Geometry and Symmetry in Physics

## **BOOK REVIEW**

*Invitation to Topological Robotics*, by Michael Farber, European Mathematical Society, Zürich Lectures in Advanced Mathematics 2000, x + 133 pp., ISBN 978-3-03719-054-8

It should be mentioned from the very beginning that the topological robotics is a new mathematical discipline studying topological problems inspired by robotics and engineering as well as problems of practical robotics requiring topological tools.

The book is outcome of a one-semester lecture course "Topics of Topological Robotics", given by the author at the ETH Zürich during April-June 2006 and as other books in this series aims that the included material be better known to a wider audience.

Here the author describes four selected mathematical stories with various interesting connections with other sciences. The first three areas concern the topology of the configuration spaces of the mechanical systems under concrete knowledge of the linkages and the last one is devoted to a new topological invariant of the topological spaces motivated by applications in robotics and engineering.

Chapter 1 - Linkages and Polygon Spaces studies configuration spaces of mechanical linkages which appear in several fields of mathematics, as well as in molecular biology and in statical shape theory. It deals with the space of closed plane linkages which is identified with a closed manifold of dimension n-3. The integral homology of the latter is evaluated and the corresponding formula for its Betti's numbers is derived. This chapter discusses also the topology of random linkages and probabilistic approach to topological spaces depending on a large number of random parameters.

In Chapter 2 - Euler Characteristic of Configuration Spaces the author describes and proves the result of Gal [1] from 2001 about the Euler characteristics of the configurational spaces F(X,n) of n distinct particles moving in a polyhedron X for all n>0. The Gal's theorem gives an explicit expression for the Euler characteristic of these configurational spaces in the form of a rational function

encoding the local topological properties of the respective polyhedron.

Chapter 3 - Knot Theory of the Robot Arm deals with the "knot theory" of a planar robot arm. The space of all admissible configurations of a robot arm with a fixed length vector is the main object of study. The main result is the so-called *unknotting theorem* for planar robot arms, proven in 2003 by Connelly, Demaine and Rote [2].

In *Chapter 4 - Navigational Complexity of Configuration Spaces* the author discusses the topological invariants which arise in connection with the problem of construction and design of robot motion planning algorithms.

At its end the book contains an useful bibliography which amounts to 103 titles. After reading it the reader will be convinced that its principal aim - to provide an easy access for students and scientists in mathematics and physics in the field with the secrets of topology and its application to robotics is largely achieved. The same refers for new-interested readers who are without a prior knowledge of the subject.

Besides, the presented material subsumes research activities and some of the recent scientific advances of this theory, and it is an useful learning tool for scientists and engineers from academia and industry.

## References

- [1] Gal S., *Euler Characteristics of Configuration Spaces of a Complex*, Colloq. Math. **89** (2001) 61-67.
- [2] Connelly R., Demaine E. and Rote G., *Straightening Polygonal Arcs and Convexifying Polygonal Cycles*, Discrete and Computational Geometry **30** (2003) 205-239.

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