Standard Generators for J_3

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CONTENTS

We present a set of generators for the sporadic group J_3 designed to allow easily reproducible computations in the group. We also discuss the relationship between J_3 and J_3:2, and the maximal subgroups of these two groups.

1. INTRODUCTION

The concept of "standard generators" for sporadic simple groups is introduced in [Wilson], as a device for improving reproducibility of computational results and for avoiding duplication of work. In fact, these standard generators have been chosen fairly arbitrarily in each case, but they are always chosen so that they are easy to reconstruct in (as far as possible) any representation.

Here we develop these ideas further, in the context of the simple group J_3. This group was chosen firstly because it has an outer automorphism group of order 2, which introduces extra complications, and secondly because it is reasonably small (it has order 50,232,960) so we can do quite a large number of calculations in the group. Our main aims at this stage are:

1. To pass from J_3:2 to J_3 and (as far as possible) vice versa.
2. To find words in the standard generators for each group, giving representatives for each of the conjugacy classes of elements.
3. To find words in the standard generators which generate representatives of each of the conjugacy classes of maximal subgroups.

Eventually we hope to develop a computerised library of sporadic simple groups [Suleiman et al.], containing matrix and permutation representations of each group, as well as their covering groups and

1058-6458/96 $0.50 per page

automorphism groups, and a whole series of procedures for finding interesting subgroups, elements, and so on. The groups J_1, M_{24}, Ru, Co_3, Co_2, J_2, M_{22} and their automorphism groups have been treated in this way by P. G. Walsh [1994]. His procedures have been implemented in Cayley [Cannon 1984], and implementations in Magma [Cannon and Playoust 1993] and GAP [Schönert et al. 1994] are planned. The present paper is designed as a case study for this development, to help clarify our ideas about what such a library should contain, what is feasible and what is not, and to explore some possible avenues for extensions to the library. See the section on Electronic Availability at the end of this article.

2. STANDARD GENERATORS FOR $J_3{:}2$

Our initial idea for standard generators for $J_3{:}2$ was to take the rationally rigid triple of conjugacy classes $(2B, 3B, 8B)$. In other words, we took $g_1 \in 2B$ and $g_2 \in 3B$ such that $g_1 g_2 \in 8B$. (Here we follow [Wilson] in using (g_1, g_2) generically to denote a pair of standard generators for whatever group is under consideration.) This defines the pair (g_1, g_2) up to conjugacy, and it can be shown that $\langle g_1, g_2\rangle = J_3{:}2$. The most obvious problem with this is that there are two classes of elements of order 8 in the outer half of $J_3{:}2$, called $8B$ and $8C$, and in some representations it is very difficult to distinguish them. We therefore abandoned the idea of using a rationally rigid triple, as rational rigidity seems to be of more theoretical than practical importance.

Instead there are two much more crucial practical issues. The first is to maximise the probability of obtaining a conjugate of (g_1, g_2) at each attempt. If $g_1 \in X$ and $g_2 \in Y$ and $x \in X$ and $y \in Y$, where X, Y are two conjugacy classes in the group G, then the probability that (x, y) is conjugate to (g_1, g_2) is just

$$\frac{|C_G(g_1)|\,|C_G(g_2)|}{|G|\,|Z(G)|}.$$

Clearly we want to maximise this probability, subject to the constraint that $\langle g_1, g_2\rangle = G$.

The second issue is to make it as easy as possible to distinguish the standard generators from any non-conjugate pair of elements of the group. This is not usually much of a problem if the first issue has been satisfactory dealt with.

After some experimentation we decided to take $g_1 \in 2B$, $g_2 \in 3A$, with $g_1 g_2$ of order 24. There are still two classes of elements of order 24 in $J_3{:}2$, but each is an algebraic conjugate of the other. The symmetrised structure constants, which may be defined by

$$\xi_G(X, Y, Z) = \sum_{\chi \in \mathrm{Irr}(G)} \frac{\chi(x)\chi(y)\chi(z)}{\chi(1)}$$

with $x \in X$, $y \in Y$ and $z \in Z$, are

$$\xi_{J_3:2}(2B, 3A, 24A) = \xi_{J_3:2}(2B, 3A, 24B) = 1.$$

(Thus these triples are rigid, but not rationally rigid.) Given a pair (a, b) of old standard generators (that is, $a \in 2B$, $b \in 3B$, with $ab \in 8B$), we found that $e = (abab^2ab(ab^2)^2)^8 \in 3A$. By making several conjugates of a and e we eventually found pairs (c, e) and (d, e), given by $c = a^{(ab)^3}$ and $d = a^{(ab)^6}$, with the following properties:

- $c \in 2B$, $e \in 3A$, ce has order 24, and $[c, e]$ has order 9.
- $d \in 2B$, $e \in 3A$, de has order 24, and $[d, e]$ has order 17.

It is easy to check also that $\langle c, e\rangle = \langle d, e\rangle = J_3{:}2$.

In this way we have found representatives for both triples of type $(2B, 3A, 24)$, and we choose arbitrarily the first to be our standard generators for $J_3{:}2$. As we shall see later, we may choose the notation so that $ce \in 24A$ and $de \in 24B$. We give in Table 1 two 18×18 matrices over GF(2), representing preimages of these (new) standard generators of $J_3{:}2$ in the smallest matrix representation of $3^{\cdot}J_3{:}2$. At the referee's suggestion, these matrices are given in a basis such that the GF(4)-structure of the representation for the subgroup $3^{\cdot}J_3$ is easily

000100000000000000	100001101111010011
001000000000000000	010011011010110010
010000000000000000	001001100110100011
100000000000000000	000111011101010010
101011000000000000	001101011110001001
111101000000000000	001011111001000111
101000110000000000	110110011011110000
111100010000000000	101101110110100000
011111111100000000	110110000100011001
100101010100000000	101101001100110111
110111110011000000	101111000011101100
011001010001000000	011010000010011000
101000111111110000	001100001101000011
111100010101010000	001000001011000010
101011001111001100	101010101001101110
111101000101000100	010101010111011001
010000110011111111	111010110100010111
100000010001010101	100101101100111110
$g_1 = c$	$g_2 = e$

TABLE 1. Standard generators for $3{\cdot}J_3{:}2$

visible. Each 2×2 block of a matrix in this subgroup can be interpreted as an element of GF(4) by the identifications $0 = \left(\begin{smallmatrix}0&0\\0&0\end{smallmatrix}\right)$, $1 = \left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$, $\omega = \left(\begin{smallmatrix}1&1\\1&0\end{smallmatrix}\right)$, $\bar\omega = \left(\begin{smallmatrix}0&1\\1&1\end{smallmatrix}\right)$, so that we get 9×9 matrices over GF(4).

3. UPWARDS COMPATIBILITY

Our old standard generators for $J_3{:}2$ have been available in a prerelease version of a computer library of groups [Suleiman et al.] for some time, and some work may have been based on them. We therefore found words in the new generators that give conjugates of the old ones, so that such work will still be easily reproducible.

Given our standard generators c, e, we found that $(cece^2)^6 \in 3B$, and if we define $a' = c^{(ce)^3}$ and $b' = ((cece^2)^6)^{(ce^2)^5}$, the pair (a', b') is conjugate to (a, b). To prove this, we took the original generators (a, b) and made new generators (a', b') from them as described. Then we put both (a, b) and (a', b') into standard form ("standard basis") as described in [Parker 1984], and observed that the representing matrices for (a', b') were identical to those for (a, b).

Also, if we define $d' = c^{(ce)^4}$ and $e' = e^{(ce^2)^5}$, then (d', e') is conjugate to (d, e).

4. STANDARD GENERATORS FOR J_3

As standard generators for the simple group J_3 we decided to take $g_1 \in 2A$ and $g_2 \in 3A$ such that $(g_1 g_2)^{19} = 1$. Since the symmetrised structure constants are

$$\xi_{J_3}(2A, 3A, 19A) = \xi_{J_3}(2A, 3A, 19B) = 2,$$

there are four such pairs of generators up to conjugacy. This reduces to just two pairs of generators up to automorphisms. We found one pair as (f, g) where $f = (ce)^{12}$ and $g = e^{cece^2}$, and the second as (f, h), where $h = g^{(fg^2)^4}$. We then have

- $f \in 2A$, $g \in 3A$, $(fg)^{19} = 1$, $[f, g]$ has order 9,
- $f \in 2A$, $h \in 3A$, $(fh)^{19} = 1$, $[f, h]$ has order 17.

We choose arbitrarily the first of these to be our standard generators for J_3.

5. FROM J_3 TO $J_3{:}2$

The problems here are of quite a different kind from those considered elsewhere in this paper. There we are working within a particular group, looking for various elements and subgroups. Here we have to go outside the starting group J_3, and look inside a larger "universal" group for an element extending J_3 to $J_3{:}2$. If we start with a permutation representation of J_3 of degree n, the appropriate universal group is the symmetric group of degree n. If we start with a matrix representation of degree d over GF(q), the appropriate group is $\mathrm{GL}_d(q)$.

Our standard generators for J_3 were defined by $f \in 2A$, $g \in 3A$, $(fg)^{19} = 1$, with $[f, g]$ of order 9. This defines the pair (f, g) uniquely up to automorphisms, but there are two conjugacy classes of such pairs in the simple group J_3. Note that $f^{-1}g^{-1} = (gf)^{-1} = ((fg)^{-1})^f$, so $f^{-1}g^{-1}$ has order 19, and

$$\begin{aligned}[f^{-1}, g^{-1}] = fgf^{-1}g^{-1} &= (f^{-1}g^{-1}fg)^{g^{-1}f^{-1}} \\ &= [f, g]^{g^{-1}f^{-1}},\end{aligned}$$

$fg(fgfg^2)^2 \in 8A$	$(fg)^4gfg(fg^2)^2 \in 10\boldsymbol{A}B$	$(fg)^3gfg(fg^2)^2 \in 15A\boldsymbol{B}$	$fg \in 19\boldsymbol{A}B$
$fgfg^2 \in 9A\boldsymbol{BC}$	$((fg)^3g(fg)^2g)^2fg \in 12A$	$(fg)^3g \in 17\boldsymbol{A}B$	

TABLE 2. Representatives of the maximal cyclic subgroups of J_3. We indicate in boldface the actual class that the given element belongs to, as determined below.

so $[f^{-1},g^{-1}]$ has order 9. Thus $f^{-1} \in 2A$, $g^{-1} \in 3A$, $(f^{-1}g^{-1})^{19} = 1$, and $[f^{-1},g^{-1}]$ has order 9, so $(f^{-1},g^{-1}) = (f,g^2)$ is also a pair of standard generators for J_3. However, if we (arbitrarily) choose the notation so that $fg \in 19A$, then $f^{-1}g^{-1}$ is conjugate to $(fg)^{-1}$, so $f^{-1}g^{-1} \in 19B$. This means that (f^{-1},g^{-1}) is automorphic to (f,g), but not conjugate to (f,g).

The crucial step in the construction of $J_3{:}2$ is therefore to find an element τ of the universal group conjugating (f,g) to (f^{-1},g^{-1}).

In the context of matrix groups, there is a well-known method, based on the standard basis concept introduced in [Parker 1984]. In essence, a matrix B is found that conjugates f and g to standard form F and G, say. Similarly, we find C that conjugates f^{-1} and g^{-1} to F and G. Then $\tau = BC^{-1}$ conjugates (f,g) to (f^{-1},g^{-1}), and $\langle f,g,\tau\rangle$ is isoclinic to $J_3{:}2$. It may not be equal to $J_3{:}2$, because we may have introduced some additional elements centralizing J_3, but it is usually straightforward to get rid of such elements. A similar idea can be used with permutation groups.

Finally, one can find words in the generators f,g,τ that are conjugate (modulo the centre of the group) to the standard generators c,e of $J_3{:}2$ defined above. For example, here we may take $c' = \tau$ and $e' = (fg^2)^{-4}g(fg^2)^4$.

6. CONJUGACY CLASSES OF ELEMENTS

First we find generators for the maximal cyclic subgroups, and later we consider problems of algebraic conjugacy.

In J_3, the maximal cyclic subgroups are as follows: $8A$, $9ABC$, $10AB$, $12A$, $15AB$, $17AB$, and $19AB$. In terms of the standard generators (f,g), we have representatives as in Table 2.

In $J_3{:}2$, the maximal cyclic subgroups are $8A$, $10AB$, $15AB$, $19AB$, $8C$, $12B$, $18ABC$, $24AB$, $34AB$. In terms of the standard generators (c,e), we have representatives as in Table 3. (Note that each of $19AB$, $15AB$ and $10AB$ is a single conjugacy class.)

If we consider only the information about conjugacy classes given in the *Atlas of Finite Groups* [Conway et al. 1985], we can choose most of these classes arbitrarily. There are just two provisos: we should be consistent between J_3 and $J_3{:}2$, and our choice should be consistent with the power maps (specifically, the square of class $10A$ and the cube of class $15A$ are both $5B$ rather than $5A$, so the choices of $10A$ and $15A$ are not independent).

In the *Atlas of Brauer Characters* [Jansen et al. 1995], however, much more precise definitions of the conjugacy classes are used (see also [Wilson 1993]). In particular, a distinction is made between the three classes $9A$, $9B$ and $9C$, and between the classes $17A$ and $17B$, in the 19-modular character table of J_3. Using the character tables in [Jansen et al. 1995] and explicitly calculating the traces of elements in the 110-dimensional representation over GF(19), we find that $(fg)^3g \in 17A$ and $fgfg^2 \in 9B$, so $((fg)^3g)^3 \in 17B$, and $(fgfg^2)^2 \in 9C$ and $(fgfg^2)^4 \in 9A$. No distinction is made between the other pairs of algebraically conjugate classes, so we can choose $fg \in 19A$ and

$(ce^2)^2(ce)^6 \in 8A$	$(ce)^4e(ce)^2e \in 10AB$	$(ce)^{10}e \in 15AB$	$(ce)^2(ce^2)^2 \in 19AB$	$(ce)^3e \in 34A\boldsymbol{B}$
$cece^2(ce)^5ce^2ce(ce^2)^4 \in 8C$	$(ce^2)^2(ce)^9 \in 12B$	$(ce)^5e \in 18A\boldsymbol{BC}$	$ce \in 24\boldsymbol{A}B$	

TABLE 3. Representatives of the maximal cyclic subgroups of $J_3{:}2$. As in Table 2, the actual class is printed in boldface.

$(fg)^4gfg(fg^2)^2 \in 10A$ without loss of generality. Then we are forced to have $(fg)^3gfg(fg^2)^2 \in 15B$, using the 18-dimensional representation over GF(9) to distinguish the two classes of elements of order five.

Similarly, in $J_3{:}2$ the three sets of classes $34A/B$, $24A/B$ and $18A/B/C$ are distinguished in the 19-modular table. We find in the same way as before that $(ce)^3e \in 34B$, $ce \in 24A$, and $(ce)^5e \in 18B$. The classes $19AB$, $15AB$ and $10AB$ are actually single classes in $J_3{:}2$, so no problem arises there. A complete set of words giving representatives for all the conjugacy classes of elements is given in Tables 4 and 5.

class	word	class	word
$1A$	f^2	$9C$	$(fgfg^2)^2$
$2A$	f	$10A$	$(fg)^4gfg(fg^2)^2$
$3A$	g	$10B$	$((fg)^4gfg(fg^2)^2)^3$
$3B$	$(fgfg^2)^3$	$12A$	$((fg)^3g(fg)^2g)^2fg$
$4A$	$(fg(fgfg^2)^2)^2$	$15A$	$((fg)^3gfg(fg^2)^2)^2$
$5A$	$((fg)^4gfg(fg^2)^2)^4$	$15B$	$(fg)^3gfg(fg^2)^2$
$5B$	$((fg)^4gfg(fg^2)^2)^2$	$17A$	$(fg)^3g$
$6A$	$(((fg)^3g(fg)^2g)^2fg)^2$	$17B$	$((fg)^3g)^3$
$8A$	$fg(fgfg^2)^2$	$19A$	fg
$9A$	$(fgfg^2)^4$	$19B$	$(fg)^{-1}$
$9B$	$fgfg^2$		

TABLE 4. Words for conjugacy classes of elements in J_3.

class	word	class	word
$1A$	c^2	$9C$	$((ce)^5e)^2$
$2A$	$(ce)^{12}$	$10AB$	$(ce)^4e(ce)^2e$
$2B$	c	$12A$	$(ce)^2$
$3A$	e	$12B$	$(ce^2)^2(ce)^9$
$3B$	$((ce)^5e)^6$	$15AB$	$(ce)^{10}e$
$4A$	$(ce)^6$	$17A$	$((ce)^3e)^6$
$4B$	$((ce^2)^2(ce)^9)^3$	$17B$	$((ce)^3e)^2$
$5AB$	$((ce)^4e(ce)^2e)^2$	$18A$	$((ce)^5e)^5$
$6A$	$(ce)^4$	$18B$	$(ce)^5e$
$6B$	$((ce)^5e)^3$	$18C$	$((ce)^5e)^7$
$8A$	$(ce^2)^2(ce)^6$	$19AB$	$(ce)^2(ce^2)^2$
$8B$	$(ce)^3$	$24A$	ce
$8C$	$cece^2(ce)^5ce^2ce(ce^2)^4$	$24B$	$(ce)^7$
$9A$	$((ce)^5e)^4$	$34A$	$((ce)^3e)^3$
$9B$	$((ce)^5e)^8$	$34B$	$(ce)^3e$

TABLE 5. Words for conjugacy classes of elements in $J_3{:}2$.

7. MAXIMAL SUBGROUPS OF $J_3{:}2$

We have already seen how to obtain standard generators for J_3 from standard generators for $J_3{:}2$. The other maximal subgroups of $J_3{:}2$ are:

$L_2(16){:}4$, $2^4(3 \times A_5).2$, $19{:}18$, $L_2(17) \times 2$, $(3 \times M_{10}){:}2$, $3^2.3^{1+2}.SD_{16}$, $2_-^{1+4}S_5$, $2^{2+4}(S_3 \times S_3)$

(see [Conway et al. 1985], or [Finkelstein and Rudvalis 1974; Wilson 1985]).

With a group of this size, the easiest way to find copies of most of these subgroups is by a random search. For example, if $x \in 2A$ and $y \in 4B$, we can estimate the probability that x and y generate $L_2(16){:}4$ as being approximately 1 in 50. An example of such a pair of generators is given in Table 6.

Similarly, $2^4(3 \times A_5).2$ and $3^2.3^{1+2}.SD_{16}$ can be generated by elements $x \in 2B$ and $y \in 4B$, with reasonable probabilities. The subgroup 19:18 is rather more difficult to find: with a random search of this kind the best we can do is to take $x \in 2B$ and y of order 9, giving a probability around 1 in 300. Similarly we can generate $2^{2+4}(S_3 \times S_3)$ by elements $x \in 4B$ and $y \in 6B$.

Two of the subgroups, namely $L_2(17) \times 2$ and $2^{1+4}S_5$, are involution centralisers, for which another method is available, which is often quicker, although it tends to produce longer words. Take for example the case $L_2(17) \times 2$, which is the centraliser of a $2B$-involution. We start with a random element of order 34, such as $(ce)^3e$, so that $((ce)^3e)^{17} \in 2B$. Then we take another involution (preferably not conjugate to the first one) such as $(ce)^{12} \in 2A$. Then these two involutions generate a dihedral group of order $4n$, for some n, and the central involution of this dihedral group clearly commutes with our original involution. In this case we found that $((ce)^3e)^{17}(ce)^{12}$ had order 8, so its fourth power is the required centralizing involution. It turned out that this element, together with the original element of order 34, was enough to generate the whole involution centraliser.

The remaining maximal subgroup is $(3 \times M_{10}){:}2$. To generate such a group we used a method similar

subgroup	generators	
J_3	$(ce)^{12}$	e^{cece^2}
$L_2(16){:}4$	$((ce(cece^2)^2)^2)^{ce}$	$(ce(cece^2)^2)^{(ce^2)^5}$
$19{:}18$	c^{ecece^2}	$(cece^2)^{((ce)^2(cece^2)^2)^6}$
$2^4{:}(3 \times A_5).2$	c	$(ce(cece^2)^2)^{(ce^2)^{12}}$
$L_2(17) \times 2$	$((ce)^{12}((ce)^3e)^{17})^4$	$(ce)^3e$
$(3 \times M_{10}){:}2$	$(uv)^2(uvuv^2)^2uv^2$	ce
$3^2.3^{1+2}.SD_{16}$	$c^{(ce)^3}$	$(ce(cece^2)^2)^{(ce^2)^3}$
$2^{1+4}S_5$	$(c^{ece^2}(ce)^{12})^{17}$	ce
$2^{2+4}(S_3 \times S_3)$	$(ce(cece^2)^2)^{(ce)^{12}}$	$((ce)^2(cece^2)^2ce^2)^{(ce^2)^8}$

TABLE 6. Words for maximal subgroups of $J_3{:}2$. The symbol u stands for $((ce)^{12})^{(ce^2)^2}$, and v stands for $(ce)^8$.

to the one we used to find involution centralisers. We took an element x of order 24, whose eighth power is an element $x^8 \in 3A$ and looked at groups $\langle x^8, y\rangle$ where y is a random element in $2A$. We found such a group of order 48, in which it was easy to find an involution z inverting x^8, such that $\langle x, z\rangle \cong (3 \times M_{10}){:}2$.

8. MAXIMAL SUBGROUPS OF J_3

The same principles apply here. All maximal subgroups were found by random searches, apart from the involution centraliser.

It is worth remarking that for the time being we have contented ourselves with finding arbitrary generators for a representative of each class of maximal subgroups. However, it is obviously desirable to have some kind of standardisation of the subgroup generators as well. This seems to be quite a tall order in general, but if the subgroup is (almost) simple, then we can go some way towards this. For example, one might wish to take generators for $L_2(17)$ and $L_2(19)$ to be images of the "standard" generators $\left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 1 & 1 \\ -1 & 0 \end{smallmatrix}\right)$ for $\mathrm{SL}_2(p)$, though we have not done this here.

It might also be worth considering more carefully which representative of a conjugacy class to give, but as yet there seem to be no clear reasons for choosing one rather than another. A related issue is compatibility between subgroups of J_3 and subgroups of $J_3{:}2$. For example, we may wish to arrange matters so that if we make the subgroups $J_3{:}2 > J_3 > L_2(17)$ and $J_3{:}2 > L_2(17) \times 2 > L_2(17)$, then we end up with the same generators for the same subgroup $L_2(17)$ in both cases. Again, we have not done this here, but these ideas are

subgroup	generators	
$L_2(16){:}2$	f	$(fg(fgfg^2)^2)^6$
$L_2(19)$	f^g	$((fgfg^2)^3)^{(fg^2)^4}$
$L_2(19)$	f^{g^2}	$((fg^2fg)^3)^{(fg)^4}$
$2^4{:}(3 \times A_5)$	$(fgfg^2)^3$	$g^{(fg^2)^8}$
$L_2(17)$	f^{gfg}	$((fgfg^2)^3)^{(fg^2)^5}$
$(3 \times A_6){:}2$	f^g	$g^{(fg^2)^2}$
$3^2.3^{1+2}{:}8$	$f^{(fg)^9}$	$(fg(fgfg^2)^2)^{(fg^2)^6}$
$2^{1+4}{:}A_5$	$(fg)^3fg^2fg(fg^2)^2$	$((fg(fgfg^2)^2)^{12}((fg)^4gfg(fg^2)^2)^{15})^3$
$2^{2+4}{:}(3 \times S_3)$	$f^{(fg)^4}$	$((fgfg^2(fg)^4(fg^2fg(fg^2)^2)^2)^2)^{(fg^2)^{10}}$

TABLE 7. Words for maximal subgroups of J_3.

explored further by P. G. Walsh [1994]. Indeed, all of these ideas could obviously be taken a lot further, but we feel at this stage it is preferable not to be too prescriptive.

9. REPRESENTATIONS OF J_3:2

The easiest place to start making representations of J_3 and of the triple cover and automorphism group, is with the 18-dimensional representation of $3^{\cdot}J_3$:2 over GF(2) given in Table 1. The skew square of this representation has degree 153, which contains the two 36-dimensional irreducibles for $3^{\cdot}J_3$:2, as well as the 80-dimensional irreducible for J_3:2. Other 2-modular representations can then be made using the Meat-Axe in usual way [Parker 1984].

Some primitive permutation representations can be made as the actions on certain orbits of vectors in the 18-dimensional space. For example, the representation of J_3 of degree 20520, on the cosets of $L_2(17) \times 2$, can be made by looking at the 61560 images of the (unique) fixed vector of an element of order 34. Similarly the representation of degree 23256, on the cosets of $(3 \times M_{10})$:2, and the representation of degree 43605, on the cosets of

$$2^{2+4}{:}(S_3 \times S_3),$$

can be found by looking at the action on suitable orbits of vectors. Once this is done, we can make representations in other characteristics by chopping up these permutation representations with the help of the condensation method [Wilson 1993].

ACKNOWLEDGEMENTS

The authors would like to thank the British Council and the Universities of Birmingham and Mu'tah for their support. This research forms part of a British Council Link Project in Algebra and Computing set up between the two universities. The computing facilities were provided by the two universities, and enhanced by a grant from the UK Science and Engineering Research Council.

REFERENCES

[Cannon 1984] J. J. Cannon, "Cayley: A Language for Group Theory", pp. 145–183 in *Computational Group Theory: Proceedings of the London Mathematical Society Symposium* (edited by M. D. Atkinson), Academic Press, London, 1984.

[Cannon and Playoust 1993] J. J. Cannon and C. Playoust, *An Introduction to MAGMA*, University of Sydney, 1993.

[Conway et al. 1985] J. H. Conway, R. T. Curtis, S. P. Norton, R. A. Parker and R. A. Wilson, *Atlas of Finite Groups: Maximal Subgroups and Ordinary Characters for Simple Groups*, Oxford University Press, Oxford, 1985.

[Finkelstein and Rudvalis 1974] L. Finkelstein and A. Rudvalis, "The maximal subgroups of Janko's simple group of order 50232960", *J. Algebra* **30** (1974), 122–143.

[Jansen et al. 1995] C. Jansen, K. Lux, R. A. Parker and R. A. Wilson, *An Atlas of Brauer Characters*, Oxford University Press, Oxford, 1995.

[Parker 1984] R. A. Parker, "The computer calculation of modular characters (The 'Meat-axe')", pp. 267–274 in *Computational Group Theory: Proceedings of the London Mathematical Society Symposium* (edited by M. D. Atkinson), Academic Press, London, 1984.

[Schönert et al. 1994] M. Schönert et al., *GAP: Groups, Algorithms, and Programming*, Lehrstuhl D für Mathematik, RWTH Aachen, Germany, 1994. Available by anonymous ftp, together with the GAP system, on the servers dimacs.rutgers.edu or math.rwth-aachen.de.

[Suleiman et al.] I. A. I. Suleiman, P. G. Walsh and R. A. Wilson, "A computerised library of sporadic groups", in preparation. See section on Electronic Availability below.

[Walsh 1994] P. G. Walsh, "Standard generators of some sporadic simple groups", M. Phil. thesis, University of Birmingham, 1994.

[Wilson 1985] R. A. Wilson, "Maximal subgroups of automorphism groups of simple groups", *J. London Math. Soc.* **32** (1985), 460–466.

[Wilson 1990] R. A. Wilson, "The 2- and 3- modular characters of J_3, its covering group and automorphism group", *J. Symbol. Comp.* **10** (1990), 647–656.

[Wilson 1993] R. A. Wilson, "The Brauer tree for J_3 in characteristic 17", *J. Symbol. Comp.* **15** (1993), 325–330.

[Wilson] R. A. Wilson, "Standard generators for sporadic simple groups", Preprint 94/21, School of Mathematics and Statistics, The University of Birmingham. To appear in *Journal of Algebra*.

ELECTRONIC AVAILABILITY

A prerelease version of the computerised library of sporadic groups [Suleiman et al.] has been used by several researchers. At present it contains over 120 representations, including at least one for every sporadic group, covering group or automorphism group, except for the Monster and the double cover of the Baby Monster.

For information, or for copies of these representations, send e-mail to R.A.Wilson@bham.ac.uk.

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Received December 24, 1994; accepted in revised form April 28, 1995