abelian groups the operators do not contribute any more to our discussion than the—purely ring theoretical—question: which properties of the integers have been really needed for our arguments? Quite naturally the answer to this question does not throw too much light on the theory of abelian groups. Still the operators have their uses: firstly a special selection of the ring of operators may produce a class of abelian groups that is amenable to treatment—a trivial example is provided by the modules over a field; and secondly they open the way to interesting applications and the author has not missed his opportunities here.

This book is written with admirable simplicity and lucidity; and it is a pleasure to be led by the author through this field, formerly so inaccessible, now so easy of access. Here it should be noted that the interesting and deep results presented in this book had been scattered through the mathematical periodicals of the world if they had been published at all. But let the reader not be misled by the apparent slimness of this monograph: There are a hundred theorems given without proof [under the misleading name of exercise] and if their proofs had been supplied [instead of the "hints" given in difficult situations] the size of the book would have been doubled at the least.

A substantial bibliography with a useful "guide to the literature"—that might be considered as a rather concentrated report on the state of the theory of abelian groups—deserves our particular attention.

We have explained why we believe that the student of the theory of abelian groups will be grateful to the author for this work. But there are various reasons why this book will be a great help to those of us who attempt to educate mathematicians and to introduce young people to the present-day ways of mathematical thinking. For instance, one likes to impress on a student's mind the central importance of structure theorems. But there do not exist many of them; and without examples of structure theorems it is impossible to explain their significance and in particular what constitutes a satisfactory structure theorem. Now these theorems form, as we mentioned before, the central theme of the book under review. Next the transfinite tools, so helpful in various branches of mathematics, are used here extensively so that the reader will learn how to avail himself of these methods. As a matter of fact this was one of the author's aims in writing this book; and in this as in his other aims he has succeeded brilliantly.

REINHOLD BAER

Handbook of elliptic integrals for engineers and physicists. By P. F. Byrd and M. D. Friedman. (Die Grundlehren der mathematischen

Wissenschaften, vol. 67.) Berlin, Springer, 1954. 13+355 pp. 36 DM.; bound, 39.60 DM.

This work is essentially a large table of elliptic integrals and, as such, is a valuable addition to the literature of the subject.

After an introduction containing a rich collection of formulas on the Jacobian elliptic functions and integrals (about 40 pages), the greater part of the book consists of 148 pages of formulas giving the reduction of more than a thousand elliptic integrals, with algebraic or elementary transcendental integrands, to (far less numerous) integrals whose integrands are rational combinations of Jacobian elliptic functions. A further section (about 30 pages) gives the expressions of the latter integrals in terms of elliptic functions and of the three classical kinds of elliptic integrals, in Legendre's notation. The last part of the book (about 70 pages) deals first with methods for evaluating the integrals of the third kind (the weak point of many works on the subject), which are generally expressed in terms of ϑ-functions. There are also a table of integrals and derivatives with respect to the modulus, an appendix on Weierstrassian functions and integrals (in the rest of the book only the notations of Legendre and Jacobi are used), and some supplements to the principal table of elliptic integrals. The book closes with about 30 pages of numerical tables, the most extensive of which gives the values of the Legendrian elliptic integrals of the first and second kinds.

In the background of the work there seems to be a certain, not unfounded, skepticism about the ability of the average applied mathematician to reduce, himself, a given elliptic integral to one of the canonical types. I myself have often had a similar feeling, considering the poor help which many "practical" books on the subject offer to an unskilled reader. With this handbook at hand, the difficulty is largely eliminated, since in most cases one will find his integral, or a similar one, in the tables.

Despite three pages of errata and some misspellings in the bibliography, the book seems to be reliable; the typography is excellent.

F. G. TRICOMI

BRIEF MENTION

The geometry of René Descartes. Trans. from the French and Latin by D. E. Smith and M. L. Latham. With a facsimile of the first edition, 1637. New York, Dover, 1954. 14+244 pp. $2.95 cloth, $1.50 paper.

This is a photographic reproduction of the 1925 edition published