Morse theory. By J. Milnor, based on lecture notes by M. Spivac and R. Wells. Annals of Mathematics Studies No. 51, Princeton University Press, Princeton, N. J., 1963. vi + 153 pp. $3.00.

The theory of Marston Morse deals with the topological analysis of a manifold or a function space together with a real function on this space. Calculus of variations in the large was originally the main purpose of the theory. A typical subject was the study of the geodesics connecting two given points in a complete Riemannian n-manifold.

In recent years a number of important applications have considerably increased the interest in Morse theory.

Along the line of calculus of variations, Bott used Morse theory to obtain his famous result that the infinite unitary group

$$U = \lim_{n=\infty} (U(n))$$

is weakly homotopy equivalent to its second loop space:

$$U \overset{h}{\sim} \Omega^{\lambda}(U),$$

and analogously for the infinite orthogonal group:

$$O \overset{h}{\sim} \Omega^{\delta}(O).$$

These theorems have many interesting consequences.

Other applications concern topological consequences of assumptions on the curvature of a Riemannian manifold.

On the other hand, Smale used nondegenerate smooth functions on closed manifolds and started a new line for the study and classification of manifolds. In particular he proved the Poincaré conjecture for smooth homotopy spheres of dimension $\geqq 5$. The interest in the theory also increased, because it has been lifted in the framework of infinite-dimensional manifolds. Eells, Palais and others use this tool.

Milnor's book is a lucid rapid introduction to the subject, with a highly geometrical flavour. It is well written, and points to many subjects of current research.

A survey of the contents is as follows.

Part I leads to the Morse inequalities for a nondegenerate smooth function on a smooth n-manifold. As an application, Lefschetz's theorem on hyperplane sections of a complex algebraic manifold is given.

Part II is called "A rapid course in Riemannian geometry," and is surprisingly rapid and efficient, indeed. The covariant derivative, Christoffel symbols, curvature tensor, completeness and exponential map are treated among others.

Part III gives the calculus of variations of the "energy" function on the (piecewise smooth) path space of a Riemannian manifold. It deals, therefore, with geodesics and Jacobi-fields. Some relations between curvature and topology are presented as applications.

In Part IV, Bott's periodicity theorems are proved. A new method suggested by the paper *Clifford modules* by Atiyah, Bott and Shapiro [Topology **3** (1964), 3–38], is presented. Observe that very recent new methods of Bott, Atiyah, and Wood avoid the use of Morse theory altogether. It is now possible to go fairly directly from a periodicity phenomenon of Clifford modules to Bott's periodicity theorems.

We hope that other lecture notes of the author will be transformed into books as well!

N. H. KUIPER