tional problems for multiple integrals of the form $\int F(x, u, u_x)dx$. The authors are able to prove acceptable formulations of the 19th and 20th Hilbert problems.

The general elliptic equation (4) is treated in Chapter 6. Derivative estimates are established under various growth assumptions on the coefficients leading to a discussion of the Dirichlet problem. Strong solutions with bounded first derivatives are shown to be smooth according to the smoothness of the data—a considerably weaker result than may be obtained for the divergence structure case mentioned above.

Many of the results of earlier chapters are extended to systems of equations in Chapters 7 and 8. However, a very specialized form of system is considered which permits an automatic extension of the earlier theory. It is on the theory of general uniformly elliptic systems where much current research interest is focused. In Chapter 9 the authors discuss alternate methods, based on works of Moser, Nirenberg, Morrey and others, for obtaining their estimates of previous chapters. A further approach to the Hölder theory has since been given by the reviewer based on Moser's work on Harnack's inequality. The final chapter treats some other boundary value problems.

The book is in a certain sense complete—the body of theory it presents is close to a final form. It would be of great use for a mathematician already somewhat expert in elliptic partial differential equations. However, for someone seeking an introduction to the theory of quasilinear elliptic equations, the present book would not be the appropriate source. A book of this kind still remains to be written.

N. S. TRUDINGER

Convex sets, by F. A. Valentine. McGraw-Hill Series in Higher Mathematics, McGraw-Hill, New York, 1964. ix+238 pp. $12.00.

In 1934 Bonnesen and Fenchel published a comprehensive survey of the geometry of convex bodies. More than half of it was directed toward quantitative aspects of the theory. By contrast, only a tenth of the present book is devoted to such aspects. This change reflects the developments of the last three decades, which have tended increasingly to emphasize the combinatorial, qualitative, and infinite-dimensional aspects of the theory.

Valentine's book consists of a preface, thirteen chapters (called "parts"), an appendix, a bibliography, and a subject index. Headings of the parts are as follows: I. Basic concepts, II. Hyperplanes and the separation theorem, III. The Minkowski metric, IV. Some characterizations of convex sets, and local convexity, V. The support function and the dual cone, VI. Helly-type theorems, VII. Further characterizations of convex sets, VIII. Properties of S determined by conditions on each set of r points of S, IX. Maximal convex subsets and upper semicontinuous decompositions, X. Convex functions, XI. Special boundary points, XII. Some properties of convex sets in L_n and E_n, XIII. Exercises, propositions, and problems. Part XIII is divided into twelve subparts, each related to an earlier part of the book and containing unsolved problems as well as improvements or relatives of earlier results. The appendix contains a list of notation, a summary of elementary material from set theory, set topology, and functional analysis used in the book, and some indication of the author's personal approach to mathematics. The bibliography lists 255 books and articles that are mentioned in the text and 211 "supplementary references," with no indication of the manner in which the latter were chosen or of their specific relevance to the text.

In the reviewer's opinion, the most significant strengths of the book are the following:

(S1) the treatment of specific areas (Parts IV–IX) to which the author himself has made fundamental contributions;

(S2) the posing of a number of unsolved problems, some of which have led to research publications by readers of the book;

(S3) the well-conceived drawings;

(S4) the author's informal asides and sense of humor.

The book's principal weakness seems to be a lack of material relating convexity to other areas of mathematics, though there are many such relations that could have been included.

Part I states that "The theorems and definitions reflect the author's primary interests in this area, and it would be most surprising if the reader did not observe the absence of a choice theorem of his own." Indeed, the author includes parts of the qualitative and combinatorial theory (for example, those in Parts VIII and IX) that make virtually no contact with the rest of convexity theory or with other areas of mathematics, while giving short shrift to other parts (most notably, the study of convex polyhedra) that make many contacts. While this militates against the use of Valentine's book as the sole text for a course in convexity, it is not necessarily bad, for some of the material treated carefully in his book is not available in any other text and the combinatorial theory of convex polytopes has been ably treated by B. Grünbaum (*Convex polytopes*, Wiley, New York, 1967).

The preface concludes as follows: ". . . Although we abhor mistakes, we also take comfort in the following thought attributed to a Chinese philosopher: 'If you write a letter to a friend, be certain to include at least one mistake. In this way you will make him feel superior'." The author does include a few minor errors—just enough to make the reader happy—but in no way do they impair the usefulness of his book.

The book is designed for a one-semester graduate course and seems, aside from the above reservation about choice of material, to be well suited to its purpose. Parts of it have also been used successfully in courses for undergraduates who have some understanding of the fundamentals of linear algebra and of the rudiments of Euclidean topology.

The book has been translated into German by E. Heil and published in 1968 by the Hochschultaschenbücher-Verlag, Bibliographisches Institut, Mannheim.

VICTOR KLEE

Nonlinear two point boundary value problems, by Paul Bailey, Lawrence Shampine, and Paul Waltman. Math. in Science and Engineering, vol. 44, Academic Press, New York, 1968. xiii+171 pp. $9.50.

In the preface, the authors state "the purpose of this book is to describe as simply as possible a number of the ideas and methods which seem to be particularly helpful in the study of nonlinear boundary value problems for differential equations of second order." The authors are primarily interested in establishing existence and uniqueness theorems for solutions of two point boundary value problems on as large an interval as possible for second order differential equations of the form

$$y'' + f(t, y, y') = 0. \tag{1.1}$$

The only two point problems considered in this monograph are the *first boundary value problem* on $[a, b]$ where the value of the unknown function $y(t)$ is prescribed at each end of the interval

$$y(a) = A, \qquad y(b) = B, \tag{1.2}$$

and the *second boundary value problems* where the value of $y(t)$ is prescribed at one end and the slope is prescribed at the other, either

$$y(a) = A, \qquad y'(b) = m \tag{1.3}$$

or