BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 6, November 1971

SOME ORDERS OF INFINITE LATTICE TYPE

BY KLAUS W. ROGGENKAMP

Communicated by Hyman Bass, May 3, 1971

Let K be a p-adic number field with ring of integers R, and let Λ be an R-order in the finite-dimensional semisimple K-algebra A. By $n(\Lambda)$ we denote the number of nonisomorphic indecomposable left Λ-lattices. In case A is commutative, an indecomposable order Λ can only be of finite lattice type, if A decomposes into at most three simple modules (cf. Dade [1]). Our main result is, that this also holds in the noncommutative case.

THEOREM I. *Let e be a primitive idempotent of Λ (i.e., Λe is a principal indecomposable Λ-module). If Ae is the direct sum of t simple A-modules with $t \geqq 4$, then $n(\Lambda)=\infty$.*

With the help of Lemma 1, the proof of Theorem I can be reduced to completely primary totally ramified R-orders Λ; i.e., Λ is completely primary and $\Lambda/J(\Lambda) \simeq R/J(R)$, where $J(S)$ denotes the Jacobson radical of S.

LEMMA 1. *Let e be an idempotent of Λ and put $\Omega = \mathrm{End}_\Lambda(\Lambda e)$. Then $n(\Omega)=\infty$ implies $n(\Lambda)=\infty$.*

Let $\mathfrak{C}$ denote the class of completely primary totally ramified R-orders in A, where A is the direct sum of t simple A-modules, $t \geqq 4$. $\mathfrak{C}$ is a partially ordered set with maximal elements and it suffices to show that for a maximal element $\Lambda \in \mathfrak{C}$ we have $n(\Lambda)=\infty$. The structure of the maximal elements in $\mathfrak{C}$ is classified by

LEMMA 2. *Let Λ be a maximal element in $\mathfrak{C}$, then $\Lambda = R + J(\Gamma)$, where Γ is a hereditary R-order in A.*

We put $\mathfrak{k} = R/J(R) = \Lambda/J(\Gamma)$; then $\Gamma/J(\Gamma)$ is a ring which is also a $\mathfrak{k}$-module, and the hypotheses on A imply that $\dim_{\mathfrak{k}}(\Gamma/J(\Gamma)) \geqq 4$.

Now a technique of Dade [1] (Drozd-Roiter [2]) allows us to conclude $n(R+J(\Gamma))=\infty$.

AMS 1970 *subject classifications*. Primary 16A18, 16A66; Secondary 16A48.
Key words and phrases. Orders, finite lattice type, completely primary orders, number of indecomposable lattices.

REFERENCES

1. E. G. Dade, *Some indecomposable group representations*, Ann. of Math. (2) **77** (1963), 406–412. MR **26** #2521.

2. Ju. A. Drozd and A. V. Roĭter, *Commutative rings with a finite number of indecomposable integral representations*, Izv. Akad. Nauk SSSR Ser. Mat. **31** (1967), 783–798 = Math. USSR Izv. **1** (1967), 757–772. MR **36** #3768.

3. K. W. Roggenkamp, *Some necessary conditions for orders to be of finite lattice type*, J. Reine Angew. Math. (1972) (to appear).

UNIVERSITÄT BIELEFELD, 48 BIELEFELD, GERMANY