square free integer, they work out the ramified, inertial and decomposed primes. Considering cyclotomic fields they derive the quadratic reciprocity law. The book closes with a theorem of Kummer.

The book should easily adapt itself as a text for a year course, early in the training of a graduate student. The reviewer feels that the average graduate student will find the book difficult to read but that his efforts will be well compensated. The book has enough rough edges so that the student will see the work and effort that go into a piece of mathematical work, something he will seldom appreciate in the very highly polished pieces of art sometimes presented to him when he is not quite ready to digest them or to benefit from them. The book is not written as a "text" book, so is devoid of problems. The book also suffers, especially in the earlier parts, from a lack of discussion of examples. Both these faults can easily be remedied by the introduction of supplementary material by the instructor. The instructor will, in many cases, feel a need to amplify the portions of the book dealing with Galois theory. Since the authors left valuation theory for a later volume one might have to supply material on this topic if it is wanted in the course.

Nowadays the fashion in the teaching of our graduate students seems to have become their introduction to highly formal gadgetry at a very early stage of the student's development. The greatest merit of this book is that it is down to earth and genuine. Its acceptance as a standard text should be a force in checking the drift to higher and higher formal structures and abstraction upon abstraction. The authors set out to fill a void in the mathematical literature. They have done this admirably.

I. N. HERSTEIN

Ordinary difference-differential equations. By Edmond Pinney. University of California Press, Berkeley 4, California, 1958. 12+262 pp. $5.00.

This work represents the only exposition, so far as the reviewer is aware, of the recent development of difference-differential equations stimulated by modern electronic control mechanisms. The author presents his own development of the subject, which in some places includes previously unpublished results. He gives a good list of references to the many recent papers published on the subject. He has not written a research treatise, in that he does not attempt even to state all the results achieved in these papers, many of which are not included in his own development. This book could serve as a text for a rather advanced graduate engineering course, although a very mathematically inclined one.

To go into more detail concerning the content of the book, a typical ordinary difference-differential equation as considered here would be the linear nth order one

$$(1) \qquad \sum_{p=0}^{n} \left\{ \sum_{k} c_{p,k}(t, \mathbf{x}) \frac{\partial^p}{\partial t^p} y(t - h_k, \mathbf{x} - {}_k\mathbf{z}) \right\} = 0$$

where t is a real variable and $\mathbf{x} = (x_1, x_2, \cdots, x_m)$ with x_i real, $y(t, \mathbf{x})$ is the unknown solution function of t and $\mathbf{x}$, $c_{p,k}(t, \mathbf{x})$ are given coefficient functions, and $\sum_k$ is a finite sum with given fixed real $h_k \geqq 0$ and real ${}_kz_i$, ${}_k\mathbf{z} = ({}_kz_1, {}_kz_2, \cdots, {}_kz_m)$. The author's use of the word "ordinary" here refers to the fact that only differentiation with respect to t arises in (1); if not all the h_k's in (1) are zero, he calls (1) mixed. By replacing $\sum_k$ in (1) by Stieltjes integrals, integro-differential equations are also considered. For clarity in this review, we will call (1) one-dimensional if $\mathbf{x}$ is suppressed in (1) (this might better be called "ordinary" and the remaining cases referred to as "partial" difference-differential equations).

With this nomenclature, the book may be described briefly as follows. The first chapter lists some facts about integral transforms, chiefly Fourier and Laplace. In the next chapter these facts are used to give existence, uniqueness, and representation theorems for systems of one-dimensional, linear difference-differential equations with constant coefficients. The representation is in terms of an infinite sum over the infinitely many zeros of the characteristic exponential polynomial, which for the one-dimensional case of (1) with $\mathbf{x}$ suppressed and $c_{p,k}(t)$ constant in t, is

$$(2) \qquad D(z) = \sum_{p=0}^{n} \left\{ \sum_{k} c_{p,k} z^p e^{-zh_k} \right\}.$$

In the third chapter the author gives a detailed analysis of the zeros of such $D(z)$, and in chapters four, five, and six he works out the details of his chapters two and three results for various special equations.

Chapters seven and eight deal with various special non-one-dimensional, nonmixed linear difference-differential equations with constant coefficients. This material has virtually no connection with the rest of the book.

In chapter nine the author presents his development of "trend equations," influenced by the heuristic Kryloff-Bogoliuboff calculations of Minorsky, to study the behavior as $t \to +\infty$ of solutions near the zero solution of one-dimensional difference-differential equations which are linear with constant coefficients except for the addition of polynomial nonlinear terms. The results of chapter nine are applied to some rather simple special cases in chapter ten, and in chapter eleven to a less simple equation of Minorsky. An appendix gives the proof of some theorems with which the author did not wish to burden the text.

Passing to an evaluation of this work, the reviewer considers it a valuable stimulus to mathematical interest in a wide open field which may perhaps be characterized as elementary but difficult, a field which in spite or perhaps because of its applied importance will probably not attract the mathematical talents it deserves from the present abstraction-mad generation of mathematicians. The mathematical level of the exposition is, unfortunately, not all that could be desired. For example, on pages 8 and 9 the author suffers from the misapprehension that uniform convergence over the infinite real line permits interchange of limits with ordinary integration from $-\infty$ to $+\infty$, despite well-known counter-examples. Again in the author's statement of the inversion of Fourier integrals, the reader must provide for himself the specification of the sense of the integral which makes the statement true. Also the use of capital O symbols in several variables at once makes a number of the author's arguments extremely confusing to follow.

The essential mathematical content of the book splits into two parts. First comes the treatment of linear, one-dimensional equations with constant coefficients by means of the characteristic exponential polynomials in chapters two and three. Although this is fairly standard material by now, the representation of the solution by a convergent infinite sum over the zeros of the characteristic exponential polynomial, instead of a finite sum plus an error estimate, was new to the reviewer at least. Secondly comes the treatment of nonlinearities, chapter nine with application to Minorsky's equation in chapter eleven, by means of "trend equations." These important tools, in origin due to the author, permit one to decide about the stability of nonzero but small periodic solutions. We remark that these results, although in agreement with, do not strictly include those of the reviewer (see references) proving the existence of (with no consideration of stability) and giving lowest order approximate formulae for small, nonzero periodic solutions by means of the Schmidt-Iglisch branching technique for integral equations. For example, by these methods of the reviewer, Equation (11.24) under Case 2, p. 215 for the periodic solution with R replaced by R_0 and for t ranging over a single period has the error estimate

$$O(\epsilon) + O\left(\frac{\epsilon^2}{\delta r}\right) + O\left(\left|\frac{(\delta r)^3}{\epsilon}\right|^{1/2}\right),$$

which is only meaningful if $\epsilon \to 0$ with $\delta r \to 0$ in such a way that $\epsilon/(|\delta r|)^{1/2} \to 0$ and $|\epsilon|/|\delta r|^3 \to +\infty$, to be replaceable by

$$O(\delta r)$$

as $\delta r \to 0$ for fixed $\epsilon \neq 0$.

F. H. Brownell

Theorie der Limitierungsverfahren. By K. Zeller. Ergebnisse der Mathematik und ihrer Grenzgebiete, New Series, vol. 15. Berlin, Gottingen, Heidelberg, Springer, 1958. 8+242 pp. DM 36.80.

This book is intended as a survey of the literature of the topic. To this end the bibliography contains over 2000 items! However the author has achieved far more than a survey, for there is running through the pages a hard core of self-contained material lying at the heart of the subject. A consistent notation and terminology are used and the contents of the bibliography are exposed by means of them. Thus it is possible to learn from the book itself without consulting the references, although at times proofs are compressed to a density rivalling that of a white dwarf. In spite of this the author has taken great pains to isolate the fundamental ideas (e.g. gleitende Buckel, Mittelwertsatz) behind the works of the giants of classical analysis such as G. H. Hardy. In addition much space is given to the important and extensive contributions of functional analysis (described somewhat condescendingly by Hardy as soft analysis).

After an interesting sketch of pre-twentieth century ideas—laced with metaphysics—on divergence, there is a section on definitions of limit. The book treats mainly those given by matrices. Multiple sequences, nonmatrix methods: strong and absolute summability, integral transforms, are dismissed with references. Section 5 lists the main problems.

In Chapter 2 the discussion proper begins. Descriptions of Banach and F spaces and their duals are given. For example proofs of the Baire category and Hahn-Banach theorems are given as well as three types of proof of the uniform boundedness principle. The FK space then appears; this highly useful object is an F sequence space with continuous coordinates. Banach algebras and Fourier trans-