## REFERENCES

1. A. Erdélyi, *Asymptotic forms for Laguerre polynomials*, J. Indian Math. Soc. **24** (1960), 225–250.
2. R. A. Handelsman and J. S. Lew, *Asymptotic expansion of Laplace transforms near the origin*, SIAM J. Math. Anal. **1** (1970), 118–129.
3. F. W. J. Olver, *Asymptotics and special functions*, Academic Press, New York, 1974.
4. F. Tricomi, *Sul comportamento asintotico dell'n-esimo polinomio di Laguerre nell'intorno dell'ascissa* $4n$, Comm. Math. Helv. **22** (1949), 150–167.
5. R. Wong (ed.), *Asymptotic and computational analysis*, Lecture Notes in Pure and Appl. Math. Conference in Honor of Frank W. J. Olver's 65th Birthday, vol. 124, Marcel Dekker, New York, 1990.

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*Generatingfunctionology*, by Herbert S. Wilf. Academic Press, Harcourt Brace Jovanovich, 182 pp., $29.95. ISBN 0-12-751955-6

I suppose that all of us had at some stage to struggle with poker hands. What is the chance of a hand of five cards containing a run of four red cards? Having gotten the answer, one was never quite sure that it was right and could easily be persuaded by a more expert friend that it should be something quite different.

If a topologist is a man who knows the difference between an orange and a bicycle tire, then a combinatoric is a man who can find the probability of a poker hand in 1-minute flat. The problems dealt with in this book are at first easy but later become quite a bit harder than the above. Consider the following from p. 65:

**Definition.** A card $C(S, p)$ is a pair consisting of a finite set $S$ (the label set) of positive integers and a picture $p \in P$. The weight of $C$ is $n = |S|$. A card of weight $n$ is called standard if its label set is $[n] = \{1, 2, \dots, n\}$. A hand is a set of cards whose label sets form a partition of $[n]$ for some $n$. The weight of a hand is the sum of the weights of the cards in the hand. A deck $\mathscr{D}$ is a finite set of standard cards, whose weights are all the same and whose pictures are all different. An exponential family $\mathscr{F}$ is a collection of decks $\mathscr{D}_1, \mathscr{D}_2, \ldots$ where for each $n = 1, 2, \ldots,$ the deck $\mathscr{D}_n$ is of weight $n$.

Is this all perfectly clear? Then you are well on the way to becoming a combinatoric and you will at least understand the problems. That is the hardest part.

However, it is only in Chapter 3 and to some extent Chapter 4, when one has to struggle with concepts such as the above. In Chapters 1,2, and 5 the author gives a beautiful account of how to solve simple and harder problems by manipulation of power series. These techniques are what the book is about.

Let $a_n$ be a sequence of numbers, generally positive integers. The author associates with $a_n$ three generating functions.

(i) The ordinary power series (ops) $f(x) = \sum_0^\infty a_n x^n$;
(ii) The exponential generating function (egf) $F(x) = \sum_0^\infty \frac{a_n}{n!} x^n$;
(iii) The Dirichlet series generating function (Dsgf) $f(s) = \sum_1^\infty a_n n^{-s}$.

If $a$ depends on several variables, e.g. $a_{m,n}$, we can have multiple series. There are simple rules on how the coefficients are affected by multiplication, differentiation, exponentiation, and other operations on the series and these techniques lead quickly to solutions of even quite complicated problems.

**Example.** Let $F_n$ be the Fibonacci sequence given by $a_0 = a_1 = 1$ and $a_{n+1} = a_n + a_{n-1}$. If $f(x) = \sum_0^\infty a_n x^n$, then $(1 - x - x^2)f(x) = 1$, and this leads via partial fractions to

$$a_n = (r_+^{n+1} - r_-^{n+1})/\sqrt{5}, \quad \text{where } r_\pm = (1 \pm \sqrt{5})/2.$$

The above series can be purely formal, i.e. they need not converge, but if they do it is a bonus.

Dsgf's are most useful in dealing with multiplicative number theoretic functions via Möbius's inversion formula.

In most of the book the emphasis is on simple exact formulae for the solutions. However in the last chapter, the generating function is used to prove asymptotic formulae for the growth by means of Cauchy's formula

$$a_n = \frac{1}{2\pi i} \int_{|z|=r} \frac{f(z)\, dz}{z^{n+1}}.$$

The author stops short of the Hardy-Littlewood method and its powerful application to Waring's problem and other number theoretic representation problems.

Wilf's book is beautifully written. The witty and slightly folksy style (one useful technique is called "the Snake Oil Method," because it has so many applications) hides the real depth of many of the results. There are masses of examples either worked out in the book or left for the reader. In the latter case the solutions are given in the back of the book. Anyone who enjoys combinatorics problems, or who likes messing about with power series and seeing what identities can be obtained that way, will get much pleasure from this book.

W. K. HAYMAN
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*Computability*; (*computable functions, logic, and the foundations of mathematics*), by Richard L. Epstein and Walter A. Carnielli. Wadsworth & Brooks/Cole, Pacific Grove, California, 1989, 295 pp. ISBN 0-534-10356-1

Euler knew perfectly well what a function was. It was defined by an expression showing the operations to be performed in order to obtain the value for a given argument. These operations could involve limits, but nevertheless the expressions had a clear computational meaning. Indeed, for Euler and his contemporaries, integrals and series made it possible to "discover" hitherto unknown functions: elliptic integrals, complex exponentials, Bessel functions. But there was a problem with trigonometric series. For example, the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

had what appeared to be a general solution as a trigonometric series, while it was evidently satisfied by

$$Af(x+ct) + Bf(x-ct)$$