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THE FUNCTIONS OPERATING ON CERTAIN ALGEBRAS OF MULTIPLIERS[1]

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In this note, we announce a new result concerning functions operating on multiplier algebras. We begin by introducing the following notation. Let G be a LCA group with dual group Γ. $M(G)$ will denote the algebra of finite, regular Borel measures on G. Let $M_0(G) = \{\mu \in M(G) | \hat{\mu}$ vanishes at ∞ on $\Gamma\}$. If $1 \leqslant p < \infty$, let $M_p(G)$ denote the class of multiplier transformations on $L_p(G)$. If $T \in M_p(G)$, $\hat{T}$ will be the unique function in $L_\infty(\Gamma)$ so that $T(f)\hat{} = \hat{T}\hat{f}$, for all integrable simple functions f. Finally, we write $C_0M_p(G) = \{T \in M_p(G) | \hat{T}$ is continuous and vanishes at ∞ on $\Gamma\}$.

Suppose that G is nondiscrete. It is well known that only entire functions operate on the Banach algebra $M(G)$ [3, Chapter 6]. This result was strengthened in [1]. There, Igari showed that only entire functions operate from $M(G)$ into the algebra $M_p(G)$, $1 < p < \infty, p \neq 2$. In [4], Varopoulos showed that for compact G, only entire functions operate on $M_0(G)$. We have the following theorems, which, in a sense, may be viewed as the L_p analogues of the aforementioned result of Varopoulos.

THEOREM 1. *Let $1 < p < \infty$ with $p \neq 2$. Suppose that F: $[-1, 1] \longrightarrow \mathbf{C}$ and that F operates on the algebra $C_0M_p(\mathbf{T}^n)$. Then F coincides with an entire function in some neighborhood of* 0.

THEOREM 2. *Let $1 < p < \infty$ with $p \neq 2$, and let G denote one of the groups $\mathbf{R}^n$ or $\mathbf{Z}^n$. Suppose that F: $[-1, 1] \longrightarrow \mathbf{C}$ and that F operates on the algebra $C_0M_p(G)$. Then F coincides with an entire function on* $[-1, 1]$.

These results complete the investigation begun by the author in [5]. We now indicate some of the ideas involved in the proof.

Assume that $G = \mathbf{T}$ and $1 < p < 2$. By standard arguments (see [1] and [3, Chapter 6]) we may assume that $F(x) = \Sigma_{k=1}^{\infty} a_k x^k$ for $|x| < \epsilon$. It then suffices to show that there exists j_ϵ such that

$$|a_j| \leqslant C_\epsilon 10^j \tag{1}$$

for all $j \geqslant j_\epsilon$. This is accomplished by studying refinements of the multipliers considered in [5]. Corresponding to the sequence $\{a_j\}$, we construct measures $\{\lambda_j\}$, λ in $M(\mathbf{T})$ so that for all j

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$$(2)\qquad \hat{\lambda}_j \text{ is real-valued and } \|\hat{\lambda}_j\|_\infty \leqslant 2^{1/2}2^{-n(j)/2},$$

$$(3)\qquad |\lambda_j| \leqslant \lambda,$$

$$(4)\qquad \left\|\sum_{k=1}^{j} a_k 2^{n(j)k/p'}\lambda_j^k\right\|_{M_p} \geqslant c(j!/j^j)2^{-j/p'}|a_j|.$$

Here $\{n(j)\}$ is a sequence of positive integers tending to infinity and $1/p + 1/p' = 1$. As in [5], the measures $\{\lambda_j\}$ and λ are essentially obtained as "generalized Riesz products" of combinations of certain Rudin-Shapiro measures. Inequalities (4) is proved by carefully studying the combinatorial properties of these Rudin-Shapiro measures. Moreover, the special properties of Rudin-Shapiro polynomials makes it possible, in essence, to "ignore" the term $\Sigma_{k=j+1}^{\infty} a_k 2^{n(j)k/p'}\lambda_j^k$ when estimating $|a_j|$. We now define $U\{f_j\} = \{2^{n(j)/p'}\lambda_j * f_j\}$. Then U is a bounded operator on $L_p(l_2)$. We construct our basic multiplier T by "cutting off" and "piecing together" the measures $2^{n(j)/p'}\lambda_j$ via the Littlewood-Paley theory (see [5] for details). The estimate (1) then follows by studying the multiplier $F(T)$, and using the properties of $\{\lambda_j\}$. This will prove Theorem 1 for the circle groups.

Theorems 1 and 2 now follow by rather standard arguments for the cases $G = \mathbf{T}^n$ or $G = \mathbf{R}^n$. However, the case $G = \mathbf{Z}^n$ (more particularly, if $G = \mathbf{Z}$) is more difficult and requires some additional ideas.

We prove the theorem for the integer group by constructing a multiplier $S \in C_0M_p(\mathbf{R})$ so that $\hat{S}$ is real-valued, supp $\hat{S} \subseteq [0, 1]$, and so that the behavior of $\hat{S}$ near the origin reflects the behavior of our basic multiplier $\hat{T}$ near ∞. The construction consists in combining the method outlined above, with a technique of Igari [2].

The proof is essentially in the same spirit as that indicated for $G = \mathbf{T}$. However, the arguments are much more involved. In particular, we introduce a vector-valued analogue of the space BMO. This allows us to obtain sharp L_p estimates for the operators involved in our constructions.

Detailed proofs of the ideas sketched here will appear in [6].

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