THE OCTOBER MEETING IN CAMBRIDGE

The five hundred twenty-seventh meeting of the American Mathematical Society was held at the Massachusetts Institute of Technology on Saturday, October 27, 1956, in conjunction with a meeting of the Society for Industrial and Applied Mathematics. About 240 persons attended, including 200 members of the Society.

An address entitled *Coverings of algebraic varieties* was presented by Professor J-P. Serre of the Collège de France and the Institute for Advanced Study by invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings. Professor Richard Brauer presided.

Sessions for contributed papers were held in the morning and afternoon, with Professors A. P. Calderón, J. B. Diaz, Dr. A. P. Mattuck, Professors L. N. Howard, D. J. Struik presiding.

Abstracts of the papers presented follow. Where a paper has more than one author, that author whose name is followed by "(p)" presented it. Those papers with "*t*" following their numbers were presented by title. Dr. Edwards was introduced by Mr. D. G. Kendall, and Mr. Manly by Mr. R. R. Christensen.

ALGEBRA AND THEORY OF NUMBERS

1. A. T. Brauer: *On the theorems of Ledermann and Ostrowski on positive matrices.*

Let A be a positive matrix of order n, and R_ν the sum of the elements of the νth row. We set $\max R_\nu = R$ and $\min R_\nu = r$. A well known theorem of Frobenius [Sitzungsberichte Preussische Akademie der Wissenschaften (1998) pp. 471–476 and (1909) pp. 514–518] states that A has a greatest positive characteristic root ω. Moreover, we have $R \geqq \omega \geqq r$. For $R = r$, these inequalities cannot be improved. Assume now that $R > r$. We denote the smallest element of A by m. W. Ledermann [J. London Math. Soc. vol. 25 (1950) pp. 265–268] and A. Ostrowski [J. London Math. Soc. vol. 27 (1952) pp. 253–256] obtained better bounds which are functions of R, r, and m. In this paper the best possible bounds of this kind and some similar results are obtained. (Received September 10, 1956.)

2*t*. Barron Brainerd: *On a class of lattice-ordered rings*, I.

An F-ring R is a σ-complete vector lattice which is also a commutative algebra with unit $1 > 0$ in which $x \geqq 0$, $y \geqq 0 \Rightarrow xy \geqq 0$ and $x \wedge 1 = 0 \Rightarrow x = 0$. Let R be regular, that is, for each $x \in R$ there exists $x^0 \in R$ such that $xx^0x = x$. The sub-F-ring $\overline{R} = \{x \mid x \in R, |x| \leqq \lambda \cdot 1 \text{ for some real } \lambda\}$ is a real Banach algebra. A (ring) ideal J of an F-ring is *closed* if $a_n \in J$ $(n \geqq 1)$ and $\sup_n a_n$ exists imply that $\sup_n a_n$ belongs to J. A maximal ideal M of R is closed if and only if $R - M$ is isomorphic to the real field. There is a one-to-one correspondence between the maximal ideals of R and those of $\overline{R}$. For a maximal ideal M of R to be closed, it is necessary and sufficient that $M \cap \overline{R}$ be a maximal ideal of $\bar{R}$. There is a one-to-one correspondence between the closed maximal ideals of R and those of $\bar{R}$. If the intersection of the closed maximal ideals of R is zero, then R is (F-ring) isomorphic to the F-ring of all $(\Phi, \mathfrak{J})$-measurable functions for a pair $(\Phi, \mathfrak{J})$ which is defined in terms of R. A real function f on Φ is $(\Phi, \mathfrak{J})$-*measurable* provided $\{x \mid f(x) \leqq \lambda\}$ belongs to the σ-algebra $\mathfrak{J}$ for each real λ. These results are obtained by exploiting the concept of projector as defined by S. Kakutani [Ann. of Math. vol. 42 (1941) pp. 223–237]. (Received August 8, 1956.)

3t. Barron Brainerd: *On a class of lattice-ordered rings*, II.

The terminology and notions of I are used. The set I of idempotents of an F-ring A forms a σ-complete Boolean algebra (σ-B.A.) under the lattice operations of A. I is the *idempotent algebra* (i.a.) of A. Every σ-B.A. I is the i.a. of a regular F-ring $R(I)$. A result of Sikorski [Fund. Math. vol. 35 (1949) pp. 247–258] is used to construct $R(I)$ which is a uniquely determined homomorph of the F-ring of all $(\Phi, \mathfrak{J})$-measurable functions for a pair $(\Phi, \mathfrak{J})$ defined in terms of I. Olmsted [Trans. Amer. Math. Soc. vol. 51 (1942) pp. 164–193] has constructed for each σ-B.A. I an F-ring $\Omega(I)$ with I as its i.a. If R is a regular F-ring with i.a. I, then R is isomorphic to both $\Omega(I)$ and $R(I)$. Every F-ring having I as i.a. is isomorphic to a sub-F-ring of $R(I)$. Let B be a semi-ordered ring with unit $1>0$ which is archimedean and in which $\inf_{\gamma\in\Gamma} a_\gamma = 0$ and $b \geqq 0 \Rightarrow \inf_{\gamma\in\Gamma} a_\gamma b = 0$. From results of Nakano [*Modern spectral theory*, Chapter V], it follows that B can be imbedded in an F-ring R. Therefore there is a σ-B.A. I such that B is isomorphic to a sub-semi-ordered ring of $R(I)$. (Received August 8, 1956.)

4. Walter Feit: *Arithmetic in algebras*. Preliminary report.

Let A be a semi-simple algebra over a field K, D an integral domain whose quotient field is K. Pick a basis of A such that the D module generated by this basis forms a ring R with a unit element. The ring R can then be written as a direct sum of a finite number of indecomposable left ideals. A left ideal is a direct summand if and only if it contains an idempotent element. If K is a p-adic field and D the ring of integers in K, then there is a natural one to one correspondence between the direct summands of R and those of the algebra R-DR. If K is an algebraic number field and D is suitably chosen, then it is possible to study arithmetical properties of A by considering the decomposition of R. In particular if A is the group algebra of a finite group G, then known arithmetical properties of the characters of G can be extended in this manner. (Received September 6, 1956.)

5. I. N. Herstein: *A theorem on the characteristic roots of symmetric matrices*.

The following theorem is proved: let A be a real symmetric matrix and let $r_1(A) \geqq r_2(A) \geqq \cdots \geqq r_n(A)$ be its characteristic roots; then each $r_i(A)$ is a monotonically increasing function of each diagonal element of A, that is $\partial r_i(A)/\partial a_{jj} \geqq 0$. If B is the Jacobian matrix of the $r(A)$'s with respect to the diagonal elements of A then B is a stochastic matrix. (Received September 7, 1956.)

6t. M. A. Hyman: *Eigenvalues and eigenvectors of real nonsymmetric matrices*. Preliminary report.

By $(N-1)(N-2)/2$ real rotations, the real nonsymmetric matrix A of order N is transformed into matrix B with all zeros above the first super-diagonal: $B = R^{-1}AR$. The eigenvalue problem for B is then $(B-\lambda I)x = 0$ where x is a column vector with components $x_1, x_2, \cdots, x_N$. Setting $x_1=1$ and guessing λ, one calculates $x_2, x_3, \cdots, x_N$ and $F(\lambda)$ recursively; $F(\lambda)$ is the characteristic polynomial, evaluated for the trial λ. By successive approximation (see abstract below) a sequence of λ's (in general complex) is obtained which approaches an eigenvalue $\bar{\lambda}$ of $B: F(\bar{\lambda})=0$. Simultaneously, the vectors x approach the eigenvector $\bar{x}$. The corresponding eigenvalue and eigenvector of A are $\bar{\lambda}$, $R\bar{x}$. Using $\bar{x}$, B, one easily obtains matrix C, in form like B but of degree $N-1$ which has the same eigenvalues as A (less $\bar{\lambda}$ once). He repeats the process above to get an eigenvalue and eigenvector of C, then obtains matrix D of order $N-2$, etc. At any stage, the corresponding eigenvector of A is obtained by a single matrix times vector multiplication. If one has some information about the location of the eigenvalues of A, he can avoid the reduction process sketched above in whole or in part; this will save time and increase somewhat the accuracy of the results. (Received September 10, 1956.)

7t. M. A. Hyman: *Finding complex roots by interpolation.* Preliminary report.

Consider $F(\lambda)$, a general function of the complex variable λ. It is not assumed that $F(\lambda)$ is known explicitly, but that, given λ, $F(\lambda)$ can be found from a rule (for example, built up by recursion). Assuming the existence of a root of $F(\lambda)$ in a certain neighborhood, one obtains from three approximations to the root, $\lambda_1, \lambda_2, \lambda_3$, the approximation λ_4; λ_4 is a particular root of the quadratic function passing through $\lambda_1, \lambda_2, \lambda_3$. From λ_4 and the "best two" (in a certain sense) of $\lambda_1, \lambda_2, \lambda_3$, one forms λ_5. Continuing, a sequence $\{\lambda_i\}$ is constructed which usually converges to the desired root $\bar{\lambda}$. The method is designed for use with automatic computing machines. It is perhaps slower than more sophisticated methods which exploit differentiability, algebraicity, etc. of $F(\lambda)$, but appears simpler to carry through and enables the machine to handle a wider class of problems without special additional programming. The method can be speeded up using "acceleration" techniques. It causes the computing machine to use "real arithmetic" when pursuing a real root, but shift automatically to "complex arithmetic" when the root is complex (thus saving computer time). This "interpolation" method can be applied using 2, 3, 4, $\cdots$ points at each step, but to date the 3-point method described here seems the most useful. (Received September 10, 1956.)

8. H. G. Jacob: *Coherence invariant mappings of symmetric transformations.*

L. K. Hua (Amer. J. Math. vol. 50 (1949) pp. 8–31) has characterized one-to-one coherence invariant mappings of the set of $n\times n$ symmetric matrices over a field. A generalization of Hua's results is obtained by considering a nonsingular hermitian scalar product (x, y) defined on a vector space $\mathfrak{X}$ of dimension $\geqq 3$ over a field of characteristic $\neq 2$. A linear transformation T on $\mathfrak{X}$ of finite rank may be written as $T=\sum_1^n x_i\otimes y_i$; i.e. $zT=\sum_1^n (z, x_i)y_i$. A transformation is called symmetric if it equals its adjoint relative to the hermitian form. Under these conditions any one-to-one coherence invariant mapping σ of the set of symmetric transformations of finite rank is of the form $T^\sigma=\sum_1^n x_i\overline{A}\otimes y_iA+S$, where A is a semi-linear transformation on $\mathfrak{X}$ with an associated automorphism which commutes with the automorphism associated with the hermitian form. $\overline{A}=cA$ where c is a scalar, and S is a fixed symmetric transformation. In the event that A has an adjoint $T^\sigma=\overline{A}^*TA+S$, where $\overline{A}^*$ is the adjoint of $\overline{A}$. (Received September 10, 1956.)

9t. J. A. Kalman: *Lattices with involution.*

I. Nondistributive (but possibly modular) lattices with involution, or "i-lattices," are classified with respect to six laws each of which, for distributive i-lattices, is equivalent to normality (cf. Bull. Amer. Math. Soc. Abstract 61-4-536). II. It is proved that for each given normal i-lattice L there exists one and to within i-isomorphism only one normal i-lattice with zero L_Ω [closed normal i-lattice L^C] having the property: there exists an i-isomorphism π of L into L_Ω [L^C] such that every i-isomorphism τ of L into a normal i-lattice with zero M [closed normal i-lattice N] is of the form $\tau = \pi\rho$ for some i-isomorphism ρ of L_Ω [L^C] into M [N]. (An "i-isomorphism" is a lattice-isomorphism which preserves the involution; a normal i-lattice L is "closed" if, in L_Ω (with involution $z \rightarrow z'$ and zero θ), $x \wedge y = \theta$ implies $x = z \vee \theta$ and $y = z' \vee \theta$ for some z). III. The i-lattice with four elements and two zeros is shown to be the only subdirectly irreducible non-normal distributive i-lattice. (Research supported by the University of New Zealand Research Fund.) (Received September 11, 1956.)

10t. Joachim Lambek: *Subgroups of the direct product of two groups.*

Given groups A, B, C, let R and S be subgroups of $A \times B$ and $B \times C$ respectively, their *relative product* $R \cdot S$ is a subgroup of $A \times C$. Let R^- denote the *converse* of R, then $R \cdot R^- \cdot R = R$. Hence $R \cdot R^-$ and $R^- \cdot R$ are congruence relations on subgroups of A and B respectively, and R induces an isomorphism between the corresponding subfactor groups. This fact appears to have been discovered by Goursat. In particular, if R and S are congruence relations on subgroups of a given group, then $R \cdot S$ induces the isomorphism of the well-known Zassenhaus lemma. These results hold not only for groups, but for all algebras with a ternary operation $f(x, y, z)$ such that $f(x, y, y) = x$ and $f(y, y, z) = z$. Examples are loops with $f(x, y, z) = (x/y) \cdot z$ and quasigroups with $f(x, y, z) = (x \cdot (y \backslash y))/(z \backslash y)$ [see A. I. Mal'cev, Mat. Sbornik N.S. vol. 35 (1954) pp. 3–20]. To obtain Goldie's generalization of the Zassenhaus lemma [Proc. London Math. Soc. vol. 52 (1950) pp. 107–131], one must replace $R \cdot S$ by its "difunctional closure" due to J. Riguet [Comptes Rendus vol. 230 (1950) pp. 1999–2000]. (Received September 10, 1956.)

11. Marvin Marcus: *Convex functions of quadratic forms.*

Let $a = (a_1 \cdots, a_n)'$ be a vector in the real n-space, E_n, and define the convex set $M(a)$ as the intersection of the half-spaces (I) $\sum_{j=1}^k t_{i_j} \leqq \sum_{j=1}^k a_j$, $1 \leqq k \leqq n-1$, $1 \leqq i_1 < \cdots < i_k \leqq n$ and the hyperplane (II) $\sum_{j=1}^n t_j = \sum_{j=1}^n a_j$. Let H be the convex hull of the points Pa as P ranges over all n-square permutation matrices. Results (i) *If f is convex on H then* $\max_{t \in M(a)} f(t) \leqq \max_P f(Pa)$. (ii) *If f is convex on E_n and nondecreasing in each variable and $N(a)$ is the convex intersection of the half-spaces* (I) *then* $\max_{t \in N(a)} f(t) \leqq \max_P f(Pa)$. (iii) *If A is n-square Hermitian with eigenvalues $\lambda_1 \geqq \cdots \geqq \lambda_n$ and $f(t_1, \cdots, t_k)$ is convex on the hypercube C defined by $\lambda_n \leqq t_j \leqq \lambda_1$, $k \leqq n$; and $\partial f/\partial t_j = 0$ holds for at most one value of $t_j \in (\lambda n, \lambda_1)$ for each fixed $t_1, \cdots, t_{j-1}$, $t_{j+1}, \cdots, t_k$ then* $\max_{(x_i, x_j) = \delta_{ij}} f[(Ax_1, x_1), \cdots, (Ax_k, x_k)] = \max_{1 \leqq i_j \leqq n} f(\lambda i_1, \cdots, \lambda i_k)$ *and a maximizing set of vectors $x_1, \cdots, x_k$ spans a k-dimensional invariant subspace of A.* For $f = \sum_{j=1}^k c_j t_j$, $c_j \neq 0$, (iii) is pertinent to a question raised by H. Wielandt (Proc. Amer. Math. Soc. vol. 6 (1955) p. 109). For $f = \sum_{j=1}^n \omega(t_j)$, (i) and (ii) follow from an inequality due to G. Polya (Proc. Nat. Acad. Sci. U.S.A. vol. 36 (1950) pp. 49–51). (Received September 7, 1956.)

12t. D. W. Robinson: *A characterization of n-groups.*

The generalized groups of W. Dörnte [*Untersuchungen über einen verallgemeinerten Gruppenbegriff*, Math. Zeit. vol. 29 (1928) pp. 1–19] are systems of elements with a polyadic operation satisfying an extension of the associativity and solvability axioms for ordinary groups. This note points out that these systems can be characterized as well by replacing the solvability axiom with a generalization of the identity—inverse axiom for groups. (Received July 11, 1956.)

13. Tadao Tannaka: *On the principal ideal problem and solution of a conjecture of Deuring.*

Professor Deuring proposed a conjecture concerning the precision of principal ideal theorem, at the conference on algebraic number theory which was held in Tokyo in September 1955. His conjecture runs as follows: Let K be the absolute class field over k, then for every k-ideal $\mathfrak{a}$ we can assign suitable element $\theta(\mathfrak{a})$ in K with $\mathfrak{a}=(\theta(\mathfrak{a}))$ and $\theta(\mathfrak{a})\theta(\mathfrak{b})\sigma(\mathfrak{a})/\theta(\mathfrak{a}\mathfrak{b}) \in k$ where $\sigma(\mathfrak{a})$ is Artin-automorphism of $\mathfrak{a}$. The author reduced this to the following lemma, which itself is a general principal ideal theorem and contains several known theorems as special cases. Let K be the absolute class field over k, K the direct compositum of cyclic intermediate fields K_i $(i=1, 2, \cdots, r)$ and $\mathfrak{p}_i$ be prime ideals in k which remain prime in K. Then there are K_i-ideals $\mathfrak{a}_i$ with $\mathfrak{p}_i \sim \mathfrak{a}_i^{1-\sigma_i}$ and product of these $\mathfrak{a}_i$ $(i=1, 2, \cdots, r)$ is principal in K. This lemma and the conjecture of Deuring are also generalized to the case of "Strahl" class field. (Received October 22, 1956.)

14t. George Whaples: *Cohomology of additive polynomial and nth power mappings of fields.*

Let K be a field of characteristic p and $f(x)$ an additive polynomial. The exact sequence $0 \rightarrow \text{kernel } f \rightarrow K^+ \rightarrow f(K^+) \rightarrow 0$ induces an exact sequence of cohomology groups. This can be used (J. T. Tate) to prove a theorem of the author's (Duke Journal 21) on the index of $f(K^+)$ in K^+. It also gives the following result: A field k has the property that $f(k)=k$ for every additive polynomial $f(x)$ ("Kaplansky's Hypothesis") if and only if k has no algebraic extension of degree divisible by p. Similar methods can be used with the multiplicative group of K and the nth power mapping, with results which are useful in local class field theory. (Received September 11, 1956.)

15. Harold Widom: *Nonisomorphic approximately finite factors.*

Given cardinals $\aleph$ and $\aleph'$ satisfying $\aleph_0 \leqq \aleph' \leqq \aleph$, a group G is found with the properties: (1) G has cardinality $\aleph$, (2) G is locally finite, (3) given a subset S of G of cardinality $<\aleph'$ there is an element of G other than the identity commuting elementwise with S, (4) there exists a subgroup G_0 of G with cardinality $\aleph'$ such that $e \neq g \in G$ implies $[g_0 g g_0^{-1}: g_0 \in G_0]$ is infinite. Let m be the type II_1 factor obtained from G in the usual way. Then m is approximately finite (A) with density character (relative to the metric $[[\ \ ]]$) equal to $\aleph$, any subfactor with density character $<\aleph'$ has a commutor in m larger than the complex numbers, but there exists a subfactor with density character $\aleph'$ whose commutor is exactly the complex numbers. Thus, for a fixed $\aleph$, the m corresponding to different $\aleph'$ are nonisomorphic. This proves the existence, on inseparable Hilbert space, of factors approximately finite (A) but not (B). (Received August 7, 1956.)

ANALYSIS

16. E. J. Akutowicz: *On the determination of the phase of a Fourier integral.* I.

Let $\hat{\phi}$ denote the Fourier transform of ϕ. The problem studied in this paper is to what extent the modulus of $\hat{\phi}$ determines ϕ? The main result is THEOREM I. *Let $C(a)$ be the class of all functions ϕ fulfilling the following conditions*: (i) $\phi \in L^1(-\infty, \infty) \cap L^2(-\infty, \infty)$, (ii) *$\phi(t)$ is equivalent to* 0 *on a half-line $t<t_0\equiv t_0(\phi)$*, (iii) $\hat{\phi}(x)\neq 0$, $-\infty<x<\infty$, (iv) *$a(x)$ is a fixed function such that $|\hat{\phi}(x)|=a(x)$, $-\infty<x<\infty$. Then if ϕ_1 and ϕ_2 belong to $C(a)$ there subsists a relation between them of the form* (1) $\exp(i\gamma+i\beta x)B_1(x)\hat{\phi}_1(x)=B_2(x)\hat{\phi}_2(x)$, *where γ, β are real numbers and $B_1(x)$, $B_2(x)$ are limits as $y\to 0+$ of certain Blaschke products in the upper half-plane. $B_1(x)$ and $B_2(x)$ are holomorphic functions of modulus identically* 1. A question of some interest is whether there exists a function $\phi\in C(a)$ such that the holomorphic extension of its Fourier transform to the upper half-plane is free of zeros. It is shown that the answer is affirmative, and an explicit construction is given, under the hypothesis that a ψ exists in $C(a)$ such that the holomorphic extension of $\hat{\psi}$ has sufficiently sparse zeros. This paper is expected to appear in a forthcoming number of Transactions of the American Mathematical Society. (Received September 6, 1956.)

17t. E. J. Akutowicz: *On the determination of the phase of a Fourier integral.* II.

This abstract is a sequel to the preceding one. What is investigated here is the amount that the nonuniqueness of phase expressed in (1) above is decreased if the assumption (ii) above is replaced by the condition (ii)*: *ϕ is equivalent to* 0 *outside a finite interval, depending upon ϕ*. In (1) there appear (the limits of) two Blaschke products which are defined in terms of their respective zeros $\{z_n^{(1)}\}$ and $\{z_n^{(2)}\}$. Under the conditions (i), (ii), (iii), (iv) these sequences are necessarily without finite limit points, but are otherwise more or less arbitrary, subject to the convergence of the products. Under the conditions (i), (ii)*, (iii), (iv) these two largely independent sequences of zeros reduce to $\{z_n^{(1)}\}$ and $\{\bar{z}_n^{(1)}\}$. Thus one can assert that the restriction (ii)*, being twice as severe as (ii), results in a reduction of leeway in the phase by fifty percent. (Received September 6, 1956.)

18t. P. R. Beesack: *A note on an integral inequality.*

The main purpose of this note is to prove the following integral inequality: *Let $F(x)$, $G(x)$, $M(x)$ be integrable functions over a measurable set A*. Let $A_1=\{x: F(x)\leqq G(x)\}$, $A_2=\{x: F(x)>G(x)\}$, *where $A=A_1\cup A_2$. Suppose that $\int_A G dx\leqq\int_A F dx$, and that either $0\leqq M(x_1)\leqq M(x_2)$ or $M(x_1)\leqq 0\leqq M(x_2)$ is satisfied for every pair x_1, x_2 of points such that $x_1\in A_1$, $x_2\in A_2$. Then $\int_A GM dx\leqq\int_A FM dx$.* This generalizes a recent theorem of Tatarkiewicz (Ann. Univ. Mariae Curie-Sklodowska, Sect. A7 (1953) pp. 83–87 (1954)) and the method of proof is the same. As an application of this inequality we prove a result comparing the first eigenvalues of two second-order linear homogeneous differential systems. (Received September 12, 1956.)

19. J. W. Brace: *The topology of almost uniform convergence.*

Consider a linear space E of scalar valued functions (for simplicity). Let $\mathfrak{A}$ be a family of subsets of the domain space for the functions. Given a positive number ϵ

and a set $A \in \mathfrak{A}$, let $V(\epsilon, A)$ be a set in E such that its complement, $\tilde{V}(\epsilon, A)$, is a maximal subset of E with the property that for every finite subset $\{f_1, f_2, \cdots, f_k\} \subset \tilde{V}(\epsilon, A)$ there exists an $x \in A$ such that $|f_i(x)| > \epsilon$ for $i = 1, 2, \cdots, k$. With the usual restrictions on the family $\mathfrak{A}$, sets of the type $V(\epsilon, A)$ form a subbase for the neighborhood system at the zero function. A net of functions converges in the topology determined by the above neighborhoods if and only if the net of functions converges almost uniformly (see author, Portugaliae Math. vol. 14 (1955) pp. 99–104) on each $A \in \mathfrak{A}$. With further restriction on the members of $\mathfrak{A}$ the neighborhood system determines a linear topology which is locally convex. This technique gives some topologies that are not directly obtainable as uniform convergence topologies. The definition of neighborhood makes it possible to carry over many of the manipulative assets of uniform convergence topologies. (Received September 12, 1956.)

20. H. J. Bremermann: *Continuation of holomorphic functions on Stein manifolds. Connectivity of the boundary of pseudo-convex domains.*

Let D be a compact subdomain of a Stein manifold. Let the boundary of D be connected. Then any function that is holomorphic on the boundary of D can be continued holomorphically into D. For the proof D is enclosed in an analytic polyhedron P which is contracted continuously onto the empty set while the "Kontinuitätssatz" is applied on the intersection $\{\text{boundary } P\} \cap D$.—For schlicht domains in the C^n this is known as the "Hartogs-Osgood theorem." It holds also if D is a bounded domain in a complex Banach space of infinite dimension. In this connection it is of interest that the boundary of a pseudo-convex domain in a complex Banach space is connected. A domain is pseudo-convex if there exists a function $V(z)$ that is plurisubharmonic in D such that for arbitrary real M the closure of the point set $\{z \mid V(z) < M, z \in D\}$ is contained in D. In particular the domains of holomorphy (existence domains of a holomorphic function) are pseudo-convex and hence their boundary connected. (Received September 13, 1956.)

21. F. E. Browder: *Eigenfunction expansions for formally self-adjoint partial differential operators.*

Let A be a formally self-adjoint partial differential operator on a domain G of Euclidean n-space, B a positive differential operator on G. Extending previous results of Garding and the writer on elliptic operators as well as results announced by Gelfand and Kostyucenko for general differential operators with self-adjoint realizations, an eigenfunction expansion theorem of the Weyl-Plancherel type is established, with an expansion in eigenfunctions of $(A - \lambda B)$ which, in general, are distributions of a certain prescribed order. (Received September 11, 1956.)

22. Chandler Davis: *A Schwarz inequality for convex operator functions.*

Let f be a real function on $[a, b]$ whose domain of definition has been extended in the usual way to include operators A on Hilbert spaces $(a \leqq A \leqq b)$. Then f is a *convex operator function* provided $f(tA + (1-t)B) \leqq tf(A) + (1-t)f(B)$ for operators in the domain and $t \in [0, 1]$. Bendat and Sherman characterized such f as real functions. Theorem: f is a convex operator function if and only if $Pf(PAP)P \leqq Pf(A)P$ for A in the domain and P a projection. Proof is brief. This paper will appear in the Proceedings. (Received September 12, 1956.)

23. D. G. Dickson: *Expansions in series of solutions of difference-differential equations.*

Let $\mathcal{H}[f(z)] = \sum_{j=1}^{n}\sum_{k=1}^{m_j} A_{jk}f^{(k)}(z+\omega_j)$, with A_{jk}, ω_j, z complex. Let $m = \max m_j$, P be the smallest closed convex set containing $\omega_1, \cdots, \omega_n$, $h(z) = \sum_{j=1}^{n}\sum_{k=1}^{m_j} A_{jk}z^k e^{\omega_j z}$, $\{\zeta_k\}$ the zeros of $h(z)$, n_k+1 the order of ζ_k, Q a translation of P. If $\phi(z)$ is analytic in the interior of Q, $\phi^{(m)}(z)$ of bounded variation and continuous on every straight line segment in Q; then there exists a series, whose terms are sums of not more than $2n$ terms of the form $p_k(z)e^{\zeta_k z}$ where $p_k(z)$ is a polynomial of degree at most n_k, that converges to $\phi(z)$ in Q minus its vertices. The convergence is uniform in any closed subset of Q excluding the vertices. The coefficients may be written explicitly and the sum at the vertices determined. If Q is a line segment on which $\phi^{(m)}(z)$ is of bounded variation, similar results are obtained. If $\mathcal{H}[\phi(z)] = 0$ in a region R and $\phi(z)$ is analytic in $R+P = \{z;\ z = r+p,\ r \in R,\ p \in P\}$, then $\phi(z)$ may be expanded in such a series converging with local uniformity in $R+P$. The coefficients are unique. Similar results are obtained if $\mathcal{H}[\phi(z)] = 0$ on a curve. The method consists in expressing the partial sums of the series in terms of closed contours tending to infinity. A study of exponential sums with asymptotically polynomial coefficients is included for contour construction and convergence. (Received June 6, 1956.)

24*t*. D. G. Dickson: *Expansions of solutions of infinite order differential equations.*

Let $\mathfrak{D}[F(z)] = \sum_{k=0}^{\infty} A_k F^{(k)}(z)$ where $d(z) = \sum_{k=0}^{\infty} A_k z^k$ is of exponential type σ. For $\tau \geqq 0$ let there exist a certain sequence $\{\Gamma_p\}$ of closed contours about the origin, tending to infinity, on which $d(z) > \exp[-(\tau+\epsilon)|z|]$ for every $\epsilon > 0$ when p is large. Let R be a region and for $r \geqq 0$ let $R \oplus r = \{z \mid z = u+v,\ u \in R,\ |v| \leqq r\}$. If $\phi(z)$ is analytic in $R \oplus (\sigma+\tau)$ and $\mathfrak{D}[\phi(z)] = 0$ in R, $\phi(z)$ may be expanded in R in a series whose terms are sums of terms $c_{kh}z^h \exp(\zeta_k z)$ where ζ_k is a zero of $d(z)$ of order greater than $h \geqq 0$. The convergence is locally uniform and the coefficients may be written explicitly. If the convergence is locally uniform in $R \oplus \sigma$, the coefficients are unique. It is always possible to pick $\tau = \sigma$. If $d(z) = \sum_{j=1}^{n} A_j z^{m_j}\{1+\epsilon_j(z)\} \exp(\omega_j z)$ where $\lim_{|z| \to \infty} \epsilon_j(z) = 0$, then τ may be taken as zero. In this case the series may be bracketed so that any term arises from at most $2n$ zeros of $d(z)$. (Received September 12, 1956.)

25. D. A. Edwards: *On the continuity properties of functions satisfying a condition of Sirvint's.*

Let X be a complex Banach space and let $\mathcal{S}$ (resp. $\mathfrak{D}$) be the class of functions $x(t)$ on the real line R to X with the property that for each compact interval $[a, b]$ there is a conditionally weakly compact (resp. bounded) set $E(a, b) \subseteq X$ (depending also on $x(\cdot)$) such that $\sum_{r=1}^{n} (x(t_{2r}) - x(t_{2r-1})) \in E(a, b)$ whenever $a \leqq t_1 < t_2 < \cdots < t_{2n} \leqq b$ $(n = 1, 2, \cdots)$. Then $\mathcal{S} \subseteq \mathfrak{D}$; in a weakly sequentially complete space $\mathcal{S} = \mathfrak{D}$; and in any complex Banach space X the strong limits $x\ (t \pm 0)$ exist at every point $t \in R$ for each function $x(\cdot) \in \mathcal{S}$, and for such a function $x(t-0) = x(t) = x(t+0)$ for all save possibly a countable set of t-values. Consequently $x(\cdot)$ is separably valued. It also follows that the classical theorem on the decomposition of a real function of bounded variation into a sum of two components, one continuous and the other a step-function, has an analogue for functions in $\mathcal{S}$. If $x(t) \equiv x(t+0)$ for a given $x(\cdot) \in \mathcal{S}$ then there exists a vector-valued measure $\mu: \mathcal{B} \to X$, where $\mathcal{B}$ is the ring of bounded Borel subsets of R, such that $\mu((a, b]) = x(b) - x(a)$ when $-\infty < a < b < \infty$. Most of these results are generalizations of previous theorems of D. G. Kendall and J. E.

Moyal, who, in a forthcoming paper, have shown that the present continuity theorems can be proved in a weakly sequentially complete space for functions of the class $\mathfrak{D}(=\mathfrak{S})$ without using the notion of compactness. (Received August 27, 1956.)

26. R. E. Fullerton: *Quasi-interior points and separation properties of cones in a linear space.*

Let X be a convex linear topological space of second category and let K be a convex cone in X with vertex at the origin. If $x \in K$ we define P_x to be the set $K \cap (x-K)$ and x is said to be a quasi-interior point of K if the linear extension $[P_x]$ of P_x is dense in X. It is shown that a necessary and sufficient condition that x be interior to K is that $[P_x]=X$ and that if K has an interior all quasi-interior points are interior points. Various properties of the set K_q of quasi-interior points of K are investigated and in particular it is shown that if two cones K and K' have no quasi interior points in common there exists a hyperplane in X separating K and K' and strictly separating K_q and K'_q. (Received September 10, 1956.)

27. D. S. Greenstein: *Approximation in norm of translates by Taylor series.* Preliminary report.

Given $f(x) \in C^\infty(-\infty, \infty)$ such that $f^{(n)}(x) \in L^p(-\infty, \infty)$ $(n \geqq 0)$, an important approximation problem is concerned with the approximation in norm of the translates $f(x+h)$ by finite linear combinations of $f(x), f'(x), f''(x), \cdots$. In the present report, the author considers representation of $f(x+h)$ by a "Taylor series in the mean"; i.e., by the expression (1) $\text{l.i.m.}_{n\to\infty} \sum_{k=0}^{n} (h^k/k!)f^{(k)}(x)$. The radius of convergence of (1) is easily computed from the norms of the derivatives, and an error estimate shows that (1) can only converge to the proper value. For (1) to have a nonzero radius of convergence, $f(x)$ must be extendable to an analytic function which is regular in an open strip containing the real line and symmetric about it and belongs uniformly to L^p in every closed substrip. The radius of convergence of (1) is equal to the half width of the maximal such strip. For the case $p=2$, this result is related to a theorem of Paley and Wiener on the Fourier transform of a function analytic in a strip and may be used to derive that theorem. (Received September 10, 1956.)

28. E. L. Griffin, Jr.: *A result on derivations of operator algebras.*

A derivation of an operator algebra is a linear endomorphism possessing the Leibnitz property. An operator T is said to generate the derivation if for each operator A in the algebra, the closure of the operator $(AT-TA)$ equals the derivative of A. Theorem: *Let the self-adjoint operator T generate a derivation of the finite ring of operators M. If T belongs to the ring M, then the derivation is continuous. In fact, the derivation can be generated by a bounded, self-adjoint operator.* The proof is based on the lemma: *If the T mentioned in the first paragraph of the above theorem annihilates a projection whose trace is positive definite, then T is already bounded.* (Received September 12, 1956.)

29. Sigurdur Helgason: *Lacunary Fourier series on noncommutative groups.*

The ordinary definition of a lacunary Fourier series $\sum a_k e^{in_k x}$, $n_{k+1}/n_k > \lambda > 1$ involves the ordering of the integers. A new definition is given which applies to all compact groups, commutative or not. The theorem of Banach stating that a lacunary L^1-series is an L^2-series is extended to noncommutative groups. According to the new definition, Fourier series of the form $\sum a_n e^{ix_n}$ ($x_1, x_2, \cdots$ are independent variables) are lacunary Fourier series on the infinite torus T^∞, and theorems of Kolmogoroff about such series are reproved and extended to groups of the form $\prod_n U(n)$ ($U(n)$ denotes the unitary group in n-dimensions). A simple application is the following theorem together with its generalization to compact abelian groups of nonzero dimension: If $\sum a_n e^{i\sigma(n)x}$ is an L^1-series for arbitrary permutation σ of the integers, then $\sum |a_n|^2 < \infty$. (Received September 14, 1956.)

30. C. S. Herz: *The closure of translations in L^p.*

A closed subset, T, of the real axis is of type U^q if $\phi \in L^q$ and $\lim_{y\to\infty} \int_{-y}^{y} \exp(-itx) \cdot \phi(x)dx = 0$ for all $t \notin T$ imply $\phi = 0$ a.e. An equivalent statement which generalizes to several variables is that there is no nontrivial $\phi \in L^q \cap L^\infty$ whose spectrum lies in T. Given $f \in L^1 \cap L^p$, $1 < p < \infty$, say that "f has property C" if its translates span L^p, "f has property U" if the set of zeros of the Fourier transform is of type U^q, $1/p + 1/q = 1$. U implies C trivially. The converse is obvious for $p \geqq 2$. We observe that if the Fourier transform of f is in Lip ϵ for some $\epsilon > 0$ then C implies U when $p < 2$. The above remarks improve and extend to several variables the results of Pollard (Proc. Amer. Math. Soc. vol. 2 (1951) pp. 100–104). (Received August 17, 1956.)

31*t*. Samuel Kaplan: *On the second dual of the space of continuous functions.*

Let C denote the Banach lattice of continuous real functions on a compact Hausdorff space X. In the paper the space of all bounded real functions on X is identified with a topological direct summand of the second dual M of C. Then each bounded function f on X represents a class or elements of M—those projecting onto f. In this class there are determined two distinguished elements, denoted by f_* and f^*, such that for each Radon measure μ on X (=an element of the first dual of C), $f_*(\mu) = \int_* f d\mu$ and $f^*(\mu) = \int^* f d\mu$, where $\int_*$, $\int^*$ are the lower and upper integrals of f. Moreover if f is integrable under every Radon measure, then $f_* = f^*$, giving a unique element whose value on each Radon measure μ is $\int f d\mu$. It is shown further that the set U of all the unique elements obtained in this way is precisely the closure of C in M under order-convergence (of general directed systems). The topology defined on M by the polars of the order-bounded sets of the first dual of C is also examined, and it is shown that C is dense in M under this topology. Finally a study is made of the specific relations between some properties of M and ordinary integration theory on X. (Received July 23, 1956.)

32*t*. D. G. Kendall and J. E. Moyal: *On the continuity properties of vector-valued functions of bounded variation.*

Suppose that X is a weakly sequentially complete complex Banach space and that $x(\cdot)$ is a function on the real line R to X such that for each compact interval $[a, b]$ a constant $K(a, b)$ exists for which $\left\| \sum_{r=1}^{n} (x(t_{2r}) - x(t_{2r-1})) \right\| \leqq K(a, b) < \infty$ whenever $a \leqq t_1 < t_2 < \cdots < t_{2n} \leqq b$. Then the strong limits $x(t \pm 0)$ exist for all $t \in R$, and $x(t-0) = x(t) = x(t+0)$ for all save possibly a countable set of t-values. It follows that $x(\cdot)$ is separably valued, and that the classical theorem on the decomposition of a real-valued function of bounded variation into two components, one continuous and the other a step-function, has an analogue for $x(\cdot)$. The Hilbert space versions of these results have applications to the expansion theorems of stochastic process theory. (Received August 27, 1956.)

33. W. T. Kyner: *An existence proof for a Stefan-like problem.* Preliminary report.

An existence proof for a quasi-linear parabolic differential equation with a free boundary has been obtained using Schauder's fixed point theorem. The maximum principal for parabolic equations was used in deriving the necessary estimates. (Received September 13, 1956.)

34. P. D. Lax: *On Cauchy's problem.* Preliminary report.

In this note it is shown that Cauchy's problem is incorrectly posed for a linear partial differential operator with variable coefficients which has a nonreal characteristic. The proof consists of demonstrating, just as Hadamard did for operators with constant coefficients, that high frequency initial data are exponentially magnified. The tool for showing this is an asymptotic expansion of solutions with such initial data. Such an expansion in the hyperbolic case may have some use in diffraction theory. (Received June 20, 1956.)

35. C. J. Lewis: *The problem of Milloux for functions analytic in an open annulus.*

Let $A=[0<s<|z|<1]$ in the complex plane; let $\mathcal{E}$ denote a set of points in A, closed relative to A, such that $[|z|=\rho]\cap\mathcal{E}\neq\emptyset$ for all ρ satisfying: $s<\rho<1$. For $0<m<1$, let $\mathfrak{F}_m$ denote the family of functions g which are defined, analytic, and such that $|g|<1$ on A, and each one of which has an associated $\mathcal{E}$-set with the property that $z\in\mathcal{E}$ implies $|g(z)|\leqq m$. The problem of Milloux is to determine, for r satisfying: $s<r<1$, max $M(g;\, r)$ for $g\in\mathfrak{F}_m$, and the corresponding extremal functions. It is shown that if f is such an extremal, and if $f(r)=M(f;\, r)$, then (a) f is unique and independent of r; (b) f has an infinite number of zeros, all simple and negative, with -1 and $-s$ as cluster points of the zeros; (c) f is real for z real; (d) on the real axis between successive zeros there exists a unique point where $f^2=m^2$; (e) $f(-s^{1/2})=m$, $f'(-s^{1/2})=0$; (f) f' has simple zeros at the points referred to in (d) and at $-s^{1/2}$; (g) $f^2=m^2$ only at the points mentioned in (f). This result is an analogue of the solution of the corresponding problem for $[|w|<1]$ due to M. Heins (Amer. J. Math. vol. 67 (1945) pp. 212–234). If f is the normalized extremal for A, ϕ is the normalized extremal for the corresponding problem for $[|w|<1]$, ψ is a certain specific (1, 2) analytic map of A onto $[|w|<1]$, then $f=\phi\circ\psi$. f may be computed by using θ-functions or S-functions of Rausenberger. (Received September 13, 1956.)

36. Walter Nef: *Axiomatic theory of invariant measure and integral.*

Necessary and sufficient conditions are given for the existence of an integral, that is a monotonous and invariant linear form on a linear set of real functions, on which a group of linear transformations is acting. Corresponding conditions for the existence of invariant measures instead of integrals are communicated. (Received October 5, 1956.)

37. S. V. Parter: *Mapping of multiply connected domains by solutions of partial differential equations.* Preliminary report.

Z. Schapiro extended the Riemann Mapping Theorem to mappings which satisfy the uniformly elliptic system of partial differential equations $u_x=a_{11}(x,\, y,\, u,\, v)v_x+a_{12}v_y$, $-u_y=a_{21}v_x+a_{22}v_y$. [Z. Schapiro, *Sur l'existence des representations quasi-*

conformes, C. R. (Doklady) Acad. Sci. URSS, vol. 30 (1941) pp. 690–692.] This result is here extended to multiply connected domains. Theorem: Given a multiply connected domain D and a smooth family F of multiply connected domains, there is a mapping of D onto one of the domains of F which satisfies the system of partial differential equations. The problem is first reduced to that of mappings onto special slit domains. The existence of solutions in that case is then shown by using strong a priori estimates on the Hölder continuity of solutions. These estimates are obtained from a representation theorem of L. Bers and L. Nirenberg [*On a representation theorem for linear elliptic systems with discontinuous coefficients and its applications*, Convegno Internazionale sulle Equazioni Derivate e Parziali, Agosto, 1954, pp. 111–140]. The concept of a smooth family of domains is rigorously defined. (Received June 14, 1956.)

38*t*. I. E. Segal: *Distributions on Hilbert space and canonical sets of operators.*

Using the formulation of (probability) distribution on a linear space given in Ann. of Math. vol. 63 (1956) pp. 160–175, absolute continuity on Hilbert space is treated and the results applied to the classification and transformation of canonical sets of quantum field operators. A correspondence between quasi-invariant (abelian) distributions (of which there exist a continuum of distinct classes for an infinite-dimensional space) and canonical sets of Bose-Einstein field operators is developed. In the case of irreducible sets arising from an isotropic normal distribution, the unitary equivalence classes are in 1-1 natural correspondence with the group of closed 1-forms on the space modulo the subgroup of exact forms, which group is nontrivial in the infinite-dimensional case. Those Fermi-Dirac field operators arising from (non-commutative) distributions are also examined. In either case the condition that there exist a unitary operator transforming one canonical set into another is determined, showing that concretely this is rarely the case. (Received September 11, 1956.)

39. V. L. Shapiro: *A best possible result in the uniqueness of double trigonometric series.*

Let $M=(m, n)$ where m and n are integers, $X=(x, y)$, $MX=mx+ny$ and $|X| =(x^2+y^2)^{1/2}$. Furthermore let T_2 be the two dimensional torus $\{(x, y), -\pi<x\leqq\pi, -\pi<y\leqq\pi\}$. Then in this paper the following theorem is proved: Let $\sum a_M e^{iMX-|M|t}$ tend to zero with t except for possibly a finite number of points in T_2. Suppose that $a_M=o(1)$ as $|M|\to\infty$. Then the a_M are identically zero and this result is a best possible one since if it is assumed that the $a_M=O(1)$, the theorem is false. The proof of the theorem follows from a judicious use of the following lemma: Let $E(t) =\sup_{0<h\leqq t} \sup_{|X-P|=h} |F_{1,X}(h)-F_{1,P}(h)|$ where $F_{1,X}(h)$ represents the mean of F in the disc of radius h with center X and $F(X)$ is the anti-Laplacian of the given double trigonometric series. Then $E(t)$ tends to zero with t. (Received August 17, 1956.)

40. G. L. Spencer, II: *An inequality and uniqueness theorem for semilinear hyperbolic systems in two independent variables.*

A comparison inequality for approximate solutions of semilinear hyperbolic systems in two independent variables is obtained analogous to the comparison inequality for ordinary first order differential equations (See Coddington and Levinson, *Theory of ordinary differential equations*, p. 8, for the inequality in the ordinary case.) The uniqueness follows directly from the inequality. (Received September 13, 1956.)

41*t*. J. L. Walsh: *On infrapolynomials with prescribed constant terms.* Preliminary report.

Every factor $z^m+\cdots$, $m\geqq 2$, of a polynomial $p_n(z)\equiv z^n+\cdots+A_n\equiv\prod_1^n(z-\alpha_k)$ of this class K on a compact point set E (containing at least n points but $0\notin E$) also belongs to K. If $p_2(z)\in K$, we have $\phi_{\alpha_1}+\phi_{\alpha_2}+\phi_0\geqq\pi$, where ϕ_z is the angle subtended at z by E; if E lies in a closed circular disc Γ not containing 0, and if one zero of $p_n(z)$ lies in no disc intersecting Γ and subtending the same angle at 0 as Γ, then all other zeros of $p_n(z)$ lie in Γ. For E in $x>0$ and symmetric in $0x$, $p_n(z)$ real, the Jensen circle on $\alpha_k\bar{\alpha}_k$ as diameter is replaced by a circle tangent to $0\alpha_k$ and $0\bar{\alpha}_k$ at α_k and $\bar{\alpha}_k$ respectively; such circles contain all nonreal zeros of $p_n(z)$. If E (real) has the convex hull H: $(0<)\ a\leqq z\leqq b$, then all zeros of a real $p_n(z)$ with $(-1)^nA_n>0$ except perhaps one lie in H, and that one lies on $(-1)^nA_n/b^{n-1}\leqq z\leqq(-1)^nA_n/a^{n-1}$; a segment of $0x$ disjoint from E contains at most one zero of $p_n(z)$. If the last ν coefficients of $p_n(z)$ are prescribed $(\nu<n)$, we deduce $\phi_{\alpha_1}+\phi_{\alpha_2}+\cdots+\phi_{\alpha_n}+\nu\phi_0\geqq\pi$. (Received September 10, 1956.)

Applied Mathematics

42. E. H. Bareiss: *The error estimation in an approximate solution of an ordinary second-order differential equation.*

This paper develops a method for estimating the error in the approximate solution of an ordinary second-order differential equation which may be given in graphical form. Although errors often cannot be detected by a point-by-point control because their magnitude lies between the limits of tolernace, they can accumulate undesirably after, say, 50 steps. A survey of the theory and a computing scheme is given by means of which any solution in graphical (or analytical) form may be checked, and the procedure is illustrated with examples for which exact solutions are known. The influence of variations in the initial conditions and in the differential equations is also considered. First-order equations are included as a special case. (Received September 12, 1956.)

43. J. B. Diaz: *On the numerical solution of* $u_{xy}=f(x, y, u, u_x, u_y)$.

The Euler-Cauchy polygon method is available for the numerical solution of the classical ordinary differential equation problem: $y_x=f(x, y)$; $y(x_0)=y_0$. An analogous method is developed for the solution of the classical hyperbolic partial differential equation problem: $u_{xy}=f(x, y, u, u_x, u_y)$; $u(x_0, y)=\tau(y)$; $u(x, y_0)=\sigma(x)$; $\sigma(x_0)=\tau(y_0)$. By means of this approach one can prove the existence theorem for this boundary value problem given by P. Hartman and A. Wintner, Amer. J. Math. (1952) and by P. Leehey, Brown University, Ph.D. thesis, 1950, which is based on the assumption that $f(x, y, z, p, q)$ satisfies a Lipschitz condition *only* with respect to p and q (this theorem generalizes the classical Picard existence theorem based on the assumption that $f(x, y, z, p, q)$ satisfies a Lipschitz condition in all three variables z, p, q). An auxiliary finite difference inequality, which plays a rôle in the present treatment similar to that played by a "convergence inequality" in the theory of the ordinary differential equation $y_x=f(x, y)$, appears to be of independent interest. (Received September 11, 1956.)

44. D. J. Dickinson: *On a generalization of the Legendre polynomials.*

Let $\{P_n^\nu(x)\}$ be a set of polynomials, each of degree precisely n, that satisfy

$(n+\nu)P_n^\nu(x)-(2n+2\nu-1)xP_{n-1}^\nu(x)+(n+\nu-1)P_{n-2}^\nu(x)=0$ for $n \geqq 1$ where $P_{-1}(x)=0$ and $P_0(x)=1$. When $\nu=0$, they are the set of Legendre polynomials. Let $\{Q_n(x)\}$ be the Legendre functions of the second kind except that $Q_{-1}(x)=1$. The $\{P_n^\nu(x)\}$ are orthogonal over any contour that includes the unit circle and with respect to the weight function $Q_\nu(x)/Q_{\nu-1}(x)$. (ν is a nonnegative integer.) (Received September 17, 1956.)

45. R. M. Durstine (p) and D. H. Shaffer: *Determination of upper and lower bounds for solutions of linear differential equations.*

Let the function u be uniquely defined by $L(u)+\phi=0$, subject to boundary conditions on u. Here L is a linear, multi-dimensional differential operator; and ϕ is a known function of the independent variables. Then, by choosing two functions w_1 and w_2, which satisfy the boundary conditions imposed on u, and certain other restrictions, a pair of linear combinations of w_1 and w_2 which bound u may be formed. The method for generating these bounding functions is restricted to those problems for which the appropriate Green's function associated with L does not change sign. Various special cases have been carried out to demonstrate this method for both ordinary and partial equations. In one such example, the true solution to the problem is generated as a result of the coincidence of the upper and lower bounds. (Received September 10, 1956.)

46*t*. P. S. Dwyer: *A theorem on the multi-index transportation problem.* Preliminary report.

In the multi-index transportation problem, and hence in the mathematically equivalent multi-index assignment problem and group assembly problem, it is known that the set of all $x_{i_1 i_2 \cdots i_j \cdots i_k}$ such that $\sum_{i_j \text{ fixed}} x_{i_1 i_2 \cdots i_j \cdots i_k} = f_{ji_j}$ with $x_{i_1 i_2 \cdots i_j \cdots i_k} \geqq 0$ and $\sum_{ij} f_{ji_j} = N$ gives a modified problem which may have extreme points which are not integral solutions of the multi-index transportation problem. A practical solution of this modified problem, with possible positive fractional elements, results from a recently developed method of reduced matrices, which differs operationally from the simplex method. The results of this solution process of the modified problem can be used to reduce the multi-dimensional transportation problem to one or more elemental modified problems. By reordering of rows, columns, etc., the extreme point solution of the modified problem can be presented in a diagonal block form in which the "off diagonal" components are zero. A solution of the original problem is obtained by solving the modified multi-index transportation problem within each block. If this solution is nonintegral, necessary conditions for finding extreme points of the original problem "adjacent" to those of the modified problem can be found from the details of the solution of the modified problem. (Received August 8, 1956.)

47. Gabriel Horvay: *Biharmonic eigenvalue problem.*

When a simply connected, homogeneous, isotropic, plane elastic body is subject over a small portion $2h$ of its boundary to self-equilibrating normal and shear tractions then the stresses in the body are presumably small at distances r greater than $2h$ from the load application region. This is the content of St. Venant's principle. In mathematical formulation the problem requires (a) determination of two complete sets of boundary tractions into which given normal and shear tractions may be expanded, (b) determination of eigensolutions $G_n(x, y)$, $H_n(x, y)$ of the $\Delta^4 G=0$ equation whose appropriate derivatives reduce to the two complete sets of boundary tractions, (c) investigation of the decay behavior, with r, of the second derivatives of G_n, H_n. This program is carried out for the wedge region $x \geqq 0$, $-y_b \leqq y \leqq y_b$, $y_b = h + x \tan \omega$ which is loaded at $x=0$; rigorously for the semi-infinite strip $\omega=0$ and the semi-infinite plane $\omega=\pi/2$, in variational approximation for $-\pi/2<\omega<\pi/2$. For $\omega=0$ the decay is of type $e^{-\alpha_n r/h}$, for $\omega \neq 0$ the decay is of type $(r/h)^{-\mu_n}$. The decay is slowest for $\omega = \pm\pi/2$. α_n and μ_n are the real parts of roots of appropriate eigenvalue equations. References: *Biharmonic Eigenvalue problem*, Quart. App. Math., 1957; *St. Venant's principle*, J. App. Mech., 1957. (Received June 8, 1956.)

48t. M. A. Hyman: *A note on best numerical integration formulas.*

The integral $I = \int_a^b F(x)dx$ is commonly approximated by $I_N = \sum_{i=1}^N A_i F(x_i)$, where the points x_i are evenly spaced on the interval $[a, b]$. There are a multitude of "quadrature rules" for assigning the A_i (trapezoidal, Simpson's, and Weddle's are well-known rules). Some rules are better than others in the sense that, for N and $F(x)$ fixed, they produce in general a smaller "truncation error" $E \equiv I - I_N$. G. Birkhoff (private communication) has remarked that, in integrating periodic functions over a full period, rules such as the trapezoidal can be expected to give truncation errors of the same order of smallness as more elaborate rules. The present note asserts the same result for "transient" functions: $F(x)=0$ for $x \leqq a$ and $x \geqq b$. Integrals of such functions occur in many applications, including the probability analysis described in the preceding abstract. Use of "simple" integration formulas somewhat facilitates machine computation. Moreover, the equality of the weights A_i, when using the trapezoidal rule, is generally desirable from the stand-point of error control. The results hold for multi-dimensional integrals. (Received September 10, 1956.)

49. M. A. Hyman: *Probability analysis of simultaneous instrument errors.*

This paper considers a system in which a physical quantity F depends on certain other quantities $\alpha, \beta, \gamma, \delta, \cdots$, whose values fluctuate within certain limits set by feed-back controllers; the probable fluctuations of F are desired. It is shown that, under rather general conditions, $F = A(\alpha)B(\beta)C(\gamma)D(\delta) \cdots$, where $A, B, C, \cdots$ are monotonic functions of their variables. Taking logarithms, $f = \log F$, $a = \log A$, $b = \log B$, etc., and choosing $a, b, c, \cdots$ as new independent variables, the problem reduces to finding the probability distribution $\phi(f)$. It is shown how $\phi(f)$ (and various cumulative distributions of f) can be accurately (and rather elegantly) calculated numerically; these computations can be readily mechanized. Usually the distributions of $a, b, c, \cdots$ within their respective ranges are not known, but it is shown how conservative results can be obtained from whatever information is available. As the number of variables increases, $\phi(f)$ approaches the Gaussian distribution. The paper concludes with certain approximations which can be used, in place of the exact analysis described above, for estimating the probable fluctuations of F. (Received September 10, 1956.)

50. L. E. Payne (p) and H. F. Weinberger: *Pointwise bounds in Neumann problems.*

Let u be harmonic in a star shaped region D with boundary B. Let the integral of u over D vanish. By an extension of previous results of the authors (Journal of Mathematics and Physics vol. 33 (1955) p. 291) a lower bound is obtained for the ratio $\int_B (\partial u/\partial n)^2 ds / \int_B u^2 ds$. (The best such lower bound is the second Stekloff eigenvalue for

D, the first being zero.) This lower bound leads to a pointwise estimate for u when $\partial u/\partial n$ is prescribed, which is simpler than that indicated in the above-quoted paper. (Received September 13, 1956.)

51. D. E. Spencer: *A postulational foundation of special theories of relativity.*

The problem is to find the most general laws of transformation between unaccelerated observers. Three postultes are taken as fundamental: I There is no privileged observer, II Relative velocities of observers are defined operationally by Doppler shifts, III A body which is unaccelerated relative to one observer A must be unaccelerated relative to all observers which are in uniform motion relative to A. From these general postulates a transformation equation is obtained which contains as special cases Galilean relativity and Einstein's special relativity, as well as many other possibilities. The role of $v=c$ as a singularity is reinterpreted. The results are in sharp contrast with the work of Pars (Phil. Mag. vol. 42 (1921) p. 249) who believed that he had proved, by a similar analysis, that Einstein's second postulate (c=const.) follows "from the mere hypothesis of relativity." Logically all of the relativities here are equally plausible. (Received August 31, 1956.)

GEOMETRY

52*t*. W. R. Abel and L. M. Blumenthal: *Metric arcs with Menger curvature.* I.

The following properties are established for any metric arc with finite Menger curvature at each point: (1) for all but a finite number of positive integers n, (i) each n-lattice L_n of the arc is a homogeneous $\lambda(n)$-chain, where $\lambda(n)$ denotes the distance of two consecutive points of L_n, and (ii) each n-lattice of the arc is unique, (2) each such arc is the sum of a finite number of metrically monotone arcs. Other theorems are (1) for any metric arc A, rectifiable or not, length $A=\lim_{n\to\infty}$ length L_n, and (2) every metric ptolemaic geodesic (that is, locally straight) arc is a metric segment. (Received September 12, 1957.)

53*t*. D. W. Crowe: *On the geometry of bicomplex numbers.*

The bicomplex numbers $z+iw=x+ju+i(y+jv)$, $i^2=j^2=-1$, can be considered as points (x, u, y, v) of euclidean 4-space, E^4. The numbers whose last two coordinates are zero constitute a plane, P. Alternately it is convenient to write $z+iw$ in the more symmetric form $pZ+qW$, where $p=(1+ij)/2$, $q=(1-ij)/2$, $Z=x+v+j(u+y)$, $W=x-v+j(u-y)$. This establishes a 1:1 correspondence between pairs (Z, W) of points of P and points of E^4. For fixed $A\in P$, the E^4 locus $A'=\{pAW+qW\colon W\in P\}$ is a plane isocline to P at (0, 0, 0, 0). The 2-parameter family of planes A' has the geometry of a 2-sphere K (Stringham, Trans. Amer. Math. Soc. vol. 2 (1901) pp. 183–214), the angle between two planes A', B' being half the central angle between their representing points A'', B'' on K. It is shown that for a natural choice of K in E^4 the mapping $m(A'')=A$ is stereographic projection. A similar result holds if the plane A' is defined by $A'=\{pA\overline{W}+qW\colon W\in P\}$. (Received August 20, 1956.)

54. L. E. Dubins: *On curves of minimal length with a constraint on average curvature, and with prescribed initial and terminal positions and tangents.*

Motivation: Let a particle pursue a continuously differentiable path from an initial point u to a terminal point v. Suppose that its speed is unity and that its velocity vectors at u and v are prescribed to be U and V respectively. Suppose its radius of curvature is never less than R. What is the nature of a path of minimal length for the particle? *Definition*: Let u, vU and V be vectors in Euclidean n space, E_n. Let $\|U\| = \|V\| = 1$, let $R > 0$. Let C be the collection of all curves X, parametrized by arc length and defined on $[0, L]$, where $L = L(X)$ varies with X, such that: $X(s) \in E_n$ for $0 \leqq s \leqq L$; $\|X'(s) - X'(t)\| \leqq R^{-1}|s-t|$ for all s and t; $X(0) = u$, $X'(0) = U$, $X(L) = v$ and $X'(L) = V$. *Proposition* 1: For any n, u, U, v, V and R, there exists an X in C of minimal length. Such a curve of minimal length is called an *R-Geodesic*. *Theorem*: *For $n = 2$, an R-Geodesic is necessarily a continuously differentiable curve which consists of not more than three pieces, each of which is either a straight line segment or an arc of a circle of radius R.* (Received September 12, 1956.)

55. David Gale: *A theorem on flows in networks.*

A *network* $[N, c]$ consists of a finite set of *nodes* N and a non-negative (real valued) *capacity function* c on $N \times N$. A *flow* on $[N, c]$ is a function f on $N \times N$ such that (1) $f(x, y) + f(y, x) = 0$ and (2) $f(x, y) \leqq c(x, y)$ for all $x, y \in N$. A function d on N is called a *feasible demand* if there exists a flow f on $[N, c]$ such that $\sum_x f(x, y) \geqq d(y)$ for all $y \in N$. *Theorem*: The function d represents a feasible demand if and only if for every subset $S \subset N$, $\sum_{y \in S} d(y) \leqq \sum_{x \notin S, y \in S} c(x, y)$. A special case of this theorem is the well-known theorem of P. Hall on systems of distinct representatives of subsets. Another is a theorem of R. Rado concerning nonnegative functions defined on oriented graphs. The theorem is also used to give a simple criterion for when it is possible to construct a matrix of 0's and 1's whose rows and columns shall have preassigned sums. The proof of the main result makes use of the minimum cut theorem of Ford and Fulkerson. (Received September 11, 1956.)

56t. Shoshichi Kobayashi: *Isotropy group and holonomy group.*

It is known that the linear isotropy subgroup H_x at x of the largest connected group of isometries of an irreducible Riemannian space M is contained in the restricted homogeneous holonomy group h_x of M with reference point x, except in the case where M is a pseudo-Kaehlerian space with vanishing Ricci curvature. It can be proved that if M is irreducible and homogeneous, then H_x is always contained in h_x. This improves a theorem of Lichnerowicz. (Received September 13, 1956.)

57t. August Newlander and Louis Nirenberg: *An almost complex manifold satisfying the integrability conditions is complex analytic.*

In terms of suitable local complex coordinates $z^j = x^j + ix^{n+j}$, $\bar{z}^j = x^j - ix^{n+j}$ on a $2n$-dimensional real manifold with almost complex structure the Cauchy-Riemann equations may be written $L_j w = w_{\bar{z}^j} - \sum_{k=1}^{n} a_j^k w_{z^k} = 0$, $j = 1, \cdots, n$, where the coefficients vanish at the center of the coordinate neighborhood. The integrability conditions being satisfied is equivalent to the statement that the L_j commute. The problem is the introduction of new variables ζ^i, $\bar{\zeta}^i$ that transform the system to $w_{\bar{\zeta}^j} = 0$, $j = 1, \cdots, n$. This requires that the z coordinates as functions of the ζ coordinates satisfy the nonlinear "characteristic" system $z^i_{\bar{\zeta}^j} + \sum_{k=1}^{n} a_k^i \bar{z}^k_{\bar{\zeta}^j} = 0$, $i, j = 1, \cdots, n$. A corresponding system of nonlinear integral equations may be set up which can be solved by iterations, assuming the a_j^i to be only finitely differentiable. (For real analytic integrable almost complex structure the problem was solved by B. Eckmann and A. Frölicher, C. R. Acad. Sci. Paris vol. 232 (1951) pp. 2284–2286.) The integrability conditions enter in showing that the resulting solution is a solution of the characteristic system. (Received October 24, 1956.)

58. S. R. Struik: *Measure theory of area and its tools.*

To express the ratio of two areas by a parallel vector pair is the basis of affine measure theory. Given two triangles, a third one, the affine reflection of the first, can be constructed with a basis equal to the basis of the second triangle, and collinear. The vertices of the second and third triangle are made to be on opposite sides of the common basis line. Their connection is divided by the basis line in the sought ratio. To construct this affine reflection necessitates: connecting points, intersecting lines, drawing parallels. Therefore a straight edge suffices as tool, besides two fixed segments $\overline{OA}$, $\overline{OB}$, on intersecting lines, with their bisecting points A', B' resp. The Desargues theorem (an axiom in affine two-space) admits of constructing through any point a parallel line to a given one (using only the straight edge) as long as the line shows one bisected segment. The above two-segment tool admits, therefore, of constructing all possible parallel lines. The three dimensional construction is analogous and simpler, because the Desargues theorem is here provable. The analogous tool is a tripod. Each leg has two equal segments. (Received September 12, 1956.)

Logic and Foundations

59. Kurt Bing: *On the axioms for order and succession.*

Hasenjaeger (Ein Beitrag zur Ordnungstheorie, Archiv fuer Mathematische Logik und Grundlagenforschung vol. 1 (1950) pp. 30–31) has answered a question raised by Hilbert and Bernays (Grundlagen der Mathematik I, Berlin, 1934, p. 279) by showing that the axiom (1) $a<b\rightarrow a'=b\vee a'<b$ can be proved, within lower predicate calculus, from the remaining axioms of the system (B) of Hilbert and Bernays, with $0=0$ replaced by $a=a$. The axioms from which (1) can be proved define a system (B_1) which is independent and is equivalent to (B). But the axiom systems obtained from (B) and (B_1) respectively by dropping the induction axiom are not comparable in strength. The proof uses the fact, established by the reduction method described by Hilbert and Bernays, that $a=a$ cannot be proved from (B) without the induction axiom. However, $\mathfrak{z}=\mathfrak{z}$ can be so proved for every numeral $\mathfrak{z}$. (Received September 10, 1956.)

Statistics and Probability

60. Bernard Friedman (p) and Ivan Niven: *The average first-recurrence time.*

A discussion of the first recurrence time of a dynamical system of k degrees of freedom, each of which is simply periodic in the time t, leads to the following problem: Suppose $\alpha_1, \cdots, \alpha_k$ are all between zero and one, and suppose t is the smallest positive integer such that, for given $\epsilon>0$, there exist integers $m_1, \cdots, m_k$ satisfying the inequalities $|t\alpha_j-m_k|<\epsilon$, $(1\leqq j\leqq k)$. Put t_{av} = average of t over all values of $\alpha_1, \cdots, \alpha_k$ in the k-dimensional unit cube. Then there exist two constants c_1 and c_2 such that $c_1\epsilon^{-k}<t_{av}<c_2\epsilon^{-k}$. If $k=1$, we prove $t_{av}=6\pi^{-2}(\log 2)\epsilon^{-1}+O(\epsilon^{-1/2})$. (Received September 12, 1956.)

61*t*. J. C. Kiefer and Jacob Wolfowitz: *On the deviations of the empiric distribution function of vector chance variables.*

Let $X_1, \cdots, X_n$ be independent vector chance variables with the common continuous distribution function (d.f.) F, and let S_n be their empiric d.f. Let G_n be the d.f. of $n^{1/2} \sup_x |S_n(x) - F(x)|$. Without loss of generality for what follows we assume that the marginal d.f.'s of each X_i are uniform in $[0, 1]$. Let A_k be the lattice points in the unit cube on the space of the X_i's whose coordinates are multiples of $1/k$. Let $H_{n,k}$ be the d.f. of $n^{1/2} \sup_{x \in A_k} |S_n(x) - F(x)|$. The authors prove: Theorem 1. $1 - G_n(r) < c_0 e^{-cr^2}$ for all $r \geqq 0$, where c_0 and c are absolute positive constants. Theorem 2. There exists a d.f. G (depending on F) such that, at every point of continuity of G, $\lim_n G_n = G$. Also $\lim_{k \to \infty} \lim_{n \to \infty} H_{n,k} = G$ at every point of continuity of G. Similar results to those of these theorems hold for the joint d.f. of the signed deviations, and also when F is not continuous. (Received August 15, 1956.)

62. Ron Manly: *Determining bounding distribution functions for nonlinear functions of random variables.*

A distribution function—(γ)—is defined as bounding another distribution function—(β)— at a point $m + \Delta$ (where m is the median for the bounded distribution function), if $\alpha(\gamma) \geqq \alpha(\beta)$, where $\alpha(\phi) = \text{sgn}\ (\Delta) \int_{m+\Delta}^{\text{sgn}(\Delta)\infty} \phi dx$. Given n independent random variables $x_1, \cdots, x_n$ with means μ_i, distribution functions F_i, and bounds $A_i \leqq x_i \leqq B_i$; if a function $g = g(x_1, \cdots, x_n)$ has continuous first and second derivatives in a region G ($G_1 \leqq g \leqq G_2$; $A_i \leqq x_i \leqq B_i$), a procedure is described for determining a bounding distribution function for the distribution function of g.

Procedures are also described for special cases of rectangular distributions, empirical data, correlation between the variables, and simple functions involving products, powers, and exponentials. The use of bounding distribution functions may be applied: (1) to determine reliability of complex systems such as nonlinear circuits in digital computers, (2) as an alternative to Monte Carlo methods, and (3) in other Systems Engineering problems involving nonlinear combinations of random variables (Received August 13, 1956.)

Topology

63. I. S. Gál: *On Lindelöf spaces and L-spaces.*

We introduce the following definitions: Let X be a general topological space and let A be an arbitrary subset of X. X is called an $L(A)$-space if every covering of A by a family of open sets contains a countable subcovering. If X is an $L(A)$-space for every A in X then X is called an L-space. X is called a P_0-space if every noncountable subset S of X has a point of accumulation in X. If every noncountable subset S of X has a point of accumulation which belongs to S then X is called a P-space. The purpose of the present paper is to study the notion of $L(X)$-, $L(A)$- and L-spaces. Some results of the theory of compact and relatively compact spaces can be duplicated for these new spaces by making simple changes in the proofs and in the additional definitions. However these spaces offer also new types of results. The main result concerns pseudo-metrizable L-spaces: For uniform spaces whose uniform structure satisfies the first axiom of countability the notions of an $L(X)$-space, of an L-space, of a P_0-space and of a P-space coincide. (Received October 3, 1956.)

64. J. P. Roth: *A combinatorial topological solution of the problem of Quine.*

The *extraction algorithm* is given for solving the problem of Quine (Bull. Amer. Math. Soc. Abstract 62-3-262); actually the problem solved is more general, allowing for "don't-care" conditions. Given a cubical complex K, an elementary cocycle is a cube which has no coface. Let Z_1 be the space of all cocycles. The *-algorithm computes Z_1. An extremal is a cocycle having a vertex unshared with any other cocycle. The $\#$-algorithm or the $\cap$-algorithm locates the subspace E_1 of extremals. In $\hat{Z}_2 = Z_1 - E_1$ a partial order $<$ is introduced; Z_2 is the set of maximal elements of $\hat{Z}_2$ under $<$. From Z_2 is extracted the subspace E_2 of 2d order extremals to form $\hat{Z}_3$. The algorithm proceeds inductively until an irreducible subspace Z_r is reached. Here the algorithm "branches" to produce an E_r, etc. until $Z_s = \phi$, $r < s$. Theorem: $E_1 + \cdots + E_s$ is a minimum cover. (Received September 12, 1956.)

65*t*. Norman Stein: *The third obstruction in complex projective spaces.*

In this paper secondary cohomology operations Ψ_n, $n \geqq 1$, are defined. The operation Ψ_1 is the Shimada-Adem operation. The domain of Ψ_n is $N_n^2(K) = \{h \in H^2(K, Z) \mid h^{n+1} = 0 \in H^{2n+2}(K, Z)\}$ where K is any complex and h^{n+1} denotes the multiple cup product. The pairing $Z \otimes Z \to Z$ for each component of the multiple cup product is given by $1 \otimes 1 \to 1$. The range of Ψ_n is $H^{2n+3}(K, Z_2)/\{H^2(K, Z) \cup H^{2n+1} \cdot (K, Z) + Sq^2 H^{2n+1}(K, Z)\}$ where the pairing $Z \otimes Z \to Z_2$ in the cup product term is the Whitehead product pairing $\pi_2(M_n) \otimes \pi_{2n+1}(M_n) \to \pi_{2n+2}(M_n)$ in which M_n denotes the complex projective n-space. It is shown that Ψ_n is the cohomology operation corresponding to the second nontrivial Postnikov k-invariant of M_n. A cochain formula is given for Ψ_n and using this the third obstruction for a map $K^2 \to M_n$ whose first two obstructions vanish is computed. It has been conjectured by I. M. James that the pairing $\pi_2(M_n) \otimes \pi_{2n+1}(M_n) \to \pi_{2n+2}(M_n)$ is trivial and he has proved this in case n is odd. No proof has been found for n even. Using this result where $n = 2k+1$ the range of Ψ_{2k+1} is seen to be simply $H^{4k+5}(K, Z_2)/Sq^2 H^{4k+3}(K, Z)$. (Received July 18, 1956.)

66*t*. Chien Wenjen: *On pseudo-compact spaces.* I.

A completely regular space is *pseudo-compact* if every real-valued continuous function defined on it is bounded. (Hewitt, Trans. Amer. Math. Soc. vol. 64 (1948) p. 67). A point in a topological space is a *limit point* of a family of disjoint open sets if every neighborhood of the point intersects infinitely many members of the family. A space is *almost compact* if every family of disjoint open sets has a limit point. A family of open sets in a topological space is an *almost covering* if the union of all the sets of the family is dense in the space. Let X be a completely regular space. The following statements are equivalent: (a) X is pseudo-compact; (b) X is almost compact; (c) Every infinite covering of X has a proper almost subcovering (cf. Arens-Dugundji, Portugaliae Math. vol. 9 (1950) p. 141); (d) Every countable covering of X has a finite almost subcovering. (Received August 20, 1956.)

67*t*. Chien Wenjen: *On pseudo-compact spaces.* II. Preliminary report.

An open net in a space is a net, the range of which consists of nonvoid open sets (instead of points of the space). An open set is a *cluster point* of an open net if every neighborhood of any point of the open set meets the net frequently. The following hold: (1) A pseudo-compact space is bicompact iff it is paracompact (cf. Arens-Dugundji, loc. cit. p. 142); (2) A completely regular space is bicompact iff each open net has a cluster point; (3) A connected, locally pseudo-compact space is metrizable iff it has a countable base; (4) A complete uniform space is pseudo-compact iff it is bicompact. (Received August 20, 1956.)

68. E. F. Whittlesey: *Classification of finite 2-complexes.*

Form a 2-complex K by "gluing" finitely many 2-cells on a connected 1-complex so that each oriented cell-boundary is mapped piecewise into a cycle in the 1-complex. Assume each 0- or 1-cell is incident with some 2-cell (but this restriction can later be removed). By "cut-and-paste" operations like those used for surfaces (see Lefschetz, *Introduction to topology*, pp. 72–84) such a 2-complex K can be reduced to canonical form K^* consisting of one or more *surface components* of the type $\alpha d_1\beta_1 d_1^{-1} \cdots d_r\beta_r d_r^{-1}$, where $\alpha = a_1b_1a_1^{-1}b_1^{-1} \cdots a_pb_pa_p^{-1}b_p^{-1}$ or $c_1c_1 \cdots c_qc_q$ according as the surface component is orientable or not and where the β's are blocks of alternate capital and small letters denoting vertex and edge *singularities* (including boundary curves as particular edge singularities). K and K^* are equivalent, combinatorially and topologically. Thus K is classified by certain invariants, partly numerical and partly of order. (Received June 25, 1956.)

69. Hidehiko Yamabe: *Hilbert fifth problem on local groups.*

Hilbert's fifth problem on local groups can be formulated as follows: "Is every locally compact connected local group locally a direct product of a local Lie group and a compact group?" First one proves the existence of Haar measure. Next, one finds a compact, normal subgroup such that the factor local group has no small subgroups, which has a locally euclidean neighborhood U of e covered by one parameter subgroups. This is proved as follows. Let $\nu(x)$ be the smallest integer such that x is outside of some neighborhood U containing no small subgroup. Suppose that the set C of points on one parameter subgroups does not cover any neighborhood of e. Then there exists a sequence x_μ converging to e, with each x_μ not in C. Take y_μ on C such that $\nu(y_\mu^{-1}x_\mu) = \sup_{y \in C} (y^{-1}x_\mu)$. By taking a subsequence if necessary, we may assume that $(y_\mu^{-1}x_\mu)^{[p_\mu r]}$ converges to $u(r)$ where $p_\mu = \nu(y_\mu^{-1}x_\mu)$. Define $z_\mu(r)$ by $\lim_m (u(r/m)y_\mu(-r/m))^m$. For large μ, $\nu(y_\mu^{-1}x_\mu) \geqq \nu(z_\mu(1/q_\mu)x_\mu)$ where $q_\mu = \nu(x_\mu)$. However $(z_\mu(1/q_\mu)x_\mu)^{q_\mu}$ converges to e which contradicts the above inequality. Thus $x_\mu \in C$ for large μ. (Received September 28, 1956.)

R. D. Schafer,
Associate Secretary