ERRATA

Correction to

SPINOR NORMS OF LOCAL INTEGRAL ROTATIONS, II

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One consequence of the 2-adic spinor norm calculations in [**3**] is an improvement of a theorem of Kneser [**5**; Satz 5] giving bounds on the power of two dividing the reduced discriminant of an indefinite quadratic **Z**-lattice having class number exceeding one (see also [**6**; p. 111] for a weaker version, and [**2**; Thm. 1.3, Chap. 11] for a restatement of Kneser's theorem).

Contrary to the claim made in [**3**], the bounds obtained in Theorem 4.2 of that paper are not best possible. The example $L \cong \langle -7, 2^2, 2^4, \ldots, 2^{2(n-1)} \rangle$ (see Remark 4.5) which purports to demonstrate that bounds attained are best possible in fact has class number one, not two as claimed. This can be seen by observing that $\theta(O^+(L_2)) \supseteq \dot{\mathbf{Q}}_2^2 \cup 5\dot{\mathbf{Q}}_2^2$ (by Proposition 1.8) and applying the argument in the last paragraph of Lemma 4.3.

We became aware of this error in reading a preliminary draft of Brzezinski's paper [**1**], where a bound better than that appearing in our Theorem 4.2 is obtained for a special class of indefinite ternary quadratic **Z**-lattices. In fact, the methods of our paper apply directly to yield this improved bound for all indefinite ternary **Z**-lattices.

Lemma 4.4 and, consequently, Theorem 4.2 of [**3**] can be improved to produce the correct best possible bounds for all ranks. The appropriate strengthened version of that lemma is:

LEMMA 4.4′. *Let L be as in* [**3**; *Thm.* 4.2]. *If $s_p < n(n-1)/2$ whenever p is odd and if $h^+(L) \neq 1$, then $s_2 \geq b_2'$, where*

$$b_2' = \begin{cases} (3n-2)(n-1)/2 & \text{if } n \text{ is odd,} \\ n(3n-5)/2 & \text{if } n \text{ is even.} \end{cases}$$

Proof. As in the proof of Lemma 4.4,

$$2^{-k}L \cong \langle \varepsilon_1, 2^{r_2}\varepsilon_2, \ldots, 2^{r_n}\varepsilon_n \rangle,$$

$0 < r_2 < \cdots < r_n$. It suffices to verify that $r_3 \geq 6$, $r_n \geq 3n-3$ for $n > 3$ odd, and $r_n \geq 3n-5$ for $n > 2$ even. Note first that if $r_s - r_t = 4$ for any s, t, then $\theta(O^+(L_2)) \supseteq \dot{\mathbf{Q}}_2^2 \cup 5\dot{\mathbf{Q}}_2^2$ (by Proposition 1.8) and $h^+(L) = 1$ follows as in the proof of Lemma 4.3. From this fact and Theorem 2.2, it can be seen that $r_2 \geq 1$ implies that $r_3 \geq 6$ under our assumption that $h(L) \neq 1$. Assume the above inequalities on r_j, $j = 1, 2, \ldots, k$. If $k+1$ is even and greater than 4, then $r_{k+1} \geq r_k + 1 \geq (3k-3)+1 = 3k-2 = 3(k+1)-5$ as desired

($r_4 \geq 7$ holds since $r_3 \geq 6$). If $k+1$ is odd, then, arguing as for r_3 above, $r_{k+1} \geq (3k-5)+5 = 3(k+1)-3$. $\square$

Theorem 4.2 remains valid with the values of b_2' given as above. Moreover, the examples $L \cong \langle 1, -2^6, 2^7 \rangle$, $L \cong \langle 1, -2^6, 2^7, \ldots, 2^{3n-3} \rangle$ if n is odd, $n \geq 5$, and $L \cong \langle 1, -2^6, 2^7, \ldots, -2^{3n-5} \rangle$ if n is even, $n \geq 4$, show that the new bounds b_2' are best possible for all $n \geq 3$. For small values of n, the last column of Table 4.8 should show the best possible bounds b_2' to be 7, 14, 26, 39, 57, 76, 100 and 125 for $n = 3, \ldots, 10$, respectively. The bounds shown for b_2'' in the third column of that table are indeed best possible (note that the final exponent in the example L of Remark 4.7 should read "$4k$", not "$2k$".

Finally, for completeness we note that in Theorem 3.14, (i) should read "If all $2^{r_i} L_i \ldots$", (ii) should read "$\ldots 2^{r_j} L_j \ldots 2^{r_k} L_k \ldots$", and the Hilbert symbol appearing in (ivb) should read "$(2^{r_k - r_{i_0}} a_{i_0} \varepsilon_k, -\det L_{i_0})$". These modifications were noted in [4].

References

[1] J. Brzezinski, *Spinor class groups of orders,* J. Algebra, **84** (1983), 468–481.

[2] J. W. S. Cassels, *Rational Quadratic Forms,* London Mathematical Society monographs, No. 13, Academic Press, 1978.

[3] A. G. Earnest and J. S. Hsia, *Spinor norms of local integral rotations,* II, Pacific J. Math., **61** (1975), 71–86.

[4] ——, *Spinor genera under field extensions,* II, Amer. J. Math., **100** (1978), 523–538.

[5] M. Kneser, *Klassenzahlen indefiniter quadratischer Formen in drei oder mehr Veranderlichen,* Arch. Math., **7** (1956), 323–332.

[6] G. L. Watson, *Integral Quadratic Forms,* Cambridge tracts in mathematics and mathematical physics, No. 51, Cambridge University Press, 1960.