containing the elements of A, those of B, and all numbers of the form a_i+b_j. Let $A(n)$ be the number of elements $a_i \leqq n$. We define the Schnirelmann density $d(A)$ as the lower bound of the quotients $A(n)/n$. Schnirelmann proved $d(A+B)=1$ for $d(A)+d(B)\geqq 1$, and Landau conjectured in 1931 that

$$d(A + B) \geqq d(A) + d(B) \text{ if } d(A) + d(B) < 1.$$

This problem was studied in a number of papers, but for some years it was proved only for special cases, and other estimates were obtained (see the report of H. Rohrbach, Jber. Deutschen Math. Verein. vol. 48 (1938) pp. 199–236). These papers were so interesting since everything had to be obtained only from the definition of Schnirelmann density. In 1941, the reviewer gave a course on additive number theory whose only aim was to develop everything then known about the problem. At the end of the semester one of the students, H. B. Mann, succeeded in proving the conjecture. His proof is very ingenious, but difficult. One year later, a simpler proof was given by Artin and Scherk. The latter proof is given in the book.

The third chapter of the book is the most valuable. It gives an elementary proof of Waring's problem, which for hundreds of years was one of the most famous unsolved problems. The proofs of Hilbert and of Hardy-Littlewood are very difficult and use complicated analytic methods. In 1942, the young Russian mathematician Linnik published an elementary proof of this theorem. But this proof was not accessible to those who cannot read Russian. While Linnik's paper is only 6 pages long, Khinchin needs 28 pages for the presentation to facilitate the understanding. It is of great importance that this proof is now available in English and German.

ALFRED BRAUER

Existence theorems for ordinary differential equations. By F. J. Murray and K. S. Miller. New York University Press, 1954. 10+154 pp. $5.00.

The following quotation from the Introduction of this book indicates the scope and level of treatment, as well as the aims of the authors: "We assume a knowledge of basic real variable theory (and for certain specialized results only, of elementary functions of a complex variable) and establish the fundamental existence theorems for ordinary differential equations which are the culmination of the nineteenth-century development. We do not consider the elementary methods for solving certain special differential equations nor the more advanced specialized topics. By restricting ourselves in this fashion we hope to obtain a logically coherent discussion which has educational value for the student in a certain stage of his mathematical development."

A brief summary of the contents, chapter by chapter, is as follows: 1. Existence theorems of Peano type; 2. Implicit function theorem, with applications to differential equations not solved explicitly for the derivatives of highest order; 3. Uniqueness of solutions of differential equations under assumption of Lipschitz condition; 4. Existence theorem of Picard-Lindelöf type, with theorems on the continuity of solutions as functions of initial values, and as functions of parameters; 5. Differentiability properties of solutions as functions of initial values, and as functions of parameters, together with an existence theorem for differential equations in the complex plane; 6. Existence theorem for a system of linear differential equations of the first order and properties of solutions of such a system, including a discussion of the monodromic group in the complex variable case. The reader is led slowly through the proofs of the general theorems; indeed, in Chapters 1, 3 and 4 the considered theorems are established firstly for a single equation of the first order, and subsequently for a system of such equations.

The text contains a few illustrative examples; there are no additional exercises for the student, however, aside from occasional theorems that are stated with the suggestion that the reader supply the proofs. The book is almost devoid of references, with the entire list consisting of three in the Introduction, scattered references in the text to Ince's *Ordinary differential equations*, and in Chapter 6 five references on the Jordan canonical form for matrices.

To the reviewer it appears highly regrettable that the authors have not seen fit to introduce vector and matrix notation prior to the last chapter, where such is used in a limited fashion; certainly this innovation of Peano in his 1888 paper has contributed greatly to the reduction of cumbersome notation. In the present treatment one searches in vain for the niceties of detail afforded by judicious use of Taylor's formula with integral form of remainder, especially in the consideration of dependence of solutions on initial values, or for the treatment of differential systems involving parameters through the adjunction of suitable differential equations. In these aspects this work is inferior to the concise treatment presented in the Appendix of G. A. Bliss' *Lectures on the calculus of variations*.

The following errors were noted: (a) the statements in Section 8 of Chapter 5 on "higher derivatives with respect to x initial conditions" are manifestly false unless hypotheses are amplified to include for the involved functions the existence and continuity of certain partial derivatives with respect to x; (b) at various places in Chapters 3, 4, 5 there are statements to the effect that for a function $f(y_1, \cdots, y_n, x)$ the existence and boundedness of partial derivatives $\partial f/\partial y_j$, $(j=1, \cdots, n)$, in an open set $\mathfrak{A}$ of $(y_1, \cdots, y_n, x)$ space implies that in $\mathfrak{A}$ the function f satisfies a Lipschitz condition in $(y_1, \cdots, y_n)$, which is clearly not true without the added assumption of convexity of the intersections of $\mathfrak{A}$ with hyperplanes $x=$constant.

W. T. REID

Analog methods in computation and simulation. By W. W. Soroka. New York, McGraw-Hill, 1954. 14+390 pp. $7.50.

Modern technology must be based on a scientific analysis of the problems considered. At first glance this might seem to require a precise solution or at least a numerically valid solution to the mathematical formulation of the problem. However, this is not quite true. The basic information necessary for technical decisions may be available from the study of an analogous system under the control of the investigator. In these circumstances mathematics plays a somewhat different role. The basic problem is not the solution of the mathematical problem, but the establishment of the analogy, that is the similarity of the mathematical equations governing the two systems. The important question is the uniqueness of the solution rather than its construction.

The principle of analogy is well established in engineering. For many years, problems in power distribution, the vibrational response of elastic structures and their stress distribution, air vehicle stability, and a whole host of model studies have been based on analogy. Since the war, there has been a considerable development of electrical analog equipment which has considerable advantages in flexibility of set up, availability, and ease of operation over most other types. It is also true, however, that the many nonelectrical analogies have continued to advance. Each of these tends to have a field of optimum application where the results obtained by the specified method are the most appropriate available.

The present book is organized to survey the field relative to the various engineering applications. There is considerable introductory material relative to the realization of mathematical operations. The major analogies treated are those based on "lumped" electrical circuit theory including commercial electronic differential analyzers and the network analogies for elasticity and the theory of structures, those associated with the finite difference expressions for partial dif-