review, not do they have quite as far-ranging reports on the literature. The authors claim that they are not specialists in the specific topic of this book—indeed, that it was their "hobby". Instead of disputing this claim, let us wish there were more such "dilettanti".

The style of the book, is readable, with some arid stretches. Very careful attention must be paid by the reader to terminology; the index is very helpful. It would be pleasant to report an absence of detected errors and misprints; unfortunately the book is riddled with them. A cursory reading of several sections detected about one per page. Most are trivial (wrong signs, 0 for ∞, misplaced exponents, etc.), quite a few are more serious (e.g., on p. 114, line 5, replace "the spectrum $\sigma(A)$ lies on the imaginary axis" by "the equation is bistable"[1]); none of those detected is crippling, but their accumulation is most annoying. More to the point, such carelessness in small things makes one wonder about the great ones that one would gladly trust.

Recommendation: a must for the specialist in stability theory (who does not need this review); an important reference book; a source—with very judicious selection—for an inspiring seminar or even a graduate course for enthusiasts.

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Indefinite inner product spaces, by János Bognár, Springer-Verlag, New York, 1974, 223 pp., \$19.70.

This is the first book on infinite-dimensional vector spaces with a signed inner product, a subject which frequently goes under the title 'spaces with an

[1] This is a translator's error.

indefinite metric.' The idea is to take a linear topological space, a Banach space, or most frequently a Hilbert space X, and then to consider a nonpositive (but nondegenerate) bilinear functional $\langle\ ,\ \rangle$ on it. One can study geometric properties such as what do $\langle\ ,\ \rangle$-orthogonal complements look like, or one can study operators on X which preserve, or diminish, or make positive the bilinear functional. The book presents basic material which is well established in the field and which one expects to have enduring value. Thus the substance of the book is ten or more years old while very complete notes direct the reader to recent work. The book is readable for someone with a little background in Banach and Hilbert space.

'Indefinite metrics' are widely studied in the Soviet Union and generally ignored here, so a discussion of what the subject entails is much in order. Historically, it began with a paper by Dirac in the early forties. Since then there have been prominent advocates of its use in quantum field theory to include Heisenberg and T. D. Lee, however, the importance here of 'indefinite metrics' is controversial. As usual, mathematicians took up the subject for entirely different reasons. About the only thing they used from quantum field theory is the name 'indefinite metrics' which would never have been coined by a mathematician since there is no way to get a metric from a signed inner product. Despite this cultural divergence the origins of mathematical work were physical. Pontryagin began the subject (1944) in studying a mechanics problem posed by Sobolev. M. G. Kreĭn and H. Langer gave a beautiful treatment of stability for the operator coefficient equation $Au_{tt}+Bu_t+Cu=0$ using indefinite metrics, while R. S. Phillips (1960) came to the area through an abstract partial differential equations problem.

There are two ways that one usually comes upon an indefinite metric problem. One has an operator, the operator preserves a bilinear form: but the form just is not positive. The other is through quadratic problems. A linear fractional map of the unit disk onto itself has four coefficients and the 2×2 coefficient matrix is symplectic; that is, it preserves a signed bilinear form. A fixed point of the linear fractional map corresponds to an invariant subspace of the matrix which is nonnegative with respect to the bilinear form. This structure carries through to very general situations including the case where the linear fractional map has operator coefficients and acts on the unit ball of operators on some Hilbert space. Whether or not a fixed point always exists is open and various conditions exist under which one does. The main result (L. S. Pontryagin-H. Langer-M. G. Kreĭn) says that if the off-diagonal entries of the coefficient matrix are compact operators, then a fixed point exists. It is an easy consequence of the Schauder-Tychonoff fixed point theorem.

The fixed point problem for linear fractional maps is the same as the problem of finding an operator root for a broad class of operator quadratic equations. Quadratic problems, even operator valued ones, are very important. For example, if we want an operator solution $U(t)=e^{Kt}$ for the partial differential equation above, then we must solve the quadratic equation $AK^2+BK+C=0$. This is the starting point for the stability analysis of Kreĭn and Langer. Other examples are plentiful and several prominent ones such as least squares quadratic control and invariant imbedding have not yet been approached by indefinite metric lovers.

As mentioned, one can consider quadratic problems or, equivalently, certain invariant subspace problems. Thus there is both an algebraic and a geometric approach to the subject. This review has leaned toward an algebraic presentation partly because the reviewer believes in its importance and partly to balance Bognár's viewpoint. His book takes the geometric view entirely. The book has an abstract flavor and goes from general to specific. The first two chapters give the elementary properties of a vector space X with indefinite bilinear form $\langle\ ,\ \rangle$ and of natural classes of operators on it (unitary, selfadjoint; Cayley transforms). The third imposes the additional structure of a locally convex topology on X which is 'compatible' with $\langle\ ,\ \rangle$. Frequently such a topology which is weak can be parlayed into one which is strong. For example a theorem (Wittstock) says that when such a topology is metrizable, then there actually exists a norm topology on X compatible with $\langle\ ,\ \rangle$. The latter part of the chapter discusses subspace complementation properties and projections on subspaces. The fourth chapter imposes a Banach space structure on X and proceeds in the same spirit as before. The fifth imposes a Hilbert space structure and gives basic geometric properties (which by this time are quite strong). The last half of the book treats operators on Hilbert spaces with an indefinite inner product and culminates in invariant subspace theorems of the type already described.

The book is carefully written, fastidiously organized, and thoroughly annotated. The approach taken has the usual virtues and flaws of going from general to specific; one can see more clearly exactly what structure is used to prove each theorem, but motivation is suppressed and the most important material which occurs at the end is less accessible. The emphasis of the book as traditional in the Ergebnisse series is not toward introductory evangelism. However, the book is well enough constructed so that a student could skip the more esoteric and abstract parts (Chapters 3 and 4) without much trouble and get a very good introduction to the subject. Bognár's book complements the excellent survey paper by Ju. L. Daleckiĭ and M. G. Kreĭn (Transl. Math. Monographs, vol. 43, Amer. Math. Soc., Providence, R.I., 1974) and a beginner should read the two together.

It is good to have the subject of indefinite inner product spaces well organized, well presented, and in a single book. Probably it will not be of much immediate interest to someone from another field whose primary concern is in quadratic problems. However, Bognár's book gives a very good treatment of the geometric aspects of the subject and it contains much of the material along that line which has reached final form.

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