A Partial Order in the Knot Table

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CONTENTS

We write $K_1 \geq K_2$ for two prime knots K_1, K_2 if there exists a surjective group homomorphism from $G(K_1)$ onto $G(K_2)$ where $G(K_1), G(K_2)$ are the knot groups of K_1, K_2, respectively. In this paper, we determine this partial order for the knots in Rolfsen's knot table.

1. INTRODUCTION

Let K be a prime knot and $G(K)$ its knot group. It is well known that a partial order can be defined on the set of prime knots as follows: for two knots K_1, K_2, we write $K_1 \geq K_2$ if there exists a surjective group homomorphism from $G(K_1)$ onto $G(K_2)$.

In this paper, we determine this partial order "$\geq$" on the set of knots in Rolfsen's knot table, which lists all the prime knots of ten crossings or less. Theorem 1.1 is the main result of this paper. The numbering of the knots follows that of Rolfsen's book [Rolfsen 03].

Theorem 1.1. *The partial order "$\geq$" on the knots in Rolfsen's table is given by*

$$\left.\begin{array}{l} 8_5, 8_{10}, 8_{15}, 8_{18}, 8_{19}, 8_{20}, 8_{21}, 9_1, 9_6, 9_{16}, 9_{23}, \\ 9_{24}, 9_{28}, 9_{40}, 10_5, 10_9, 10_{32}, 10_{40}, 10_{61}, 10_{62}, \\ 10_{63}, 10_{64}, 10_{65}, 10_{66}, 10_{76}, 10_{77}, 10_{78}, 10_{82}, \\ 10_{84}, 10_{85}, 10_{87}, 10_{98}, 10_{99}, 10_{103}, 10_{106}, 10_{112}, \\ 10_{114}, 10_{139}, 10_{140}, 10_{141}, 10_{142}, 10_{143}, 10_{144}, \\ 10_{159}, 10_{164} \end{array}\right\} \geq 3_1,$$

$$\left.\begin{array}{l} 8_{18}, 9_{37}, 9_{40}, 10_{58}, 10_{59}, 10_{60}, 10_{122}, 10_{136}, \\ 10_{137}, 10_{138} \end{array}\right\} \geq 4_1,$$

$$10_{74}, 10_{120}, 10_{122} \geq 5_2.$$

In Section 2, we construct explicitly a surjective homomorphism for any pair of knots that belongs to the list in Theorem 1.1. In Section 3, we give the definition and results of the twisted Alexander invariants for knots. In Section 4, we prove the nonexistence of surjective homomorphisms by using the twisted Alexander invariants. This completes the proof of Theorem 1.1. In

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Section 5, we give the tables of data that are needed to prove Theorem 1.1.

2. CONSTRUCTION OF SURJECTIVE HOMOMORPHISMS

In this section, we construct a surjective homomorphism between the groups of each pair of knots that appears in the list of Theorem 1.1. For any knot K, we always take a Wirtinger presentation of its knot group $G(K)$ and denote it as follows:

$$G(K) = \langle x_1, \dots, x_n \,|\, r_1, \dots, r_{n-1} \rangle.$$

We denote by $\bar{x}$ the inverse of x in $G(K)$. Further, for simplicity, we write a number representing a generator of $G(K)$ in Tables 1, 2, and 3. For example, we write $1, 2, \dots, 9, 10$ for the generators $x_1, x_2, \dots, x_9, x_{10}$ and $12\bar{1}\bar{1}\bar{0}$ means a relator $x_1 x_2 \bar{x}_1 \bar{x}_{10}$.

Proposition 2.1. *There exists a surjective homomorphism $G(K_1) \to G(K_2)$ for any pair (K_1, K_2) of knots in Theorem 1.1.*

Proof: First, we consider surjective homomorphisms onto the knot group of the trefoil knot 3_1. The knot group of 3_1 admits a presentation:

$$G(3_1) = \langle x_1, x_2, x_3 \,|\, x_3 x_1 \bar{x}_3 \bar{x}_2,\ x_1 x_2 \bar{x}_1 \bar{x}_3 \rangle.$$

Table 1 gives the relators of each knot group $G(K)$ and the images of the generators of $G(K)$ in $G(3_1)$. We can check easily that the mappings are surjective homomorphisms.

Next, we construct surjective homomorphisms onto the knot group of the figure eight knot 4_1. The knot group of 4_1 has a presentation:

$$G(4_1) = \langle x_1, x_2, x_3, x_4 \,|\, x_4 x_2 \bar{x}_4 \bar{x}_1,\ x_1 x_2 \bar{x}_1 \bar{x}_3,\ x_2 x_4 \bar{x}_2 \bar{x}_3 \rangle.$$

Similarly, Table 2 gives surjective homomorphisms to $G(4_1)$.

Finally, we fix a presentation of $G(5_2)$:

$$G(5_2) = \langle x_1, x_2, x_3, x_4, x_5 \,|\, x_4 x_1 \bar{x}_4 \bar{x}_2,\ x_5 x_2 \bar{x}_5 \bar{x}_3,\ x_2 x_3 \bar{x}_2 \bar{x}_4,\ x_1 x_4 \bar{x}_1 \bar{x}_5 \rangle.$$

Surjective homomorphisms to $G(5_2)$ are described in Table 3. □

3. TWISTED ALEXANDER INVARIANTS OF KNOTS

In this section, we recall briefly the definition and some properties of the twisted Alexander invariants for knots. See [Wada 94] and [Kitano et al. 04] for a more precise definition in the general case of a finitely presentable group.

Let us take a Wirtinger presentation of a knot group $G(K)$ as follows:

$$G(K) = \langle x_1, x_2, \dots, x_u \,|\, r_1, r_2, \dots, r_{u-1} \rangle.$$

In this paper, the integers $\mathbb{Z}$ are identified with the multiplicative cyclic group $\langle t \rangle$. Then, by mapping each generator x_i to t, the abelianization

$$\alpha : G(K) \to \mathbb{Z} \simeq \langle t \rangle$$

is obtained. Now, we fix a prime integer p and take a representation

$$\rho : G(K) \to SL(2; \mathbb{F}_p).$$

Here, $\mathbb{F}_p$ is the finite field $\mathbb{Z}/p\mathbb{Z}$. The two maps ρ and α induce ring homomorphisms $\tilde{\rho} : \mathbb{Z}[G(K)] \to M(2; \mathbb{F}_p)$ and $\tilde{\alpha} : \mathbb{Z}[G(K)] \to \mathbb{Z}[t, t^{-1}]$, respectively. Then we get the tensor representation

$$\tilde{\rho} \otimes \tilde{\alpha} : \mathbb{Z}[G(K)] \to M(2; \mathbb{F}_p[t, t^{-1}]).$$

From a fixed Wiritinger presentation, a natural homomorphism $\mathbb{Z}[F_u] \to \mathbb{Z}[G(K)]$ is induced where F_u is the free group on generators $\{x_1, \dots, x_u\}$. Then a ring homomorphism

$$\Phi : \mathbb{Z}[F_u] \to M(2; \mathbb{F}_p[t, t^{-1}])$$

is defined by taking the composition of the above natural homomorhpism and $\tilde{\rho} \otimes \tilde{\alpha}$.

Now we define the matrix

$$M \in M((u-1) \times u; M(2; \mathbb{F}_p[t, t^{-1}]))$$

to be the $(u-1) \times u$ matrix whose (i, j)-component is $\Phi\left(\frac{\partial r_i}{\partial x_j}\right) \in M(2; \mathbb{F}_p[t, t^{-1}])$. Here $\partial/\partial x_j$ denotes the Fox derivation $\partial/\partial x_j : \mathbb{Z}[F_u] \to \mathbb{Z}[F_u]$ for each x_j.

Further, we write M_1 for the $(u-1) \times (u-1)$ matrix obtained from M by removing the first column. This matrix M_1 can be considered as a $2(u-1) \times 2(u-1)$-matrix whose entries belong to $\mathbb{F}_p[t, t^{-1}]$. Here, $\Delta^N_{K,\rho}(t)$ denotes the determinant of M_1 and $\Delta^D_{K,\rho}(t)$ the determinant of $\Phi(x_1 - 1)$. By using these polynomials, now we define the following:

Definition 3.1. The twisted Alexander invariant of $G(K)$ for a representation $\rho : G(K) \to SL(2;\mathbb{F}_p)$ is defined to be

$$\Delta_{K,\rho}(t) = \frac{\Delta^N_{K,\rho}(t)}{\Delta^D_{K,\rho}(t)} = \frac{\det M_1}{\det \Phi(x_1 - 1)}.$$

Remark 3.2. $\Delta_{K,\rho}(t)$ does not depend on the choice of Wiritinger presentation, up to a factor t^k ($k \in \mathbb{Z}$). Namely, the twisted Alexander invariant $\Delta_{K,\rho}(t)$ is well defined as an invariant of a triple (K, ρ, α). See [Wada 94].

By using the twisted Alexander invariants, a criterion for the existence of a surjective homomorphism between two knot groups is given as follows (it is proved in a more general setting in [Kitano et al. 04]): let K_1 and K_2 be two knots and α_1, α_2 surjective homomorphisms from the knot groups $G(K_1), G(K_2)$, respectively, to $\mathbb{Z}$. Suppose that there exists a surjective homomorphism $\varphi : G(K_1) \to G(K_2)$ such that $\alpha_1 = \alpha_2 \circ \varphi$.

Theorem 3.3. (Kitano-Suzuki-Wada.) *For any representation $\rho_2 : G(K_2) \to SL(2;\mathbb{F}_p)$ and $\rho_1 = \rho_2 \circ \varphi$, $\Delta_{K_1,\rho_1}(t)$ is divisible by $\Delta_{K_2,\rho_2}(t)$. More precisely, $\Delta^N_{K_1,\rho_1}(t)$ is divisible by $\Delta^N_{K_2,\rho_2}(t)$ and $\Delta^D_{K_1,\rho_1}(t) = \Delta^D_{K_2,\rho_2}(t)$.*

Remark 3.4. The corresponding fact about the classical Alexander polynomial is well known. Namely, if there exists a surjective homomorphism from $G(K_1)$ to $G(K_2)$, then the Alexander polynomial of K_1 is divisible by that of K_2. See [Crowell and Fox 77].

4. NONEXISTENCE OF SURJECTIVE HOMOMORPHISMS

In this section, we prove the nonexistence of a surjective homomorphism between the groups of any two knots except for the pairs listed in Theorem 1.1.

First, for the pairs of knots that do not appear in Theorem 1.1 or in Table 4, we can show easily that there exists no surjective homomorphism between their groups by using only the classical Alexander polynomial. Therefore, we need to consider only the pairs of knots in Table 4.

Theorem 4.1 is a direct consequence of Theorem 3.3.

Theorem 4.1. *If there exists a representation $\rho_2 : G(K_2) \to SL(2;\mathbb{F}_p)$ such that for any representation $\rho_1 : G(K_1) \to SL(2;\mathbb{F}_p)$, $\Delta^N_{K_1,\rho_1}(t)$ is not divisible by $\Delta^N_{K_2,\rho_2}(t)$ or $\Delta^D_{K_2,\rho_2}(t) \neq \Delta^D_{K_1,\rho_1}(t)$, then there exists no surjective homomorphism from $G(K_1)$ onto $G(K_2)$.*

By applying this theorem with the aid of a computer, we can prove that there exists no surjective homomorphism between any pair of knots in Table 4. This completes the proof of Theorem 1.1.

We describe how to read Table 4. First, there are knots with a prime integer in each row of Table 4. For example, $8_{11}(5)$ in the row of 3_1 means that the nonexistence of a surjective homomorphism from $G(8_{11})$ onto $G(3_1)$ is checked by using the twisted Alexander invariants of $SL(2;\mathbb{F}_5)$-representations.

All twisted Alexander invariants that we use to check Table 4 are listed in Table 5. To prove $K_1 \not\geq K_2$ by $SL(2;\mathbb{F}_p)$-representations, Table 5 shows $\Delta^N_{K_1,\rho_1}(t)$ and $\Delta^D_{K_1,\rho_1}(t)$ for all $SL(2;\mathbb{F}_p)$-representations of $G(K_1)$. Furthermore, $\Delta^N_{K_2,\rho_2}(t)$ and $\Delta^D_{K_2,\rho_2}(t)$, for a certain $SL(2;\mathbb{F}_p)$-representation of $G(K_2)$, are placed under $\Delta^N_{K_1,\rho_1}(t)$ and $\Delta^D_{K_1,\rho_1}(t)$. For any pair of $\Delta^N_{K_1,\rho_1}(t)$ and $\Delta^D_{K_1,\rho_1}(t)$ in each list, we can see that $\Delta^N_{K_1,\rho_1}(t)$ is not divisible by $\Delta^N_{K_2,\rho_2}(t)$ or that $\Delta^D_{K_2,\rho_2}(t) \neq \Delta^D_{K_1,\rho_1}(t)$. Then there exists no surjective homomorphism from $G(K_1)$ onto $G(K_2)$.

Remark 4.2. We note that the twisted Alexander invariants are invariant under changing a representation to any conjugate representation in the set of $SL(2;\mathbb{F}_p)$-representations. Therefore, we consider only conjugacy classes of representations.

By a similar argument to that of [Kitano 96] and [Kirk and Livingston 99], it is proved easily that the twisted Alexander invariant for an $SL(2;\mathbb{F}_p)$-representation of a knot is symmetric up to a factor t^k. It is clear that its denominator is symmetric, because it is the characteristic polynomial of the matrix $\rho(x_1)$. Hence, its numerator is also symmetric. So finally we obtain

$$\Delta^D_{K,\rho}(t) = t^{k_1}\Delta^D_{K,\rho}(t^{-1}),\ \Delta^N_{K,\rho}(t) = t^{k_2}\Delta^N_{K,\rho}(t^{-1}).$$

Therefore, in Table 5, an expression $a_0 + a_1 + a_2 + \cdots + a_n$ represents the symmetric polynomial $a_0 + a_1(t^{-1} + t) + a_2(t^{-2} + t^2) + \cdots + a_n(t^{-n} + t^n)$.

Remark 4.3. All the twisted Alexander invariants in Table 5 were calculated using the second author's computer program, and some of them (the numerators of the twisted Alexander invariants) were also calculated using the Kodama Knot program [Kodama 04].

5. TABLES

Tables 2 and 3 are included here in their entirety. Tables 1, 4, and 5 are included only partially here. The complete Tables 1–5 can be found at http://www.expmath.org/expmath/volumes/14/14.4/KitanoSuzuki/tables.pdf.

K	relators
	surjective homomorphism to 3_1
8_5	$72\bar7\bar1, 83\bar8\bar2, 64\bar6\bar3, 15\bar1\bar4, 36\bar3\bar5, 47\bar4\bar6, 28\bar2\bar7$
	$1 \mapsto 3, 2 \mapsto 2, 3 \mapsto 1, 4 \mapsto 3, 5 \mapsto 3, 6 \mapsto 2, 7 \mapsto 1, 8 \mapsto 3$
8_{10}	$72\bar7\bar1, 42\bar4\bar3, 63\bar6\bar4, 85\bar8\bar4, 35\bar3\bar6, 17\bar1\bar6, 28\bar2\bar7$
	$1 \mapsto 3, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 3, 5 \mapsto 3, 6 \mapsto 1, 7 \mapsto 13\bar1, 8 \mapsto 3$
8_{15}	$41\bar4\bar2, 82\bar8\bar3, 53\bar5\bar4, 24\bar2\bar5, 75\bar7\bar6, 16\bar1\bar7, 37\bar3\bar8$
	$1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 3, 4 \mapsto 13\bar1, 5 \mapsto 1, 6 \mapsto 2, 7 \mapsto 3, 8 \mapsto 3$
8_{18}	$41\bar4\bar2, 53\bar5\bar2, 63\bar6\bar4, 75\bar7\bar4, 85\bar8\bar6, 17\bar1\bar6, 27\bar2\bar8$
	$1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 1, 4 \mapsto 3, 5 \mapsto 3, 6 \mapsto 13\bar1, 7 \mapsto 3, 8 \mapsto 1$
8_{19}	$52\bar5\bar1, 83\bar8\bar2, 64\bar6\bar3, 15\bar1\bar4, 36\bar3\bar5, 17\bar1\bar6, 58\bar5\bar7$
	$1 \mapsto 3, 2 \mapsto 3, 3 \mapsto 1, 4 \mapsto 3, 5 \mapsto 3, 6 \mapsto 2, 7 \mapsto 1, 8 \mapsto 13\bar1$
8_{20}	$51\bar5\bar2, 72\bar7\bar3, 13\bar1\bar4, 75\bar7\bar4, 35\bar3\bar6, 47\bar4\bar6, 58\bar5\bar7$
	$1 \mapsto 2, 2 \mapsto \bar232, 3 \mapsto 3, 4 \mapsto 1, 5 \mapsto 3, 6 \mapsto 3, 7 \mapsto 2, 8 \mapsto 1$
8_{21}	$81\bar8\bar2, 73\bar7\bar2, 13\bar1\bar4, 74\bar7\bar5, 16\bar1\bar5, 86\bar8\bar7, 57\bar5\bar8$
	$1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 3, 4 \mapsto 1, 5 \mapsto 2, 6 \mapsto 2, 7 \mapsto 3, 8 \mapsto 1$
9_1	$61\bar6\bar2, 72\bar7\bar3, 83\bar8\bar4, 94\bar9\bar5, 15\bar1\bar6, 26\bar2\bar7, 37\bar3\bar8, 48\bar4\bar9$
	$1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 3, 4 \mapsto 1, 5 \mapsto 2, 6 \mapsto 3, 7 \mapsto 1, 8 \mapsto 2, 9 \mapsto 3$

TABLE 1. Surjective homomorphisms to 3_1.

K	relators
	surjective homomorphism to 4_1
8_{18}	$41\bar4\bar2, 53\bar5\bar2, 63\bar6\bar4, 75\bar7\bar4, 85\bar8\bar6, 17\bar1\bar6, 27\bar2\bar8$
	$1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 4, 4 \mapsto 1, 5 \mapsto 2, 6 \mapsto 3, 7 \mapsto 4, 8 \mapsto 1$
9_{37}	$81\bar8\bar2, 72\bar8\bar3, 94\bar9\bar3, 34\bar3\bar5, 16\bar1\bar5, 56\bar5\bar7, 27\bar2\bar8, 49\bar4\bar8$
	$1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 14\bar1, 4 \mapsto 3, 5 \mapsto 1, 6 \mapsto \bar141, 7 \mapsto 4, 8 \mapsto 1, 9 \mapsto 4$
9_{40}	$81\bar8\bar2, 73\bar7\bar2, 64\bar6\bar4, 24\bar2\bar5, 16\bar1\bar5, 96\bar9\bar7, 57\bar5\bar8, 49\bar4\bar8$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 2, 7 \mapsto 4, 8 \mapsto 1, 9 \mapsto \bar141$
10_{58}	$81\bar8\bar2, 42\bar4\bar3, 104\overline{10}\bar3, 24\bar2\bar5, 75\bar7\bar6, 97\bar9\bar6, 57\bar5\bar8, 18\bar1\bar9, 610\bar6\bar9$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 21\bar2, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 14\bar1, 7 \mapsto 4, 8 \mapsto 1,$ $9 \mapsto 1, 10 \mapsto 3$
10_{59}	$52\bar5\bar1, 93\bar9\bar2, 63\bar6\bar4, 15\bar1\bar4, 76\bar7\bar5, 36\bar3\bar7, 48\bar4\bar7, 108\overline{10}\bar9, 210\bar2\bar9$
	$1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 4, 4 \mapsto 1, 5 \mapsto 1, 6 \mapsto 3, 7 \mapsto 14\bar1, 8 \mapsto 4, 9 \mapsto 3, 10 \mapsto 2$
10_{60}	$51\bar5\bar2, 13\bar1\bar2, 93\bar9\bar4, 24\bar2\bar5, 36\bar3\bar5, 106\overline{10}\bar7, 68\bar6\bar7, 48\bar4\bar9, 79\bar7\overline{10}$
	$1 \mapsto 4, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 4, 7 \mapsto 1, 8 \mapsto 2, 9 \mapsto 2, 10 \mapsto 3$
10_{122}	$91\bar9\bar2, 83\bar8\bar2, 104\overline{10}\bar3, 14\bar1\bar5, 26\bar2\bar5, 46\bar4\bar7, 38\bar3\bar7, 59\bar5\bar8, 69\bar6\overline{10}$
	$1 \mapsto 2, 2 \mapsto 1, 3 \mapsto 1, 4 \mapsto 4, 5 \mapsto 3, 6 \mapsto 2, 7 \mapsto 1, 8 \mapsto 1, 9 \mapsto 4, 10 \mapsto 3$
10_{136}	$52\bar5\bar1, 63\bar6\bar2, 24\bar2\bar6, 94\bar9\bar5, 86\bar8\bar5, 37\bar3\bar6, 107\overline{10}\bar8, 19\bar1\bar8, 29\bar2\overline{10}$
	$1 \mapsto 21\bar2, 2 \mapsto 1, 3 \mapsto 4, 4 \mapsto \bar141, 5 \mapsto 2, 6 \mapsto 3, 7 \mapsto \bar323, 8 \mapsto 2\bar323\bar2,$ $9 \mapsto \bar121, 10 \mapsto 2$
10_{137}	$51\bar5\bar2, 13\bar1\bar5, 103\overline{10}\bar4, 24\bar2\bar5, 36\bar3\bar5, 86\bar8\bar7, 108\overline{10}\bar7, 18\bar1\bar9, 49\bar4\overline{10}$
	$1 \mapsto 2, 2 \mapsto 23\bar2, 3 \mapsto 3, 4 \mapsto 3, 5 \mapsto 21\bar2, 6 \mapsto 2, 7 \mapsto 1, 8 \mapsto 4,$ $9 \mapsto 3, 10 \mapsto 3$
10_{138}	$51\bar5\bar2, 13\bar1\bar2, 84\bar8\bar3, 24\bar2\bar5, 36\bar3\bar5, 106\overline{10}\bar7, 68\bar6\bar7, 39\bar3\bar8, 79\bar7\overline{10}$
	$1 \mapsto 4, 2 \mapsto 1, 3 \mapsto 2, 4 \mapsto 2, 5 \mapsto 3, 6 \mapsto 4, 7 \mapsto 1, 8 \mapsto 2, 9 \mapsto 2, 10 \mapsto 3$

TABLE 2. Surjective homomorphisms to 4_1.

K	relators
	surjective homomorphism to 5_2
10_{74}	$61\bar{6}\bar{2}, 42\bar{4}\bar{3}, 84\bar{8}\bar{3}, 104\bar{10}\bar{5}, 95\bar{9}\bar{6}, 16\bar{1}\bar{7}, 27\bar{2}\bar{8}, 39\bar{3}\bar{8}, 59\bar{5}\bar{10}$
	$1 \mapsto \bar{2}12, 2 \mapsto 2, 3 \mapsto 12\bar{1}, 4 \mapsto 1, 5 \mapsto 12\bar{1}, 6 \mapsto 4\bar{1}2, 7 \mapsto \bar{2}52, 8 \mapsto 5,$ $9 \mapsto 13\bar{1}, 10 \mapsto 5$
10_{120}	$51\bar{5}\bar{2}, 92\bar{9}\bar{3}, 13\bar{1}\bar{4}, 74\bar{7}\bar{5}, 35\bar{3}\bar{6}, 106\bar{10}\bar{7}, 47\bar{4}\bar{8}, 68\bar{6}\bar{9}, 29\bar{2}\bar{10}$
	$1 \mapsto 3, 2 \mapsto 4, 3 \mapsto 5, 4 \mapsto 1, 5 \mapsto 2, 6 \mapsto 3, 7 \mapsto 4, 8 \mapsto 5, 9 \mapsto 1, 10 \mapsto 2$
10_{122}	$91\bar{9}\bar{2}, 83\bar{8}\bar{2}, 104\bar{10}\bar{3}, 14\bar{1}\bar{5}, 26\bar{2}\bar{5}, 46\bar{4}\bar{7}, 38\bar{3}\bar{7}, 59\bar{5}\bar{8}, 69\bar{6}\bar{10}$
	$1 \mapsto 2, 2 \mapsto 2, 3 \mapsto 1, 4 \mapsto 5, 5 \mapsto 25\bar{2}, 6 \mapsto 5, 7 \mapsto 5, 8 \mapsto 4, 9 \mapsto 2, 10 \mapsto 3$

TABLE 3. Surjective homomorphisms to 5_2.

3_1	$8_{11}(5), 9_{29}(3), 9_{38}(3), 10_{59}(3), 10_{113}(3), 10_{122}(5), 10_{136}(3), 10_{147}(5)$
4_1	$8_{21}(3), 9_{12}(3), 9_{24}(3), 9_{39}(3)$
5_1	$10_{21}(5), 10_{62}(5), 10_{100}(5), 10_{132}(5)$
5_2	$9_{12}(5), 10_{65}(17), 10_{67}(5), 10_{77}(7), 10_{95}(5), 10_{111}(7)$
6_1	$8_{11}(7), 9_{37}(7), 9_{46}(11), 10_{21}(7), 10_{67}(7), 10_{74}(11), 10_{87}(7), 10_{98}(7), 10_{147}(11)$
6_2	$10_{111}(7), 10_{123}(7)$
6_3	$10_{95}(5), 10_{100}(5), 10_{159}(5)$
7_2	$8_{15}(3), 9_{39}(3)$
7_3	$9_{16}(3)$
7_4	$9_2(5), 9_{23}(7), 10_{120}(7)$

TABLE 4. Nonexistence of surjective homomorphism.

$(K_1 \ngeq K_2, p)$	$\left(\Delta^N_{K_1,\rho_1}(t), \Delta^D_{K_1,\rho_1}(t)\right)$
	$\left(\Delta^N_{K_2,\rho_2}(t), \Delta^D_{K_2,\rho_2}(t)\right)$
$(8_{11} \ngeq 3_1, 5)$	$(4+1+2+2, 1+1), (4+4+2+3, 4+1), (0+0+1+0+1, 0+1),$ $(4+1+4+2+2, 4+1), (4+4+4+3+2, 1+1), (1+0+3+0+4, 0+1),$ $(3+0+1+1+4, 4+1), (2+1+0+2+4, 3+1), (2+4+0+3+4, 2+1),$ $(3+0+1+4+4, 1+1)$
	$(2+2+1, 2+1)$
$(9_{29} \ngeq 3_1, 3)$	$(0+0+0+0+1+0+1, 0+1), (2+1+2+2+0+1+1, 2+1),$ $(1+1+0+0+1+1+1, 2+1), (2+2+2+1+0+2+1, 1+1),$ $(1+2+0+0+1+2+1, 1+1)$
	$(2+1+1, 1+1)$
$(9_{38} \ngeq 3_1, 3)$	$(1+0+0+0+1, 0+1), (0+0+2+0+1, 1+1), (0+0+2+0+1, 2+1),$ $(2+0+2+1+1, 1+1), (2+0+2+2+1, 2+1)$
	$(2+1+1, 1+1)$
$(10_{59} \ngeq 3_1, 3)$	$(1+0+2+0+1+0+1, 0+1), (2+1+1+2+1+1+1, 1+1),$ $(1+2+1+0+2+1+1, 1+1), (2+2+1+1+1+2+1, 2+1),$ $(1+1+1+0+2+2+1, 2+1)$
	$(2+1+1, 1+1)$
$(10_{113} \ngeq 3_1, 3)$	$(1+0+0+0+2+0+1, 0+1), (0+2+0+1+0+1+1, 1+1),$ $(0+1+0+2+0+2+1, 2+1), (2+0+0+0+0+0+2, 1+1),$ $(2+0+0+0+0+0+2, 2+1)$
	$(2+1+1, 1+1)$
$(10_{122} \ngeq 3_1, 5)$	$(2+2+4+2+0+1, 3+1), (4+4+4+0+4+1, 3+1),$ $(2+3+4+3+0+4, 2+1), (4+1+4+0+4+4, 2+1),$ $(3+0+0+0+0+0+4, 0+1), (2+0+0+0+3+0+4, 0+1),$ $(3+0+4+3+2+1+4, 3+1), (2+3+2+0+3+2+4, 1+1),$ $(4+1+1+3+4+2+4, 1+1), (2+2+2+0+3+3+4, 4+1),$ $(4+4+1+2+4+3+4, 4+1), (3+0+4+2+2+4+4, 2+1)$
	$(2+1+1, 1+1)$
$(10_{136} \ngeq 3_1, 3)$	$(1+0+0+0+1, 0+1), (2+2+1+1+1, 2+1), (2+0+2+1+1, 1+1),$ $(2+1+1+2+1, 1+1), (2+0+2+2+1, 2+1)$
	$(2+1+1, 1+1)$

TABLE 5. Twisted Alexander invariants.

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