let $f_i \in M_{\lambda_{i+1}}$, $f_i \notin M_{\lambda_i}$ and let $f(X) = \sum_{i=1}^\infty f_i X^i$; then $f(X) \in P[[X]]$ but $f(X) \notin P \cdot V[[X]]$.

MARSHALL UNIVERSITY

Correction to

COHOMOLOGY OF FINITELY PRESENTED GROUPS

P. M. CURRAN

Volume 42 (1972), 615–620

In the second paragraph of the abstract, p. 615, the first sentence, "If G is generated by n elements, $\cdots$" should read "If G is a residually finite group generated by n elements, $\cdots$".

Correction to

COMMUTANTS OF SOME HAUSDORFF MATRICES

B. E. RHOADES

Volume 42 (1973), 715–719

In [2] it is shown that, for A a conservative triangle, B a matrix with finite norm commuting with A, B is triangular if and only if

(1) for each $t \in l$ and each n, $t(A - a_{nn}I) = 0$ implies t belongs to the linear span of $(e_0, e_1, \cdots, e_n)$. On page 716 of [2] it is asserted that

(2) $(U^*)^{n+1}(A - a_{nn}I)U^{n+1}$ of type M for each n is equivalent to

(1). This assertion is false. Condition (2) is sufficient for (1) but, as the following example shows, it is not necessary.

EXAMPLE 1. Let A be a triangle with entries $a_{nk} = p_{n-k}$, $n, k = 0, 1, \cdots$, where $p_k = 2^{-k}$. Then, for each n, $A - a_{nn}I = (U^*)^{n+1}(A - a_{nn}I)U^{n+1} = B$, where $b_{nn} = 0$ for each n, and $b_{nk} = a_{nk}$ otherwise. The only solutions of $tB = 0$ for $t \in l$ lie in the linear span of e_0, so that A satisfies (1). However, B is not of type M.

Since (2) is not equivalent to (1), some of the material on pages 717 and 718 of [2] must now be reworked.

We establish the following facts:

1. If a factorable triangular matrix A contains at least two zeros on the main diagonal, then $\mathrm{Com}(A)$ in $\Delta \neq \mathrm{Com}\, A$ in Γ.

2. If A is not factorable, then the number of zeros on the main diagonal gives no information about the size of $\mathrm{Com}(A)$ in Γ.

3. Having distinct diagonal entries is necessary but not sufficient for a conservative Hausdorff matrix H to satisfy $\mathrm{Com}(H)$ in $\Delta = \mathrm{Com}(H)$ in Γ.

Proof of 1. Let n and k denote the smallest integers for which $a_{kk} = a_{nn} = 0$, $n > k$. Then the system $t(A - a_{kk}I) = 0$ clearly has a solution in the space spanned by $(e_0, e_1, \cdots, e_n)$. It remains to show that there is a solution not in the subspace spanned by $(e_0, e_1, \cdots, e_k)$. Since A is factorable, either the kth row or the kth column of A is zero. In either case we can obtain a solution of the system using $t_n = 1$, $t_k = 0$ for $k > n$, which can be used to construct a non-triangular conservative matrix B which commutes with A.

Proof of 2. Define $D = (d_{nk})$ by $d_{n0} = 1$, $d_{nk} = 0$ otherwise. Then $D \leftrightarrow B$ implies that $(DB)_{nk} = b_{0k}$, whereas $(BD)_{n0} = \sum_j b_{nj}$, and $(BD)_{nk} = 0$ for $k > 0$. Thus, if B is any matrix satisfying (i) $b_{0k} = 0$ for all $k > 0$ and (ii) $\sum_{j=0}^{\infty} b_{nj} = b_{00}$ for all $n > 0$, then $B \leftrightarrow D$. For example, if $b_{00} = 1$, $b_{nk} = 2^{n-k-1}$, $k \geqq n > 0$ $b_{nk} = 0$ otherwise, then B is row infinite and commutes with D. D is factorable, but $A = D + I$ is not. Moreover, since $\mathrm{Com}(I)$ in $\Gamma = \Gamma$, $\mathrm{Com}(A)$ in $\Gamma = \mathrm{Com}(D)$ in $\Gamma \neq \mathrm{Com}(D)$ in Δ.

Example 1 with $p_0 = 0$ is a nonfactorable matrix with an infinite number of zeros on the main diagonal, and yet $\mathrm{Com}(A)$ in $\Gamma = \mathrm{Com}(A)$ in Δ.

The following examples establish 3.

EXAMPLE 2. Let H be the Hausdorff matrix generated by $\mu_n = -2n(n-1)/(n+1)(n+2)$, $B = (b_{nk})$ with $b_{0k} = b_{1k} = 1$ for all k, $b_{nk} = 0$ otherwise. Then $B \leftrightarrow H$, but $B \notin \Delta$ since $b_{01} \neq 0$.

EXAMPLE 3. Let H be generated by $\mu_n = n(n-1/2)/(n+1)(n+2)$. We can regard H as the product of two Hausdorff matrices H_α and H_β, with generating sequences $\alpha_n = (n-1/2)/(n+1)$ and $\beta_n = n/(n+2)$, respectively. From Theorem 1 of [1], the sequence $t = \{t_n\}$, with $t_0 = 1$, $t_n = (-1)^n(1/2)(-3/2)\cdots(-n+3/2)/n!$, $n > 0$ satisfies $tH_\alpha = 0$. Therefore $tH = 0$. Let B be the matrix with the sequence t as each row. Then

$$(HB)_{nk} = \sum_{j=0}^{n} h_{nj}b_{jk} = t_k \sum_{j=0}^{n} h_{nj} = t_k\mu_0 = 0 \ ,$$

and

$$(BH)_{nk} = \sum_{j=k}^{\infty} b_{nj}h_{jk} = \sum_{j=k}^{\infty} t_j h_{jk} = 0, \quad \text{so that} \quad B \longleftrightarrow H \, .$$

REFERENCES

1. B. E. Rhoades, *Some Hausdorff matrices not of type M*, Proc. Amer. Math., Soc., **15** (1964), 361-365.

2. ———, *Commutants of some Hausdorff matrices*, Pacific J. Math., **42** (1972), 715-719.

INDIANA UNIVERSITY

Corrections to

VERSUM SEQUENCES IN THE BINARY SYSTEM

CHARLES W. TRIGG

Volume 47 (1973), 263-275

Line 12 should read "the universal verity of the conjecture [5, 6]". Instead of the universal verity of the conjecture [1, 2].

The first page should be 263 instead of 163.