

shown to be rigid in 5.2, this is correct. Thus the applications in the remainder of the proof of Proposition 5.3 are valid.

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CORRECTION TO  
SUMS OF PRODUCTS OF POWERS  
OF GIVEN PRIME NUMBERS

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Lemma 3(b) is false and hence the proof of Theorem 3 needs revision. We present a corrected version of Lemma 3(b) and a proof of Theorem 3 based on it.

LEMMA 3(b). *If  $3^b \mid 2^a + 1$ , then  $a \geq 3^{b-1}$ .*

*Proof.* If  $3^b \mid 2^a + 1$ , then  $2^{2a} - 1 = (2^a + 1)(2^a - 1) \equiv 0 \pmod{3^b}$ . Since 2 is a primitive root of  $3^b$  for any  $b \in \mathbf{N}$ ,  $\varphi(3^b) \mid 2a$  where  $\varphi(x)$  is the Euler's function. Hence  $3^{b-1} \mid a$ .  $\square$

*Proof of Theorem 3.* Without loss of generality we may assume that  $x \geq 1$ ,  $y \geq 0$ ,  $z \geq 2$ ,  $w \geq 1$ . By (1.3) and Lemma 3(b), we have  $x \leq z$  and  $z \geq 3^{\min(y,w)-1}$ . We derive from (1.3) that  $2^x \mid 3^w - 1$  and therefore  $2^{x-2} \leq w$ . Hence

$$x < (\log 2)^{-1} \log w + 2.$$

We distinguish between two cases.

*Case 1.*  $y \leq w$ . Since (1.3) implies  $3^w < 2^z$ , we have  $w < 0.631z$  and

$$(1.11) \quad |2^z - 3^w| < 2^x 3^y < 4w3^y < 2.524z3^y.$$

If  $z > 11$ , then, from (1.11) and Lemma 1, we obtain for nonexceptional pairs  $(z, w)$ ,

$$\frac{\exp(3^{y-1}(\log 2 - 0.1))}{3^{y-1}} \leq \frac{\exp(z(\log 2 - 0.1))}{z} < 2.524 \times 3^y.$$

Thus we have  $3^y < 11.2y$  and hence  $y \leq 3$ . From (1.11) and Lemma 1 we see that

$$z(\log 2 - 0.1) < \log z + 4.3$$

and so  $z \leq 11$ , which yields a contradiction. For each exceptional pair  $(z, w)$ , the number  $2^z - 3^w + 1$  has some prime factor greater than 3. Thus there are no solutions in this case with  $z > 11$ .

If  $2 \leq z \leq 11$ , then  $0 \leq w < 0.631z < 6.95$ , hence  $1 \leq x \leq 4$ . By checking these ranges for  $x, y, z, w$  we find the solutions:  $(1, 0, 2, 1)$ ,  $(1, 1, 3, 1)$ ,  $(1, 1, 5, 3)$ ,  $(3, 0, 4, 2)$ ,  $(3, 1, 5, 2)$ ,  $(4, 1, 7, 4)$ ,  $(4, 3, 9, 4)$ .

*Case 2.*  $w < y$ . It follows from (1.3) that

$$|2^{z-x} - 3^y| \leq |3^w - 1|/2.$$

If  $z - x > 11$ , then we obtain from Lemma 1 for non-exceptional pairs  $(z - x, y)$  that

$$(z - x)(\log 2 - 0.1) \leq w \log 3,$$

and so

$$3^{w-1} \leq 2(w + \log w + 1).$$

Thus  $w \leq 3$ , and  $x \leq 3$ ,  $|2^{z-x} - 3^y| \leq 13$ . Therefore

$$z \leq \frac{3 \log 3}{\log 2 - 0.1} + x < 9,$$

which yields a contradiction. It is easy to check that  $|2^{z-x} - 3^y| > 13$  for each exceptional pair  $(z - x, y)$ . Thus each solution of (1.3) in this case satisfies  $z - x \leq 11$ , hence  $z \leq 14$ . If  $2 \leq z \leq 14$ , then by (1.3),  $0 \leq y \leq 9$ . We find only one solution with  $y > w$ , namely  $(1, 5, 9, 3)$ .

We conclude that (1.3) has exactly eight non-trivial solutions  $(x, y, z, w) \in \mathbf{N}_0^4$ , namely

$$(1.12) \quad (1, 0, 2, 1), \quad (1, 1, 3, 1), \quad (1, 1, 5, 3), \quad (1, 5, 9, 3), \\ (3, 0, 4, 2), \quad (3, 1, 5, 2), \quad (4, 1, 7, 4), \quad (4, 3, 9, 4).$$

The argument for solutions with some negative values is similar to that in the proof of Theorem 1. Using (1.12) we obtain only one additional non-trivial solution in  $\mathbf{Z}^4$ , namely  $(3, -1, 1, -1)$ .  $\square$

Finally we note that the following should be added to reference [3] (W. J. Ellison): *On a theorem of S. Sivasankaranarayana Pillai*, Same Séminaire, Exp. 12, 10 pp.