## CORRECTION: DERIVATIVES OF INFINITE ORDER

R. P. BOAS, JR., AND K. CHANDRASEKHARAN

It has been pointed out to us by S. Mandelbrojt that our statement<sup>1</sup> that  $M_{n+1}^o/M_n^o$  is nondecreasing is incorrect except on the interval  $(-\infty, \infty)$  (where  $M^o$  must be replaced by  $M^o$ ) and that for a finite interval there are in fact quasianalytic classes  $C\{M_n\}$  which do not contain the class  $C\{1\}$ . However, Mandelbrojt has shown that our Theorem 2 is nevertheless correct; with his permission, we give his proof here. Theorem 2 states that, if f(x) belongs to a quasianalytic class  $C\{M_n\}$  in a < x < b and if  $f^{(n)}(x_0) \to L$  for one  $x_0$  in (a, b), then f(x) is analytic in (a, b) and consequently  $f^{(n)}(x) \rightarrow Le^{x-x_0}$  in a < x < b. There are two cases: either  $\lim \inf M_n^{1/n} > 0$  or  $\lim \inf M_n^{1/n} = 0$ . In the first case  $C\{1\} \subset C\{M_n\}$  trivially and our original proof applies. In the second case, let  $\{n_j\}$  be a sequence such that  $M_j^{1/n_j} \rightarrow 0$ . Since  $|f^{(n_i)}(x_0)| < k^{n_i} M_{n_i} \rightarrow 0$  and  $f^{(n)}(x_0) \rightarrow L$ , we must have L = 0. Given  $\epsilon > 0$ , there exist p and i such that  $|f^{(n)}(x_0)| < \epsilon$  for n > p and  $k^{n_i} M_{n_i}$  $\langle \epsilon \text{ for } j \rangle i$ . For n > p let j > i and  $n_j > p$ ; then for x in (a, b) and  $|x-x_0| < 1$ 

$$f^{(n)}(x) = f^{(n)}(x_0) + (x - x_0)f^{(n+1)}(x_0) + \cdots + f^{(n_j)}(x')(x - x_0)^{n_j - n}/(n_j - n)!,$$

where x' is between  $x_0$  and x. Then  $|f^{(n)}(x)| \le \epsilon \sum_{k=0}^{\infty} |x-x_0|^k/k! + \epsilon = \epsilon(e^{|x-x_0|}+1)$ , which shows that  $f^{(n)}(x) \to 0$  uniformly between  $x_0$  and x (and so, by a repetition of the argument, if necessary, in (a, b)), and also that f(x) is analytic.

In line 9 of page 523, replace  $ae^x$  by  $ke^x$ .

Brown University and Institute for Advanced Study

Received by the editors September 10, 1948.

<sup>&</sup>lt;sup>1</sup> R. P. Boas, Jr. and K. Chandrasekharan, Derivatives of infinite order, Bull. Amer. Math. Soc. vol. 54 (1948) pp. 523-526; p. 524.