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GEOMETRIC TOPOLOGY AND SHAPE THEORY: A SURVEY OF PROBLEMS AND RESULTS

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1. Introduction. The theory of shape, introduced by Karol Borsuk [**2**] in 1968, has developed extremely rapidly in the intervening years. Much of the recent work (with the notable exception of that of Borsuk and his students and colleagues in Warsaw) has concentrated on the pro-homotopy, categorical aspect of the theory. I think this may well prove ultimately to be the most important part of the theory–indeed, it may already be so–but nevertheless there remain interesting unsolved problems of a more geometric nature, problems which might be accessible through the more primitive techniques of geometric or general topology. The purpose of this paper is to give a brief (and necessarily incomplete) survey of results of this sort, and to call attention to a number of unsolved problems in this area. With one exception, all the problems listed have appeared in print and are identified with appropriate references. It happens that more than half of the problems listed are due to Borsuk, and a quarter of the total can be found either in his book [**15**] or in the survey article [**14**]. These two references are sources of many other problems as well.

2. Basic definitions for compact metric spaces. There are two basic approaches to the shape theory of compact metric spaces: the "fundamental sequences" of Borsuk's initial paper [**2**] and the "ANR-systems" used by Mardešić and Segal [**38**], [**39**]. Since fundamental sequences seem conceptually more geometric in nature, they will be used primarily here. It should be pointed out, however, that some of the results mentioned in this paper were obtained only with the aid of the other approach (which surely is a part of geometric topology, too). Moreover, and more importantly, replacing ANR-*sequences* by ANR-*systems* extends the notion of shape to all compact Hausdorff spaces.

Let Q denote the Hilbert cube, and consider a sequence $\mathbf{f} = \{f_k\}_{k=1}^{\infty}$ of maps (continuous functions) of Q into Q. If X and Y are compact subsets of Q, the triple $(\mathbf{f}, X, Y)$ will be called a *fundamental sequence from X to Y* (in Q) provided that for every neighborhood V of Y, there is a neighborhood U of X such that $f_i|U \simeq f_j|U$ in V for almost all integers i, j. (Note that X and Y are not uniquely determined by the sequence $\mathbf{f}$; indeed, it is evident that if $(\mathbf{f}, X, Y)$ is a fundamental sequence, then for any compacta X', Y' in Q with $X' \subset X$ and $Y \subset Y'$, $(\mathbf{f}, X', Y')$ is a fundamental sequence. Thus "restricting the domain and enlarging the range" of a fundamental sequence yields another fundamental sequence. A similar assertion holds for continuous functions, of course.)

Let $\mathbf{i} = \{i_k\}_{k=1}^{\infty}$, where $i_k = \mathrm{id}_Q$ for all k. Then for any compact subset X

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of Q, $(\mathbf{i}, X, X)$ is a fundamental sequence; $(\mathbf{i}, X, X)$ is called the *identity fundamental sequence* on X, and is denoted by $\mathbf{i}_X$.
if X is homotopically equivalent to Y then X is fundamentally equivalent to Y. Moreover, for the class of (compact) ANR's in Q, homotopy and fundamental domination coincide, as do homotopy and fundamental equivalence. To a large extent, the aim of geometric shape theory is to discover relations among arbitrary compacta, relative to these extended notions of equivalence and domination, that are analogous to theorems of homotopy theory for ANR's.

Since every compact metric space can be embedded in Q (and [**2**] the relations of fundamental equivalence and fundamental domination are independent of the embedding), there is no loss in assuming that all compacta considered are subsets of Q. It is worth noting, however, that Q can be replaced in the above definitions by any absolute retract for metrizable spaces which contains X and Y [**10**]. Thus, for example, in considering shapes of compact subsets of E^2, the fundamental sequences may be taken to consist of maps of E^2 into itself, rather than having to extend these to all of Q.

3. Fundamental retracts. Fundamental retractions and fundamental retracts (or neighborhood retracts) are defined in complete analogy with the usual definitions of retractions and retracts, using fundamental sequences instead of maps. Specifically, if X and A are compact subsets of Q with $A \subset X$, then a fundamental sequence $(\mathbf{r}, X, A)$ is called a *fundamental retraction* of X to A if $r_k|A = \mathrm{id}_A$ for all k; A is said to be a *fundamental retract* of X if there is a fundamental retraction of X to A, and A is a *fundamental neighborhood retract*

Two fundamental sequences $(\mathbf{f}, X, Y)$ and $(\mathbf{g}, X, Y)$ are said to be *homotopic* (denoted $(\mathbf{f}, X, Y) \simeq (\mathbf{g}, X, Y)$) if for every neighborhood V of Y, there is a neighborhood U of X such that $f_k|U \simeq g_k|U$ in V for almost all k.

If $\mathbf{f} = \{f_k\}_{k=1}^{\infty}$ and $\mathbf{g} = \{g_k\}_{k=1}^{\infty}$ are sequences of maps of Q into Q, then the sequence $\{g_k f_k\}_{k=1}^{\infty}$ is called the *composition* of $\mathbf{f}$ with $\mathbf{g}$, and is denoted by $\mathbf{gf}$. It is easy to see that if $(\mathbf{f}, X, Y)$ and $(\mathbf{g}, Y, Z)$ are fundamental sequences, then so is $(\mathbf{gf}, X, Z)$; this fundamental sequence is called the *composition* of $(\mathbf{f}, X, Y)$ with $(\mathbf{g}, Y, Z)$.

It is easily verified that homotopy of fundamental sequences is an equivalence relation, and that it is *compositive*; i.e., if $(\mathbf{f}, X, Y) \simeq (\mathbf{f}', X, Y)$ and $(\mathbf{g}, Y, Z) \simeq (\mathbf{g}', Y, Z)$, then $(\mathbf{gf}, X, Z) \simeq (\mathbf{g}'\mathbf{f}', X, Z)$.

Two compact subsets X, Y of Q are said to be *fundamentally equivalent*, or to *have the same shape*, if there exist fundamental sequences $(\mathbf{f}, X, Y)$ and $(\mathbf{g}, Y, X)$ which are *homotopy inverses*; i.e., $(\mathbf{gf}, X, X) \simeq \mathbf{i}_X$ and $(\mathbf{fg}, Y, Y) \simeq \mathbf{i}_Y$. The class of all compact subsets of Q having the same shape as a given compactum X in Q is denoted by $\mathrm{Sh}(X)$. Thus two compacta X, Y in Q have the same shape if and only if $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$, or, equivalently, if $Y \in \mathrm{Sh}\, X$.

If only the first of the two homotopy relations above is postulated (i.e., $(\mathbf{gf}, X, X) \simeq \mathbf{i}_X$), then X is said to be *fundamentally dominated* (or *shape dominated*) by Y; this is expressed notationally by $\mathrm{Sh}(X) \leqslant \mathrm{Sh}(Y)$. A property α of compacta is called a *shape invariant (hereditary shape invariant)* provided that if Y has property α, so does every compactum X which has the same shape as Y (respectively, which is shape dominated by Y).

The relations of fundamental equivalence and fundamental domination are extensions of the concepts of homotopy equivalence and homotopy domination, in the sense that for any two compact subsets X, Y of Q, if X is homotopically dominated by Y, then X is fundamentally dominated by Y and of X if there exist a closed neighborhood N of A in X and a fundamental retraction of N to A. The definitions of *fundamental absolute retract* (FAR) and *fundamental absolute neighborhood retract* (FANR) are entirely analogous to the corresponding definitions of AR's and ANR's, using fundamental retractions in place of retractions. Among the basic properties of FAR's and FANR's are the following, all due to Borsuk [3].

(3.1) Every AR in Q is an FAR and every ANR in Q is an FANR.

(3.2) Every fundamental retract of an FAR (FANR) is an FAR (FANR).

(3.3) A compactum in Q is an FAR (FANR) if and only if it is a fundamental retract of an AR (ANR).

(3.4) All Betti groups of an FANR are finitely generated and almost all are trivial. (In particular, an FANR can have only a finite number of components. It easily follows that every component of an FANR is an FANR.)

(3.5) A compactum X in E^2 is an FANR if and only if X has only a finite number of components, and each component of X has only a finite number of complementary domains in E^2.

It is also true [9] that a compactum X in Q is an FAR if and only if X has trivial shape (i.e., the shape of a point). Thus X is an FAR if and only if X has the shape of an AR, and it is natural to conjecture that X is an FANR if and only if X has the shape of an ANR, or of a polyhedron [**14**]. This conjecture has recently been disproved: Edwards and Geoghegan [**22**] have given an example of a (2-dimensional) continuum X which is an FANR but does not have the shape of any finite complex; moreover, West [**54**] has shown that every ANR has the homotopy type (and therefore the shape) of a finite complex, so X does not have the shape of any ANR. It follows from (3.5) that every FANR lying in E^2 does have the shape of a polyhedron, and it is known [**51**] that every 1-dimensional FANR has the shape of a plane compactum. So the question of which FANR's have polyhedral shape is completely settled with respect to dimension: all 0- and 1-dimensional ones do, those of dimension 2 or more need not. However, the following questions remain open.

Problem 1 [**15**, p. 350]. Does every FANR in E^3 have the shape of a polyhedron?

Problem 2 [**15**, p. 357]. Does every movable (definition in §4) compactum in E^3 with finite Betti numbers have the shape of a polyhedron?

In analogy with well-known results for AR's, Borsuk [3] proved that if X, Y and $X \cap Y$ are FAR's, then $X \cup Y$ is an FAR, and Chapman [**18**] has shown that if $X \cap Y$ and $X \cup Y$ are FAR's, then X and Y are FAR's. It is natural to inquire whether similar results hold for FANR's.

Problem 3 [3], [**13**]. If X, Y and $X \cap Y$ are FANR's, must $X \cup Y$ be an FANR?

It is known that the intersection of any decreasing sequence of AR's is an FAR, but it is easy to see that the corresponding result for ANR's is false. Borsuk has shown, however, that if $Y_1 \supset Y_2 \supset \dots$ is a sequence of ANR's such that Y_{k+1} is a deformation retract of Y_k for all k, then $\bigcap_{k=1}^{\infty} Y_k$ is an FANR.

PROBLEM 4 [**3**]. If $Y_1 \supset Y_2 \supset \ldots$ is a sequence of ANR's such that Y_{k+1} is a retract of Y_k for each k, is $\bigcap_{k=1}^{\infty} Y_k$ an FANR?

PROBLEM 5 [**3**]. If $Y_1 \supset Y_2 \supset \ldots$ is a sequence of ANR's and Y_{k+1} is a retract of Y_k for each k, must Y_{k+1} be a deformation retract of Y_k for all sufficiently large k?

PROBLEM 6 [**3**]. If X is an FANR and $Y_1 \supset Y_2 \supset \ldots$ is a sequence of fundamental retracts of X, is $\bigcap_{k=1}^{\infty} Y_k$ a fundamental retract of X? (It is known [**20**] that this is false without the hypothesis that X be an FANR.)

4. Movability. The notion of a movable space seems to be one of the most useful ideas in shape theory. The basic definitions are due to Borsuk ([**4**], [**8**], [**9**]).

A compact set X in Q is said to be *movable* if for every neighborhood U of X, there is a neighborhood U_0 of X which can be deformed into any neighborhood of X by a homotopy in U; i.e., for every neighborhood V of X, there is a homotopy φ_V: $U_0 \times I \to U$ such that $\varphi_V(x, 0) = x$ and $\varphi_V(x, 1) \in V$ for every $x \in U_0$. If the homotopy φ_V can always be chosen so that $\varphi_V(x, 1) = x$ for every $x \in X$, then X is said to be *strongly movable*. If $x_0 \in X$, the pair (X, x_0) is *pointed movable*, or X is *movable with respect to* x_0, if φ_V can be chosen so as to leave x_0 fixed at all times t.

Related to the idea of pointed movability is that of *shapes of pointed compacta*, or *pointed shapes*. These are defined and their basic properties demonstrated on pp. 243–246 of [**2**], and will not be repeated in detail here. Suffice it to say that for a *pointed fundamental sequence* from (X, x_0) to (Y, y_0) one requires a sequence $\mathbf{f} = \{f_k\}_{k=1}^{\infty}$ of maps of (Q, x_0) into (Q, y_0); moreover, all homotopies in the definition of "pointed fundamental sequence" are required to be rel x_0, as are the homotopies involved in the definition of homotopic pointed fundamental sequences. (Recall that a map φ: $A \times I \to B$ is a homotopy rel x_0, where $x_0 \in A$, if for each $t \in I$, $\varphi(x_0, t) = \varphi(x_0, 0)$.) The *pointed shape*, $\mathrm{Sh}(X, x_0)$, of a pointed compactum is defined analogously to $\mathrm{Sh}(X)$, and similarly for the relation $\mathrm{Sh}(X, x_0) \leqslant \mathrm{Sh}(Y, y_0)$. (Perhaps it is well to point out that $\mathrm{Sh}(X, x_0)$ is not the same as $\mathrm{Sh}(X, \{x_0\})$, where $\mathrm{Sh}(X, \{x_0\})$ is the (relative) shape of the pair (X, X_0), defined earlier in [**2**], with $X_0 = \{x_0\}$.)

It has been shown by Borsuk ([**4**], [**8**]) that movability (pointed movability) is an hereditary invariant of shape (pointed shape).

Other results on movability include the facts [**4**] that every ANR, or more generally, every FANR is movable, that every plane compactum is movable (in fact [**8**], pointed movable with respect to each of its points), that solenoids are not movable, and that a compactum is movable if each of its components is movable (but not conversely). Additionally [**8**], if (X, a) is movable, so is (X, b) for any point b in the same component of X as a; from this hypothesis ((X, a) movable and b in the component of X containing a) it follows that $\mathrm{Sh}(X, a) = \mathrm{Sh}(X, b)$. This latter result does not hold in general without the requirement that (X, a) be movable. It was shown in [**3**] that every FANR is strongly movable (without using that term), and in [**9**] it is shown that strong movability in fact characterizes FANR's. The question of whether, or when, movability implies pointed movability is of considerable interest; the question has been raised independently by Borsuk [**8**], McMillan [**41**] and Krasinkiewicz [**31**]. Though phrased in point set topology terms, this question seems to be essentially algebraic in nature. Nevertheless, because of its importance to many shape theory results, it is included here.

PROBLEM 7 [**8**], [**41**], [**31**]. Is every movable continuum X necessarily pointed movable with respect to one (or to each) of its points?

A related result, given by McMillan [**41**], is that if X is a movable continuum and (X, x_0) is pointed 1-movable, then (X, x_0) is movable.

Mardešić has shown [**34**] that every n-dimensional LC^{n-1} continuum is movable. Borsuk [**5**] gave an example of a nonmovable locally connected (LC^0) continuum X_0 in E^3, with $\dim X_0 = 2$. The continuum X_0, indicated in Figure 1, is locally E^2 at all points except a.

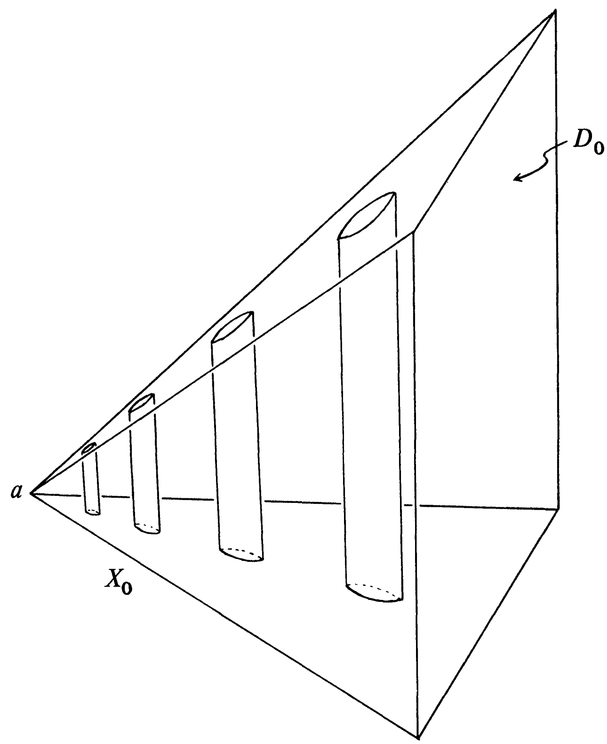

FIGURE 1

Borsuk showed that $X_0 - \operatorname{Int} D_0$ has the shape of a plane continuum, and hence is movable.

PROBLEM 8 [**5**]. Is every closed proper subset of X_0 movable?

In connection with the preceding example of Borsuk, we note that McMillan [**42**] has recently constructed a locally connected nonmovable continuum in E^3 which does not separate E^3.

Of course, there exist many *non*locally connected continua which *are* movable (for example in E^2), but there are no known movable continua which do not at least have the shape of a locally connected continuum. This is an example of what Borsuk calls the problem of "reasonable representatives of shapes."

PROBLEM 9 [**15**, p. 357]. Does every movable continuum have the shape of a locally connected continuum?

In an interesting recent paper [**31**], J. Krasinkiewicz has shown, among many other results, that every 1-dimensional continuous image of a tree-like continuum is movable. He poses several questions, of which the following two seem most relevant here.

PROBLEM 10 [**31**]. Is every 1-dimensional continuous image of a movable continuum movable?

PROBLEM 11 [**31**]. Are all arcwise connected 1-dimensional continua movable?

A compact subset X of Q is said to be *n-movable* [**11**] if for every neighborhood U of X, there is a neighborhood U_0 of X such that any n-dimensional subset of U_0 can be deformed into an arbitrary neighborhood of X by a homotopy with values in U. Borsuk shows that n-movability is an hereditary shape invariant, that a compactum is n-movable if each of its components is n-movable, and that the suspension of any n-movable compactum is n-movable. He poses the following question.

PROBLEM 12 [**11**]. If X is n-movable and Y is m-movable, must $X \times Y$ be $(n + m)$-movable?

A partial answer to Problem 12 has been given by Kodama and Watanabe [**29**], who show that, under the given hypothesis, $X \times Y$ is k-movable where $k = \min(n, m)$. Kodama and Watanabe also answer another question of Borsuk by showing that there is a nonmovable continuum which is n-movable for every n; the same example was also given independently by Kozlowski and Segal [**30**].

McMillan [**41**] has introduced an interesting variant of 1-movability, which he calls "nearly 1-movable." McMillan indicates that solenoids fail to be nearly 1-movable, as does the Case-Chamberlain curve, and shows how to construct an example of a 1-dimensional continuum which is nearly 1-movable but not 1-movable. He also proves that every continuous image of a nearly 1-movable continuum is nearly 1-movable, and that every continuous image of a pointed 1-movable continuum is pointed 1-movable.

PROBLEM 13 [**41**]. Is every continuous image of a 1-movable continuum necessarily 1-movable?

The next question, a special case of Problem 6, is also due to McMillan.

PROBLEM 14 [**41**]. Is every 1-movable continuum necessarily pointed 1-movable (with respect to at least one of its points)?

Another result from [**41**] is that if A and B are nondegenerate continua such that $A \times B$ is embeddable in a PL 3-manifold, then each of A and B is pointed 1-movable with respect to each of its points. This suggests the following question.

PROBLEM 15 [**41**]. If A and B are nondegenerate continua such that $A \times B$ is embeddable in a PL 3-manifold, must each of A and B be movable?

N. Shrikandhe [**49**] has shown that if X is a compactum in E^n, then E^n/X is locally simply connected if and only if X is nearly 1-movable. McMillan and Shrikandhe [**43**] have used this theorem to obtain a number of additional results on the simple connectivity of quotient spaces.

5. Fundamental dimension and Euclidean coefficients. The *Euclidean coefficient*, $e(X)$, of a compactum X is the smallest n ($n = 0, 1, \ldots, \infty$) such that $\mathrm{Sh}(X) \leqslant \mathrm{Sh}(Y)$ for some compactum $Y \subset I^n$. This definition of $e(X)$ is (equivalent to) that given in [**15**]. In [**14**], $e(X)$ was defined to be the smallest n such that $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$ for some $Y \subset I^n$; here we will let $e'(X)$ denote this latter number. It is clear that $e(X) \leqslant e'(X)$ and that $\mathrm{Sh}(X) \leqslant \mathrm{Sh}(Y)$ implies $e(X) \leqslant e(Y)$. The following problems remain open, however.

PROBLEM 16 [**15**, p. 354]. Is $e(X) = e'(X)$ for every compactum X?

PROBLEM 17 [**14**]. Does $\mathrm{Sh}(X) \leqslant \mathrm{Sh}(Y)$ imply $e'(X) \leqslant e'(Y)$?

Several other interesting questions concerning the numbers $e(X)$ and $e'(X)$ are given in [**14**] and [**15**].

The *fundamental dimension* $\mathrm{Fd}(X)$ of a compactum X was introduced in [**7**]; it is the smallest n ($n = 0, 1, \ldots, \infty$) such that $\mathrm{Sh}(X) \leqslant \mathrm{Sh}(Y)$ for some n-dimensional compactum Y. (Equivalently [**45**], [**28**], $\mathrm{Fd}(X)$ is the smallest n such that $\mathrm{Sh}(X) = \mathrm{Sh}(Y)$ for some n-dimensional Y.)

The fundamental dimension has been studied extensively by S. Nowak ([**45**], [**46**], [**47**]). The following are some of his results.

(5.1) For any compactum $X \subset Q$, $\mathrm{Fd}(X) \leqslant n$ if and only if X is deformable inside each of its neighborhoods to a set of dimension $\leqslant n$.

(5.2) For any compactum $X \subset Q$, $\mathrm{Fd}(X) \leqslant n$ if and only if $\mathrm{Fd}(C) \leqslant n$ for every component C of X.

(5.3) Every compact, connected n-manifold with nonempty boundary has fundamental dimension $\leqslant n - 1$.

(5.4) Every compact proper subset of a PL n-manifold has fundamental dimension $\leqslant n - 1$; in particular, $\mathrm{Fd}(X) \leqslant n - 1$ for every compact subset of E^n.

PROBLEM 18 [**45**]. Does every compact proper subset of an n-manifold have fundamental dimension $\leqslant n - 1$?

PROBLEM 19 [**46**]. If $\mathrm{Fd}(X) > \mathrm{Fd}(X \cap Y)$, is $\mathrm{Fd}(X \cup Y) \geqslant \mathrm{Fd}(X)$?

PROBLEM 20 [**46**]. Let C be the well-known Case-Chamberlain curve. Is $\mathrm{Fd}(C^n) = n$?

The following question is due to Borsuk.

PROBLEM 21 [**14**], [**15**]. For every compactum X, is $\mathrm{Fd}(X \times S^1) = \mathrm{Fd}(X) + 1$? More generally, is $\mathrm{Fd}(X \times S^n) = \mathrm{Fd}(X) + n$?

Kodama [**28**] has obtained a number of results on fundamental dimension, and has defined the fundamental dimension $\mathrm{Fd}(X, A)$ for a pair of compacta as the minimum dimension of a compactum Y having a closed subset B such that $\mathrm{Sh}(X, A) \leqslant \mathrm{Sh}(Y, B)$. (Here $\mathrm{Sh}(X, A)$ is the relative shape defined in [**2**].) Kodama shows that if X is an AR and A is a closed subset of X, then $\mathrm{Fd}(A) \leqslant \mathrm{Fd}(X, A) \leqslant \mathrm{Fd}(A) + 1$. He asks the following question.

PROBLEM 22 [**28**]. For every compact pair (X, A), is $\mathrm{Fd}(X, A) \leqslant \max(\mathrm{Fd}(X), \mathrm{Fd}(A) + 1)$?

6. Stability. A space X is said to be *homotopically stable* (or *H-stable*) if for each closed proper subset Y of X, no map of X into Y is homotopic in X to id_X. Similarly, X is R-stable if no proper subset of X is a deformation retract of X. Borsuk [**13**] has studied analogs of these notions in shape theory and has raised a number of interesting questions concerning them.

A compact subset X of Q is *fundamentally stable*, or *F-stable*, if for no closed proper subset Y of X is there a fundamental sequence $(\mathbf{f}, X, Y)$ with $(\mathbf{f}, X, X) \simeq \mathbf{i}_X$. Similarly, X is *fundamentally R-stable,* or *FR-stable*, if no closed proper subset of X is a fundamental deformation retract of X (i.e., for no closed proper subset Y of X is there a fundamental retraction $(\mathbf{r}, X, Y)$ with $(\mathbf{r}, X, X) \simeq \mathbf{i}_X$). Finally, X is *shape stable*, or *S-stable*, if no closed proper subset of X has the same shape as X. (Shape stable compacta are called "primitive" in [**15**].) Borsuk obtains a number of theorems related to these notions, including in particular the result that every FANR X contains an FR-stable compactum Y with $\mathrm{Sh}(Y) = \mathrm{Sh}(X)$. He poses, among others, the following similar questions.

PROBLEM 23 [**13**]. Does every compactum X contain an FR-stable (or an F-stable) compactum Y with $\mathrm{Sh}(Y) = \mathrm{Sh}(X)$?

PROBLEM 24 [**13**]. Does every continuum X contain an S-stable continuum Y with $\mathrm{Sh}(Y) = \mathrm{Sh}(X)$?

An affirmative answer to Problem 24 for the case $\mathrm{Fd}(X) < \infty$ has been given by Cook, Feuerbacher and Kuperberg [**19**].

PROBLEM 25 [**15**, p. 357]. Is the product (or the one-point union) of two S-stable compacta necessarily S-stable?

7. Shapes and complements. The beautiful result of Chapman [**16**] that two Z-sets in Q have the same shape if and only if their complements in Q are homeomorphic has inspired a number of investigations concerning similar theorems for compacta in E^n or S^n.

First, Chapman [**17**] proved a finite dimensional version of his theorem, but needed strong codimension requirements as well as a fairly complex embedding condition. Geoghegan and Summerhill [**25**] reduced the codimension requirements to the trivial range, the best possible for arbitrary compacta, and replaced Chapman's embedding condition with the more familiar 1-ULC complement property. Rushing [**48**], considering embeddings in S^n, $n \geqslant 5$, showed that if one of the compacta is S^k ($k \neq 1$) and the other is globally 1-alg, no further restriction is needed to obtain the equivalence of "having the same shape" and "having homeomorphic complements" (except for the case $k = n - 2$, which can be handled by adding an embedding restriction). Hollingsworth and Rushing [**27**] show that, for compacta in E^n, $n \geqslant 5$, with dimensions in the trivial range, it is sufficient that X and Y satisfy the "small loops condition." Additional results of this kind have been obtained by Liem [**32**] and Venema [**52**], [**53**]. Coram, Daverman and Duvall [**20**] have obtained results relating shapes and embeddings in E^n of a different sort, showing that, under suitable dimensional and embedding restrictions, a compactum in E^n which has the shape of a complex K must have arbitrarily close neighborhoods which are regular neighborhoods of a copy of K in E^n.

It seems likely that further results relating shapes of compacta in E^n to properties of their embeddings will be developed, but apparently no specific problems in this area have been posed in the literature.

8. Compact Hausdorff spaces. As indicated in §2, the theory of shape can be extended to the class of all compact Hausdorff spaces by the method of Mardešić-Segal, based on inverse systems of ANR's. Since the definitions are fairly involved and only a few problems in this area will be mentioned, the definitions will not be repeated here. The interested reader is referred to [**38**] and [**39**].

Gordh and Mardešić [**26**], using categorical techniques, obtain a number of results on movable Hausdorff curves (1-dimensional continua). They ask several questions, including the following.

PROBLEM 26 [**26**]. Is every locally connected (hereditarily locally connected) Hausdorff curve movable?

Mardešić [**33**] has generalized the notion of FANR to compact Hausdorff spaces, and uses the terminology *absolute neighborhood shape retract* (ANSR) for this generalized concept.

Mardešić has also [**35**] extended the definition of strong movability to compact Hausdorff spaces. He shows, among other results, that every ANSR is strongly movable. Borsuk [**9**] proved that strong movability (of compact metric spaces) characterizes FANR's, and Mardešić asks whether the analogous result holds in his more general setting.

PROBLEM 27 [**35**]. Is every strongly movable compact Hausdorff space an ANSR?

The following question was asked by D. A. Edwards in a casual conversation. It probably has not been seriously considered by anyone, but would be interesting if true.

PROBLEM 28. If X is an open n-cell and βX is the Čech compactification of X, is $\mathrm{Sh}(\beta X - X) = \mathrm{Sh}(S^{n-1})$?

9. Noncompact spaces. There are several ways of extending shape theory to noncompact spaces; there seems to be general agreement that the approach of Mardešić [**36**] (see also Morita [**44**]) or an equivalent method is the most appropriate. This treatment applies to arbitrary topological spaces, and is highly abstract and nonintuitive. If one is willing to restrict attention to a smaller class of spaces, however, more geometrically oriented theories are available.

For metrizable spaces, the shape theory developed by Fox ([**23**], [**24**]) does not completely obscure the geometry involved (and is equivalent [**37**] to Mardešić's theory restricted to metrizable spaces).

Borsuk's "weak shape" for metrizable spaces [**12**], [**15**] and the related notion of "position" (of a set in a space) are quite geometrically oriented. There are many appealing problems here.

The theory of proper shape for locally compact metric spaces, introduced by the author and R. B. Sher [**1**], was geometrically motivated. It was an attempt to carry over to a class of noncompact spaces the property of Borsuk's shape theory for compacta of reflecting the global geometric similarities of spaces while ignoring their local complexities. It might be worth further development.

10. Concluding remarks. It has not been possible here to give a truly comprehensive survey of geometrically appealing results and problems of shape theory. Many topics have been treated less thoroughly than they deserve, and many others have been omitted altogether. However, in addition to the references cited explicitly, I have attached a supplementary bibliography listing many other papers that seem to fall into the category of "geometric shape theory." I am sure there are omissions in this listing, but I hope it will prove useful to those who wish to undertake a systematic study of the area. In this connection, I should remark that Jack Segal [55] has compiled an essentially complete bibliography of shape theory.

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