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Design of "Random Walker" for Monte-Carlo Method Part II (Electronic Device)

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1. Introduction.

In the previous report, Part I,[1] our colleagues, Hiroshi SUGIYAMA and Osamu MIYATAKE have explained the outline of mathematical principles of "Random Walker". In Part II, we shall report on the mechanisms of this electronic computing machine.

As a first step, we have constructed an experimental model, in which case problems to be solved were limited to case of two dimensions. Recently, we have extended the ability of the computer to solve problems in case of three dimensions. The speed of operation of the computer has been increased remarkably by improving the random number generator to handle the mean pulse rate of 500 pulses per second.

As seen in Part I, this computer can solve eigenvalue problem, Dirichlet problem, Poisson's equation etc., in a domain with an arbitrarily shaped boundary by Monte Carlo method, but a large number of data must be arranged and be put in order, to obtain the final solutions. For this purpose, we have designed a data processor which stores the data in a magnetic drum.

To perform "random walk", the experimental model has utillized the spot on a Braun tube and some electronic counters, while the full-scale computer makes use of a ferrite core matrix and "Parametron" logical circuits for both cases of two and three dimensions.

2. Random Walker for Case of Two Dimensions (Experimental Model[2]).

Fig. 1 shows the block diagram of the random walker. The plane of random walk is realized on the surface of a Braun tube. The random number generator consists of a key-puncher and a tape-transmitter used in telegraph communications. In this tape, four different numbers are punched according to a random number table. The tape is transmitted through the tape-transmitter at a rate of four to eight numbers per second. The logical circuits generate random pulses at four output terminals according to the instruction sent from the tape, selecting any one of four terminals at each time. The four output terminals are connected to the following X and Y counters. The X and Y counters are binary reversible counters which count the number of steps positively or negatively in accordance

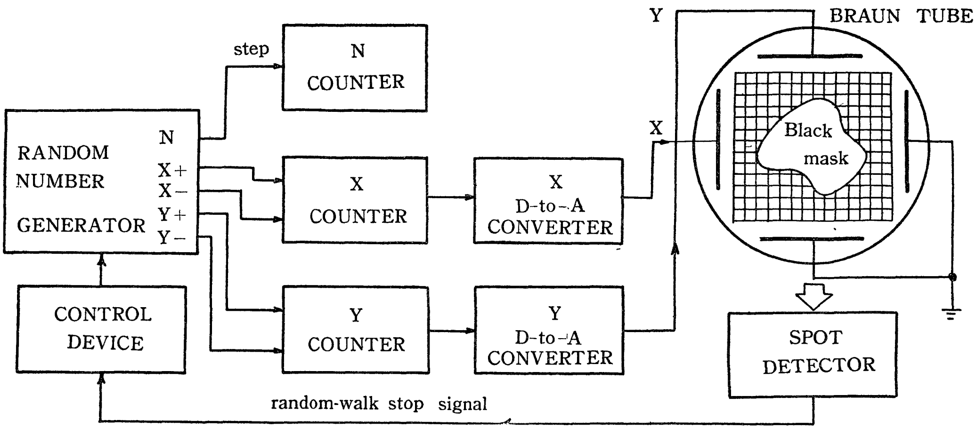


Fig. 1 Block diagram of "Random Walker" for case of two dimensions.

with the pulses sent from the four terminals, $X+$, $X-$, $Y+$ and $Y-$ respectively. The N counter is also a binary counter which counts all the pluses from the random number generator, thus, indicates all the steps of random walk. These counters are constituted with flip-flop circuits of electronic tubes.[3)]

X D-to-A and Y D-to-A converters are digital to analogue converters which convert the binary numbers to the coresponding analogue voltages. They consist of circuits weighting each binary digit and integrators, as well known in an analogue technique. The output voltages of the X and Y D-to-A converters are impressed to the X and Y deflection plates of a Braun tube, respectively. Then the spot walks stepwise on the surface of the tube at random, a process being known as " random walk ".

A black mask, which is formed similarly to the boundary shape of the problem, is set on the surface of the tube, and when the spot appears from the back of this mask, the spot detector which is set in front of the surface of the tube catches this spot and generates a stop pulse. The spot detector is consituted with a multiplier-phototube. The control devices receive this pulse and generate the stop pulse which controls the random number generator to stop the random walk, and after a short time, generate pulses to reset all the counters and to start the random walk again.

In case of eigenvalue problem, as explained in part I, it is necessary to count the number of random paths which had the same definite number of steps to reach the boundary. For this purpose, our computer provides an ordinary counter which counts the number of random paths, and the N counter which counts the number of steps of random walks. From these data, we can estimate the eigenavalues and the relative value of an eigenfunction at any starting point of random walks.

For Dirichlet problem, it is necessary to calculate the probability $\omega(P, Q)$

defined in the previous report.[1] The probability $\omega(P, Q)$ may be calculated by counting the number of times of random walks terminating at any point Q on the bounary among the total number of radom walks which start from a fixed point P in the domain. For this purpose, our computer provides the X and Y counters which represent the coordinates of terminal points of random walks.

3. Random Walker for Case of Three Dimensions (Full-scale Model).

In case of three dimensions, a large number of lattice points are required, and so the use of the spot on Braun tube as the random-walk-point inevitably necessitates to utilize a number of tubes corresponding to the partition of Z-axis, which is practically expensive. To avoid such an uneconomical method, the design has been to use a plane matrix of ferrite cores upon which is projected each two dimensional section of a body corresponding to each partition point of Z-axis, and "Parametron"[4] logical circuits for random walks has been adopted. To speed up the computation, we have improved the random number generator and provided the data processor. The block diagram of this computer is shown in Fig. 2. The details of the random number generator will be reported in the succeeding report III given by T. Oshio. The ferrite core matrix is constituted with 1024 ferrite cores which are connected to the X and Y counters. The informations from two 5-digits binary counters are transmitted to any one position of matrix cores. The plug-board is for the programming of any boundary. The method of setting the boundary in case of three dimensions is shown in Fig. 3-a.

A three dimensional boundary is cut by planes parallel to the X-Y plane which pass through the lattice points on the Z-axis, each section being projected

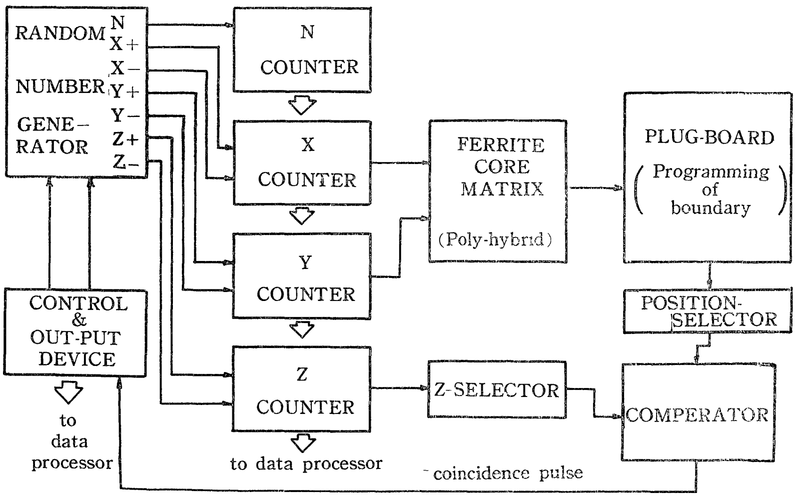
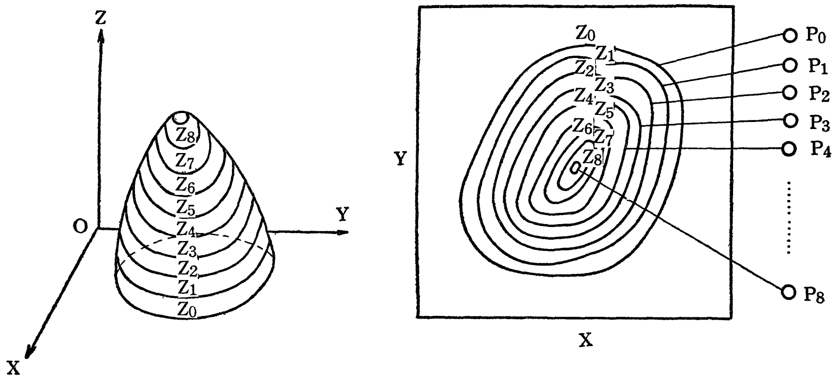


Fig. 2 Block diagram of "Random Walker" for case of three-dimensions.



(a) (b)

Fig. 3 Programming in case of three dimensions.

onto the X–Y plane. (Fig. 3–b) These projected boundaries are named as Z_0, Z_1, Z_2, $\cdots$, according to the number of lattice points in the direction of Z-axis, respectively. Each boundary Z_0, Z_1, Z_2, $\cdots$, thus projected on the X–Y plane is connected to the Z-axis position selector, P_0, P_1, P_2, $\cdots$, respectively.

The Z selector is a logical circuit consisting of parametrons in which the information from 5-digits of the Z counter is transmitted to any one position of 32 positions of the Z selector. The comparator consists of parametron " AND " gates which compares the 32 parametric informations of the position selector with those of the Z selector. If any of 32 signals on both the position selector and the Z selector coincides, the comparator sends out a stop singnal to the control device. At this moment, the random walk has reached a boundary point. The control device controls the random number generator with stop and start signals and the output device transmits the informations indicated in the X, Y, Z and N counters to the data processor at the instant when the random walk has reached the boundary.

4. Data Processor[5)]

The data thus obtained from the random walker are the number of steps of each random walk and the coordinates of the terminal points of random walk, on the boundary. When problems are to be solved by Monte Carlo method, the larger the number of random paths the better is the accuracy of the solution. Thus, the number of random paths must be more than a few thousands. As previously stated, the data must be rearranged to get the final results, and this is a troublesome work with manual operation. We have therefore provided a data processor which can rearrange the numerous data to solve the problems easily. The constitution of this processor is shown by the block diagram in Fig. 4. The main part of this processor is a magnetic drum which is 284 millimeters

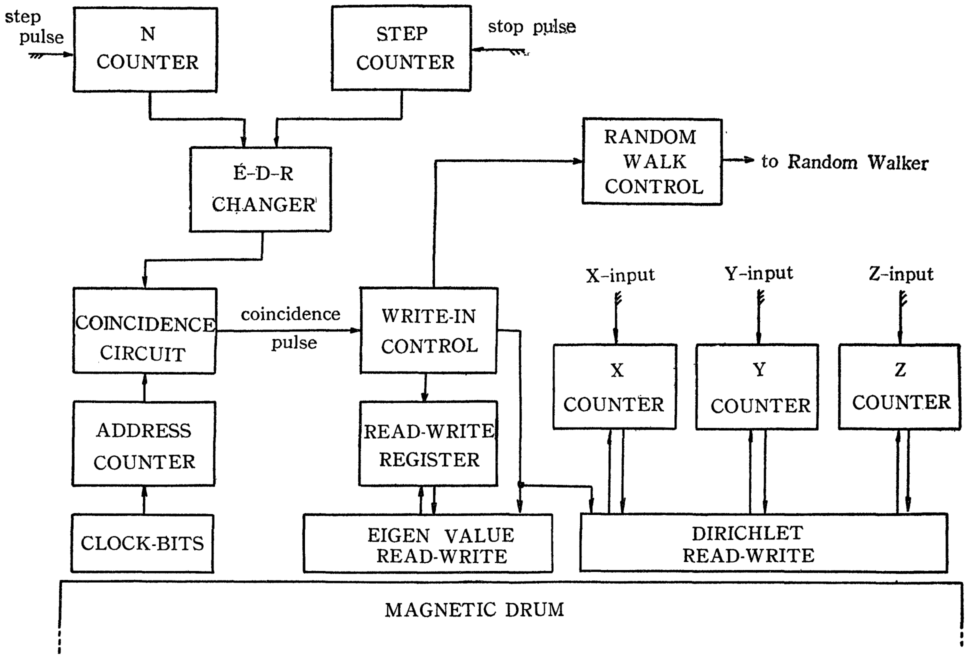


Fig. 4 Block diagram of Data Processor.

in diameter and 360 millimeters in length, and has a capacity of 675,000 bits (3,000 bits per track and 225 heads). The address counter is the one which counts the clock bits recorded on the surface of the magnetic drum. The read-write register can read the number of steps out of the drum, and write the registered number on the drum.

For eigenvalue problem, it is necessary that the number of steps of random walks are coincident with address numbers on the periphery of the drum, and the recurrent number of random walks of the same step number is recorded on the drum. The recording process of the number of recurrence on the drum is as follows: First, at the instant of the stop of random walks, the read register reads the number of recurrence which has been recorded on the drum: Second, the already registered number is added successively by the write-in control circuits: Third, after one revolution of the drum, the registered number is rewritten on the same address of the drum.

For Dirichlet problem, the coordinates of terminal points which are represented in the X, Y and Z counters, are recorded successively on the drum. The E-D-R changer is a switch that changes the logical process of the processor. At the E position and the D position of the switch, the processor records the data for eigenvalue problem, and for the Dirichlet problem respectively, and at the R position, the processor may read the data for both of the above problems.

Coincidence circuits are employed to write the data on the drum retaining the coincidence between the number in the address counter and the number in the N counter for eigenvalue problem or between the number in the address counter and the number in the stop counter which counts the stop pulses sent from the random walker for Dirichlet problem. For reading the data out of the drum, it is necessary to set the N counter manually until the set number of the N counter reaches the desired address number. As components for logical circuits of this processor, germanium diodes and transistors have been adopted.

5. Conclusion.

A simple computing apparatus to solve the differential equations in a domain with an arbitrarily shaped boundary by Monte Carlo method has been designed in case of two dimensions. Results obtained have been reported in part I. The random walker for three dimensional problems has been completed recently and it is currently under test. The random number generator is constituted with Cs^{137} and electronic tubes, the random walker with prarametron-logical-circuits, and the data processor with a magnetic drum and transistor circuits. Connections between these three parts involve many difficulties, but most part of their difficulties have been solved, and the results obtained with this computer will be shown later.

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