10. ———, *On a definition of the integral in topological spaces.* I, II, Časopis Pěst. Mat. 89 (1964), 129–147, 257–277.

11. ———, *A note on the lower derivate of a set function and semihereditary systems of sets*, Proc. Amer. Math. Soc. 18 (1967), 1020–1025.

12. ———, *On the lower derivate of a set function*, Canad. J. Math. 20 (1968), 1489–1498.

13. ———, *The integral in topological spaces.* I and II (to appear).

14. ———, *Singular integrals are Perron integrals of a certain type* (to appear).

15. W. F. Pfeffer and W. J. Wilbur, *A note on cluster points of a semihereditary stable system of sets*, Proc. Amer. Math. Soc. 21 (1969).

16. ———, *On the measurability of Perron integrable functions* (to appear).

17. W. J. Wilbur, *On non-absolute integration in topological spaces*, Ph.D. thesis, University of California, Davis, Calif.

18. S. Saks, *Theory of the integral*, Hafner, New York, 1937.

UNIVERSITY OF CALIFORNIA, DAVIS, CALIFORNIA 95616

ON GENERALIZED COMPLETE METRIC SPACES

BY JAMES D. STEIN, JR.

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The following remarks are of interest in connection with the research announcement [1]:

LEMMA. *A generalized metric space is the disjoint union of metric spaces such that each metric space is infinitely distant from every other metric space.*

PROOF. Note that $d(x, y) < \infty$ is an equivalence relation, and the equivalence classes obtained are metric spaces. Also, if the generalized space is complete, so is each metric space. Q.E.D.

Let $M = \bigvee_{\alpha \in A} M_\alpha$ denote the above partitioning. The Banach contraction principle becomes

PROPOSITION 1. *Let T be a strict contraction of a generalized complete metric space* $M = \bigvee_{\alpha \in A} M_\alpha$, $0 \leqq q < 1$, $d(x, y) < \infty \Rightarrow d(Tx, Ty) \leqq qd(x, y)$. *For each* $\alpha \in A$, $\exists \beta \in A$ *such that* $T(M_\alpha) \subseteq M_\beta$. *There is a unique periodic point of order n in each* M_α *such that* $T^n(M_\alpha) \subseteq M_\alpha$.

PROOF. Let $x, y \in M_\alpha$, $Tx \in M_\beta$. Then $d(x, y) < \infty \Rightarrow d(Tx, Ty) < \infty \Rightarrow Ty \in M_\beta$. Since T^n is a strict contraction of the complete metric space M_α, it has a unique fixed point, which is a periodic point of order n for T. Q.E.D.

The local contraction principle becomes

PROPOSITION 2. *Let T be a local contraction* $(d(x, y) \leqq C \Rightarrow d(Tx, Ty) \leqq qd(x, y))$ *of a complete generalized metric space. For each* $\alpha \in A$, $x, y \in M_\alpha$, *define* $x \sim y$ *iff* $\exists x_0, \cdots, x_n \in M_0$ *such that* $x = x_0$, $y = x_n$, $d(x_i, x_{i+1}) \leqq C$ *for* $0 \leqq i \leqq n-1$. *Then* $\sim$ *is an equivalence relation on each* M_α; *call the equivalence classes thus obtained C-components. T maps each C-component of* M_α *into a C-component of some* M_β. *There is a unique periodic point of order n in each C-component N of* M_α *such that* $T^n(N) \subseteq N$.

PROOF. Clearly $x \sim y$ is an equivalence relation; and if $x, y \in M_\alpha$ and $x \sim y$, let $x = x_0, \cdots, x_n = y$ be the chain. Then $d(Tx_i, Tx_{i+1}) \leqq qd(x_i, x_{i+1}) \leqq C$, and so $Tx \sim Ty$ in some M_β. The remainder is Theorem 1.4 of Bonsall (or Edelstein) of *On some fixed point theorems of functional analysis*. Q.E.D.

Several of Edelstein's and Rakotch's results go over analogously.

REFERENCE

1. J. B. Diaz and Beatriz Margolis, *A fixed point theorem of the alternative, for contractions on a generalized complete metric space*, Bull. Amer. Math. Soc. **74** (1968), 305–309.

UNIVERSITY OF CALIFORNIA, LOS ANGELES, CALIFORNIA 90024