cover all $(4n-1)$-primes up to, and inclusive of, $p=347$. Strictly speaking[3] Mersenne's numbers end with $p=257$. For the second sequence 4, 14, 194, $\cdots$ we have $s_9=26$ 21634 65049 27851 45260 59369 55756 30392 13647 87755 95245 45911 90600 53495 55773 83123 69350 15956 28184 89334 26999 30798 24186 64943 27694 39016 08919 39660 72975 85154. This term would be applicable to *all*[4] odd primes inclusive of $p=479$.

There now remain just two numbers of the form 2^p-1 in the Mersenne range whose character has not been investigated. These are M_{193} and M_{227}. The writer has begun the study of M_{227} with the sequence 4, 14, 194, $\cdots$.

YALE UNIVERSITY

[3] R. C. Archibald, *Mersenne's numbers*, Scripta Mathematica vol. 3 (1935) pp. 112–119.

[4] D. H. Lehmer, *On Lucas's test for the primality of Mersenne's numbers*, J. London Math. Soc. vol. 9–10 (1934–1935) pp. 162–165.

ON THE FACTORS OF $2^n \pm 1$

D. H. LEHMER

A recent investigation concerning the converse of Fermat's theorem disclosed that the fundamental table of Kraitchik [1][1] giving the exponent of 2 modulo p for $p<3\cdot 10^5$ contains numerous errors[2] in the previously unchecked region above 10^5. Hence it was decided to make an independent examination of primes, considerably beyond 10^5, having small exponents. As a by-product of this search the following new factors of $2^n\pm 1$ $(n\leqq 500)$ were discovered. This list is intended to supplement the fundamental table of Cunningham and Woodall [1]. The entries can be inserted in the blank spaces provided in that table. It is believed that all factors under 10^6 have now been found.[3] Moreover, any further factors of 2^n-1 for $n\leqq 300$ or of 2^n+1 for $n\leqq 150$ lie beyond 4538800. The methods used to obtain these results will be described elsewhere.

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[1] Numbers in brackets refer to the bibliography at the end of the paper.

[2] A partial list of these will appear shortly in Mathematical Tables and Other Aids to Computation.

[3] Including, of course, the previously published addenda to Cunningham and Woodall [1] which are to be found in Kraitchik [3] and [6].

n	Factor of 2^n-1
113	1868569
115	4036961
123	3887047
135	348031
143	724153
151	2332951
161	3188767
163	704161
167	2349023
173	1505447
181	1164193
187	707983
189	1560007
205	2940521
223	1466449·2916841
225	617401
229	1504073
233	622577
255	949111
277	1121297
279	1437967
291	272959
301	490631
315	870031·983431
335	464311
351	446473
353	931921
359	855857
381	349759
405	537841
441	309583
489	836191

n	Factor of 2^n+1
91	1210483
100	340801·2787601
123	165313
136	383521
139	4506937
147	748819
170	550801
196	1007441
208	928513
214	843589
220	109121
237	647011
238	823481
239	340337
280	557761·736961
285	1101811
286	958673
288	816769
289	1077971
294	540961
297	694387
305	331841
316	504337·994769
327	666427
333	304363
341	647219
357	428401
390	468781
397	321571·476401
412	454849·667441
414	318781·853669
417	441187
420	127681
430	370661
436	598193
438	1013533
441	311347
444	532801·854257
449	194867
450	695701
465	316201

n	Factor of 2^n+1
494	515737
499	825347
500	1074001

The following new complete factorizations of $2^n \pm 1$ result from entries in the above list. The large residual factors, in each case, have been proved prime by a direct test of primality. This includes the large factor of $2^{170}+1$ credited to Kraitchik by Cunningham and Woodall [1].

(1) $2^{91} + 1 = 3\cdot 43\cdot 2731\cdot 224771\cdot 1210483\cdot 25829691707$

(2) $2^{100} + 1 = 17\cdot 401\cdot 61681\cdot 340801\cdot 2787601\cdot 3173389601$

(3) $2^{113} - 1 = 3391\cdot 23279\cdot 65993\cdot 1868569\cdot 1066818132868207$

(4) $2^{115} - 1 = 31\cdot 47\cdot 14951\cdot 178481\cdot 4036961\cdot 2646507710984041$

(5) $2^{123} + 1 = 3^2\cdot 83\cdot 739\cdot 165313\cdot 8831418697\cdot 13194317913029593$

(6) $2^{123} - 1 = 7\cdot 13367\cdot 3887047\cdot 164511353\cdot 177722253954175633$

(7) $2^{135} - 1 = 7\cdot 31\cdot 73\cdot 151\cdot 271\cdot 631\cdot 23311\cdot 262657\cdot 348031 \cdot 49971617830801$

(8) $2^{170} + 1 = 5^2\cdot 41\cdot 137\cdot 953\cdot 1021\cdot 4421\cdot 26317\cdot 550801\cdot 23650061 \cdot 7226904352843746841.$

Incorrect factorizations of the first and last of these numbers are given in Kraitchik [1]. The composite numbers

$$31266402706564481 = 1210483\cdot 25829691707$$

and

$$13026477248861 = 550801\cdot 23650061$$

are given as primes. These errors are perpetuated in Kraitchik [3, 4, 5] and Cunningham and Woodall [1]. The second factorization is incorrectly given in Kraitchik [6] where the number

$$301477472991078323800 1 = 340801\cdot 2787601\cdot 3173389601$$

is listed as a prime. The factorization (6) might have been completed 20 years ago had not Kraitchik [2] given the residue index of 165313 incorrectly as 96, instead of 672. The results (2) and (7) are due to Paul Poulet and were communicated in January 1946 by letter just before his death.

Eleven of the new factors given above pertain to Mersenne numbers 2^p-1, p a prime not greater than 257. In particular the factors given for $2^{167}-1$ and $2^{229}-1$ confirm positively the composite character of these numbers, whose tests for primality have been announced recently by Uhler and Barker (Bull. Amer. Math. Soc. vol. 51 (1945) p. 389, vol. 52 (1946) p. 178, Mathematical Tables and Other Aids to Computation vol. 2, p. 94). The present state of our knowledge about the 55 Mersenne numbers may be summarized as follows:

p	Character of 2^p-1
2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127	Prime
11, 23, 29, 37, 41, 43, 47, 53, 59, 67, 71, 73, 79, 113	Composite and completely factored
151, 163, 173, 179, 181, 223, 233, 239, 251	Two or more prime factors known
83, 97, 131, 167, 191, 197, 211, 229	Only one prime factor known
101, 103, 109, 137, 139, 149, 157, 199, 241, 257	Composite but no factor known
193, 227	Character unknown

What progress has been made in the last fourteen years may be seen by comparing this table with a similar one in Bull. Amer. Math. Soc. vol. 38 (1932) p. 384.[4]

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5. *Les grands nombres premiers*, Sphinx vol. 3 (1933) pp. 99–101.
6. *Factorisation de* $2^n \pm 1$, Sphinx vol. 8 (1938) pp. 148–150.

UNIVERSITY OF CALIFORNIA

[4] The composite character of $2^{199}-1$ has recently been determined by Uhler, *On Mersenne's number* M_{199} *and Lucas's sequences*, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 163–164.