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CONVERGENCE OF FOURIER SERIES ON COMPACT LIE GROUPS[1]

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Let G be a compact connected semisimple Lie group. Fix a maximal torus T and denote its Lie algebra by $\mathfrak{T}$. The irreducible unitary representations of G are indexed by a semilattice L of dominant integral forms on $\mathfrak{T}$. For each λ in L let χ_λ and d_λ be the character and degree of the representation corresponding to λ.

By the Fourier series of a function f on G we mean the formal series $\Sigma_{\lambda \in L} d_\lambda \chi_\lambda * f$. In this paper we announce results concerning the convergence properties (both mean and pointwise) of polyhedral partial sums of these Fourier series. Details and proofs will appear elsewhere.

Let P be an open, convex polyhedron in $\mathfrak{T}$ centered at the origin. Assume P is Weyl group invariant. Let $RP = \{RX | X \in P\}$ and $S_R f(g) = \Sigma_{\lambda \in RP} d_\lambda \chi_\lambda * f(g)$.

THEOREM A. *If $p \neq 2$ there is an f in $L^p(G)$ such that $S_R f$ does not converge to f in the L^p norm.*

An immediate corollary of this theorem is that when $p < 2$ almost everywhere convergence fails for some f in $L^p(G)$. However, the convergence behaviour of Fourier series of functions having invariance properties, in particular class functions, is markedly different.

A class function is a function f such that $f(gxg^{-1}) = f(x)$ for all g in G and almost all x in G. Let $L_I^p(G)$ denote the p-integrable class functions. For f in $L_I^p(G)$,

$$d_\lambda \chi_\lambda * f(g) = \left(\int f(x) \overline{\chi_\lambda(x)}\, dx \right) \chi_\lambda(g).$$

Let $n = \dim G$ and $l = \operatorname{rank} G = \dim T$.

We now assume that G is a simple, simply connected compact Lie group.

THEOREM B. *If $p > 2n/(n + l)$ and f is in $L_I^p(G)$ then $S_R f(g)$ converges to $f(g)$ for almost all g.*

THEOREM C. *If $p < 2n/(n + l)$ or $p > 2n/(n - l)$ there is an f in $L_I^p(G)$ such that $S_R f$ does not converge to f in the L^p norm.*

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REMARKS. Clerc [1] has proved Theorem A for spherical partial sums. The argument for polyhedral partial sums involves a reduction to the rank one case. If the rank of G is one, Theorem B is due to Pollard [4] while Theorem C was obtained by Wing [6]. For general rank a slightly weaker version of Theorem C was obtained by Stanton [5]. The proofs of our results are extensions of the rank 1 arguments coupled with Fefferman's results [2], [3]. A calculation of the integrability of powers of Weyl's Δ-function and a related function provided the critical indices.

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