followed. The arrangement of bibliographies and the references make it difficult in some cases to find out exactly which work is the source for a given result, and therefore where further material may be found.

The above short outline does not suggest the remarkable amount of material in this book. The author's compact style enables him to include an extraordinary mass of ideas and theorems in a systematic and coherent fashion. At each stage he usually succeeds in achieving the utmost in generality, both in the large and in minor details—for example in allowing infinite-valued expectations wherever possible. This book will be a standard reference text for years, and students will find it indispensable, although difficult.

J. L. Doob

General topology. By J. L. Kelley. New York, Van Nostrand, 1955. 14+298 pp. $8.75.

The appearance of a comprehensive treatise in English on present day set-theoretic topology is an important event. The rapid progress in set-theoretic topology during the past 20 years, and the ever increasing applications of this discipline to analysis, make the appearance of the volume under review particularly appropriate at the present time. Professor Kelley has set himself the task of producing a book useful for both students and specialists, and he has succeeded to a remarkable extent in reaching both of these somewhat inconsistent goals. Like many other texts of this genre, the present volume could be understood, at least in theory, by any intelligent person who can read English. All of the machinery is supplied; but a knowledge of the real numbers and elementary abstract algebra as set forth for example in Birkhoff-MacLane (*A survey of modern algebra*, rev. ed., New York, Macmillan, 1953) and a thorough knowledge of elementary analysis are certainly minimal prerequisites for appreciating this book.

The author's style is spirited, to say the least. The atmosphere of an informal and humorous lecture pervades the book, especially in the first part. An essential difference between oral and written communication is well illustrated here, for some sallies that would clearly enliven a lecture are less felicitous in print. (For example, a very special case of Fubini's theorem is designated without further comment as Fubinito (p. 78), and a *topologist* is defined as a man who doesn't know the difference between a doughnut and a coffee cup (p. 88).) An occasional solecism was noted (e.g., p. 135, line 7), and one might wish that the author adhered to the "which" and "that" precepts of Fowler. However, these are but trifling criticisms of a large and serious mathematical enterprise. We proceed to a discussion of the subject matter.

Chapter 0 and Appendix. The necessary facts about the algebra of sets, relations, functions (i.e., single-valued functions), orderings, ordinary algebra, the real numbers, cardinal and ordinal numbers, and the axiom of choice and certain useful equivalents such as Zorn's lemma, are collected here. Many of the proofs are postponed to the Appendix, where an excellent account of elementary set theory is given. The real numbers are defined as an ordered field in which every set bounded above has a supremum, and brief but unexceptionable proofs are given for a few important facts about real numbers. The treatment of the various equivalents of the axiom of choice (maximum principles and the like) is extremely clear.

Chapter 1. Topological spaces. The material in this chapter is quite standard. A topology on a set X is defined as the family $\mathfrak{T}$ of open sets. Closed sets, closure operators, and (in an exercise) interior operators are discussed. Hausdorff's neighborhood axioms appear, in slightly altered form, in an exercise. Since definition of bases at each point is one of the commonest methods of defining a topology, a more detailed discussion of this matter seems called for. Throughout the book, the exercises are called upon to carry a large part of the load of exposition. Examples, counter-examples, minor theorems, and occasionally very major theorems appear with hints of proof as exercises.

Chapter 2. Moore-Smith convergence. This chapter contains an excellent discussion of convergence of functions on directed sets, embodying among other things the author's own contributions. The axioms for convergence that generate a topology are given. Filters, which are of course in their applications equivalent to functions on directed sets, are touched on only briefly in an exercise. The exercises also contain a discussion of maximal ideals in lattices, a miniature integration theory, a treatment of Riemann integration on an interval $[a, b]$, and a proof of the semi-simplicity of Boolean rings (Boolean rings are unnecessarily postulated to satisfy the identity $r+r=0$).

Chapter 3. Product and quotient spaces. Continuous functions and topological equivalence are introduced here for the first time. The treatment of products and quotients is lucid and standard.

Chapter 4. Embedding and metrization. The following are proved: Tychonoff's theorem on embedding completely regular spaces (here called Tychonoff spaces) in products of closed unit intervals; Urysohn's lemma; Urysohn's metrization theorem; the general metrization theorem of Nagata, Smirnov, and Bing.

Chapter 5. Compact spaces. A lucid treatment of the basic facts about compact and locally compact spaces is given. The Alexandroff and Stone-Čech embedding theorems are proved. The main facts about paracompact spaces (including the theorem of A. H. Stone that every metric space is paracompact) are established in a brief and very readable section. Wallman's compactification appears in an exercise.

Chapter 6. Uniform spaces. This chapter contains the best account the reviewer has seen of the theory of uniform spaces. Every essential fact is found here. The closed graph theorem and the Banach-Steinhaus theorem are exercises.

Chapter 7. Function spaces. This chapter is couched in such generality as to lose considerable interest. The function spaces considered are mostly sets of mappings of a topological space into a uniform space. The important fact that the uniform limit of a sequence of continuous real-valued functions is continuous appears in concealed form and is explicitly mentioned only in a footnote to an exercise. The section dealing with a concept called even continuity seems out of place in a general text. The Stone-Weierstrass theorem appears only in an exercise. In view of the huge importance of this theorem, more extended treatment is surely called for. The complex form of this theorem is not mentioned at all.

The clarity of the author's thought and the carefulness of his exposition make reading this book a pleasure. He makes a large concession to human weakness in frequent repetition of definitions and even theorems. A few remarks on the subject matter may be in order. The author avoids the use of separation axioms whenever possible. For example, many assertions ordinarily proved only for metric spaces or Hausdorff spaces with a uniformity are established here for spaces that need not satisfy even the T_0 axiom. It would be useful, however, to have a more thorough discussion of the various separation axioms and a clear statement of their importance in applications. In particular, the reduction of arbitrary topological spaces to T_0-spaces should be given. The reviewer would have been glad to see some treatment of spaces of dimension zero (perhaps in exercises) and a discussion of Borel and Baire sets, including the theory of functions of the various Baire classes. No reasonable person would alter established terminology without grave reasons. While recognizing that the author undoubtedly has such reasons, the reviewer takes exception to the new definitions given for complete regularity (p. 117) and compacta (implicitly on p. 229) and to the introduction of the new term "Tychonoff space."

The reviewer noted a few inaccuracies and other defects of exposition, as follows. The term "nowhere dense" is defined in two quite different ways on pages 145 and 201 (the latter being the customary one). The attribution to Tong [1] on p. 167 should be to Hewitt [2]. The reference to Loomis [2] on p. 248 is inexact: Loomis deals with complex function spaces and homomorphisms onto the complex numbers. The terms "real linear function" and "linear functional" are used interchangeably, to the probable confusion of unsophisticated readers (pp. 108 and 241). The author cites Banach spaces (p. 110) and Haar measure (pp. 166 and 210) without explanation. Such references are bound to be unintelligible to many student readers. Only a few misprints, all of them inessential, were noted. The convenience of the book would be enhanced by numbering the chapter at the top of each page, or by designating theorems, etc., as they appear, with the number of the chapter containing them. As it is, locating back references in the text is troublesome.

Comparison of Professor Kelley's treatise with those of Kuratowski and Bourbaki is of some interest. Kuratowski's work (*Topologie* I and II, Monografie Matematyczne, Warszawa, 1948–1950) is an encyclopedic affair, strictly for professionals, and overlapping Kelley's book very little in subject matter. Bourbaki's work (*Topologie générale*, Actualités Sci. et Ind., Nos. 858–1142, 916, 1029, 1045, 1084, Paris, Hermann, 1940–1949) covers a good deal of the same ground as Kelley's, but in an austere manner very far from the sprightly style used by Professor Kelley. The treatments are in some ways complementary, and an abstract analyst could profitably be acquainted with both. In the United States, at least, Kelley's book will undoubtedly be more widely read than Bourbaki's, and it may be expected to exert a decisive and beneficent effect upon the future development of set-theoretic topology.

EDWIN HEWITT

Theory of functions of a real variable. By I. P. Natanson. Trans. by L. F. Boron, with the editorial collaboration of and with annotations by E. Hewitt. New York, Ungar, 1955. 277 pp. $6.50.

This volume differs from the original Russian book on which it is based (*Teoriĭ funktsiĭ veščestvennoĭ peremennoĭ*, Moscow-Leningrad, 1950) and from the more or less literal German translation (*Theorie der Funktionen einer reellen Veränderlichen*, Berlin, 1954 (reviewed in this Bulletin, vol. 61 (1955) p. 346) in two major respects. These changes appear to have been made by the editor to fit the book more closely to a course in Lebesgue measure and integration on the real