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Schauder bases in Banach spaces of continuous functions, by Zbigniew Semadeni, Lecture Notes in Mathematics, vol. 918, Springer-Verlag, Berlin, 1982, v + 135 pp., \$9.80. ISBN 3-5401-1481-5

Let K be a metrizable compact set and let $C(K)$ denote the space of continuous functions on K endowed with the supremum norm. $C(K)$'s are, together with the Hilbert space, the most "popular" Banach spaces, applied and studied in practically every branch of analysis. For example, recall that every separable Banach space is linearly isometric to a subspace of, e.g. $C([0, 1])$; a stochastic process can be thought of as a probability measure on $C([0, T])$. (More specifically, let us mention Ciesielski's construction of the Brownian motion using the Faber-Schauder basis of $C([0, 1])$; see [2]; more about this later.)

Both for numerical and theoretical purposes the need arises to consider approximations of continuous functions. This is done by using *Schauder bases* (such as spline bases, etc.). Recall that a sequence (f_n) of elements of a Banach space X is called a Schauder basis iff every element f of X can be uniquely written as the sum (convergent in X) $\sum_{n \in \mathbb{N}} t_n f_n$, t_n being scalars. The partial sums $S_N f = \sum_{n=1}^N t_n f_n$ are then the successive approximations of f .

It has been known for quite a while (some 30 years) that every $C(K)$ -space has a Schauder basis; moreover, a complete isomorphic classification of $C(K)$'s as Banach spaces has been available for almost as long [1, 5, 7, 8]. More recently, it has been shown that in fact any separable space from a larger class of \mathcal{L}^∞ -spaces (or \mathcal{L}^p , $1 \leq p \leq \infty$) has a basis [6]; this, however, is not discussed in the book. The above contrasts with negative results for general Banach spaces: some of them do not even possess the much weaker approximation property (Enflo [3]).

The question of existence being settled, the emphasis switches to looking for bases with "nice" properties and to studying specific bases. To these the bulk

of the book is devoted, with most of the attention focused on the so-called interpolating bases. A basis (f_n) of $C(K)$ is called *interpolating* with nodes $t_n \in K$, $n = 1, 2, \dots$, if, for any $f \in C(K)$, its N th approximation $S_N f$ coincides with f at the points t_1, t_2, \dots, t_N (for all $N \in \mathbf{N}$). A typical example is the classical Faber-Schauder system (perhaps more often quoted as the Schauder basis) in $C([0, 1])$, the nodes are then the naturally ordered dyadic numbers from $[0, 1]$ and the basis elements are “hat” functions defined by: $f_n(t_n) = 1$, $f_n(t_k) = 0$ for $1 \leq k < n$, f_n piecewise linear between t_k 's, $1 \leq k \leq n$. Note that in particular the diameters of supports of f_n 's tend to 0.

Let us mention here that the space $C([0, 1])$ has also another entirely different basis: the orthonormal Franklin system, which is the result of applying the Gram-Schmidt orthonormalization process to the Faber-Schauder basis. In contrast to the latter, the Franklin system is noninterpolating; in fact it is not obvious that it is a basis at all.

The situation gets more complicated in the case $C([0, 1]^d)$, $d \geq 2$. Although the space $C([0, 1]^d)$ may be thought of as the injective tensor product of d copies of $C([0, 1])$ and therefore the corresponding products of the Faber-Schauder bases in the copies of $C([0, 1])$ form a (interpolating) basis of $C([0, 1]^d)$ [4], this approach has some drawbacks. For example, the diameters of supports of the elements of such a basis do not tend to 0, and therefore it is hard to estimate the rate of convergence. To avoid this one introduces the so-called regular pyramidal and squew (why not “skew”?) pyramidal bases. The book describes these bases in great detail—analytically rather than geometrically, with specific formulas given. Apparently this is the first such presentation available; it should be very helpful in applications.

The book is augmented with carefully described interpolating bases of $C([0, 1]^\omega)$ (continuous functions on the Hilbert cube) and other $C(K)$'s, including a “generic” $C(K)$ -space, introduction to construction of bases in various spaces of differentiable and analytic functions and a survey of general results about bases of $C(K)$'s.

The book is generally “reader friendly” (with the possible exception of parts of Chapter 4, which contain a number of misprints and an apparently missing picture at the bottom of p. 92); especially transparent is the indexing of elements of an interpolating basis by its nodes rather than positive integers. The text should be useful both for the student who will find a general, not too technical, introduction to the subject as well as exercises after most of the sections, and for the mathematician who utilizes bases of the spaces of continuous functions for theoretical or numerical purposes.

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