$+\sum_{s=1}^{m}(A_{2s}^{(k)}\delta_0^{2s}+B_{2s}^{(k)}\delta_1^{2s})]+R_{2m}$. This article tabulates: I. $A_{2s}^{(2)}$, $B_{2s}^{(2)}$, exact values for $2s=0, 2, \cdots, 20$ and sixteen decimal places for $2s=22, \cdots, 48$. II. $A_{2s}^{(k)}$, $B_{2s}^{(k)}$, $k=3(1)6$, $2s=2, 4, \cdots, 20$, eight significant figures (but exactly for $s=0$). Simple recursion formulas are obtained for $A_{2s}^{(2)}$ and $B_{2s}^{(2)}$ in terms of $M_{2s}\equiv 2A_{2s}^{(1)}$, an application of which is the expression of (1) in terms of δ_0^{2s} and $\delta_{1/2}^{2s+1}$, analogous to the forward version of the Newton-Gauss interpolation formula. Expressions are derived for $A_{2s}^{(2)}$ and $B_{2s}^{(2)}$ in terms of $B_\nu^{(n)}(x)$, Bernoulli polynomials of degree ν and order n. Cumulative recursion formulas are derived for $A_{2s}^{(k)}$ in terms of $B_{2s}^{(i)}$, and for $B_{2s}^{(k)}$ in terms of $A_{2s}^{(i)}$, $i=1, 2, \cdots, k$. (Received November 22, 1945.)

29. H. E. Salzer: *Table of coefficients for obtaining the first derivative without differences.*

When a function $f(x)$ is known for n equally spaced arguments at interval h, an approximation to the derivative at a point $x=x_0+ph$ may be obtained, by the differentiation of the well known Lagrangian interpolation formula, in the form $f'(x_0+ph)\sim(1/hC(n))\sum_{i=-[(n-1)/2]}^{i=[n/2]}C_i^{(n)}(p)f(x_0+ih)$, where $[m]$ denotes the largest integer in m, $C_i^{(n)}(p)$ are polynomials in p of the $(n-2)$th degree, and $C(n)$ denotes the least positive integer which enables $C_i^{(n)}(p)$ to have integral coefficients. The present table gives the exact values of these polynomials $C_i^{(n)}(p)$, for p ranging from $-[(n-1)/2]$ to $[n/2]$. For $n=4$, 5 and 6, the polynomials $C_i^{(n)}(p)$ are tabulated at intervals of 0.01; for $n=7$, they are tabulated at intervals of 0.1. (Received November 6, 1945.)

30. A. C. Sugar: *On the numerical treatment of forced oscillations.*

The solution of the equation $\ddot{x}+\omega^2x=a(t)$, $x(0)=0=\dot{x}(0)$, is given by $x=(1/\omega)\cdot\int_0^t a(\tau)\sin\omega(t-\tau)d\tau$. In this paper a simple approximation of $x$and hence of $\ddot{x}$ is found. Easy vector methods of obtaining max $|x|$ and max $|\ddot{x}|$ are discussed. (Received November 17, 1945.)

Geometry

31. L. M. Blumenthal: *Metric characterization of elliptic space.*

In this paper the first characterizations of finite and infinite dimensional elliptic spaces to be expressed wholly and explicitly in terms of distance relations are obtained. The characterizations are secured by direct, elementary geometric arguments. Only the simplest properties of elliptic space are used and no reference whatever is made to topological theorems. (Received October 3, 1945.)

32. S. C. Chang: *A new foundation of the projective differential theory of curves in five-dimensional space.*

As a preliminary a covariant triangle of reference and unit point for a plane curve is determined in an elementary and geometric manner using neighborhoods of order six. For a point P on a curve Γ in five dimensions a covariant triangle PP_1P_2 and unit point is first determined for the curve of intersection C of osculating plane and developable hypersurface of Γ. PP_1P_2 are three vertices of a quadrilateral Q on a covariant quadric generated by certain Bompiani osculants. The fourth vertex of Q is chosen as P_3. Similarly P_4, P_5 can be defined leading to a covariant pyramid for Γ. The Frenet-Serret formulas for the cases of P an ordinary and a k-ic ($k=6$, 7, 8) point follow from the corresponding canonical expansions. The method has the advantage of facilitating the geometric interpretation of the projective invariants of Γ. (Received November 1, 1945.)

33. John DeCicco: *Differential geometry in the Kasner plane.*

A horn angle is defined as the configuration formed by two curves which pass through a point in a common direction. The qualitative and quantitative aspects of a horn angle have been under consideration since ancient times. In discussions from 1909–1912, Kasner showed that each category has an invariant under the conformal group, essentially unique for each category. To study a horn angle of category n, it has been found convenient to introduce an auxiliary $(n+1)$-dimensional space, termed the Kasner space K_{n+1}, where the metric corresponds to the measure of a horn angle. Consider the simplest case $n=1$. The metric in the Kasner plane is $ds=dx^2/dy$. In the present paper, the author begins the study of the differential geometry of the Kasner plane. The concept of curvature and the theory of evolutes and involutes are developed. Also the arc curvature is considered. This is the limit of the ratio of arc to chord. (Received October 11, 1945.)

34. G. B. Huff: *Inequalities connecting solutions of Cremona's equations.*

An integer solution $x=(x_0; x_1, \cdots, x_\rho)$ of Cremona's equations $x_0^2-x_1^2-\cdots-x_\rho^2=d+p-1$, $3x_0-x_1-\cdots-x_\rho=d-p+1$ is said to be *proper* if x_0 is the order and $x_1, \cdots, x_\rho$ are the multiplicities, at a set of ρ points in the plane, of a *complete* and *regular* linear system of plane curves of dimension d and genus p. Inequalities of the following type are established. If x is a solution of Cremona's equations with $x_0>0$, $p\geqq 0$, $d\geqq 0$ and c is a proper characteristic for $p=0$, $d=2$, then $c_0x_0-c_1x_1-\cdots-c_\rho x_\rho\geqq 0$. If f is a proper characteristic for $p=d=0$ and x is a characteristic of $x_0>0$ and p, d non-negative and not both zero and $x_0\leqq f_0$, then $f_0x_0-f_1x_1-\cdots-f_\rho x_\rho\geqq 0$. These inequalities lead to criteria concerning proper solutions. (Received October 16, 1945.)

35. Edward Kasner and John DeCicco: *Problems on perspective maps.*

The authors present some theorems in the perspective mapping of a surface upon a plane from a given point. The only perspective conformalities upon a plane are Ptolemy's stereographic projection of a sphere, and the obvious limiting case of a parallel plane. If more than $3\infty^1$ geodesics are projected into straight lines, then all geodesics become straight lines, and the surface is a sphere, or any plane, parallel or not; furthermore the point of perspectivity is at the center of the sphere. The surfaces are classified with respect to the maximum number of geodesics which project into straight lines. There are four distinct classes. The class for which area preserving perspectivities exist is discussed. The complete integrals are cylinders whose directorial curves are defined by a Tchebycheff integral of non-elementary character. Other solutions may be found as envelopes of these cylinders. The sphere is not a solution. (November 20, 1945.)

36. C. E. Springer: *Rectilinear congruences whose developables intersect a surface in its lines of curvature.*

A congruence of lines is referred to a surface S, and the equation of the net of curves in which the developables of the congruence intersect the surface is obtained. This net is called the *intersector net* on S. It is shown that the exhibition of a congruence relative to S for which the intersector net coincides with the lines of curvature net on S requires a solution of a partial differential equation of Laplace. It is also demonstrated that the specification of a congruence for which the intersector net coincides with the asymptotic net on S requires a solution of two partial differential equations of parabolic type. (Received October 11, 1945.)

37. T. Y. Thomas: *Absolute scalar invariants and the isometric correspondence of Riemann spaces.*

Necessary and sufficient conditions for the isometric correspondence of Riemann spaces R_n and $\bar{R}_n$ are given in terms of the equality of absolute scalar invariants of the spaces. In the general case for which the spaces admit a complete set of n functionally independent scalars it is proved that these and a certain derived set of scalars suffice for the solution of the problem. The solution of the correspondence problem is given for spaces of two dimensions which do not admit two functionally independent scalars. (Received October 10, 1945.)

38. T. Y. Thomas: *Topological theory of dynamical systems.*

The projective or topological theory of dynamical systems is concerned with the study of the trajectories independently of their time parameterization. This paper deals with the possible changes in the invariants determining the system which leave the trajectories unaltered. The case of the conservative system is of especial interest and is treated in detail. Under the assumption that the dynamical systems admit essentially a single quadratic or energy integral it is proved that the most general transformation on the coefficients $g_{\alpha\beta}$ of the expression for the kinetic energy and the potential function V is given by $g_{\alpha\beta}=(cw+d)h_{\alpha\beta}$ and $V=(aw+b)/(cw+d)$ where the a, b, c, d are constants. It is shown, moreover, that the property of a conservative system of possessing essentially only one energy integral is invariant under this transformation. The methods can be applied to systems which are not conservative. (Received October 10, 1945.)

Logic and Foundations

39. A. R. Schweitzer: *On the genesis of number systems.* I.

This paper aims to effect a gradual transition from the foundations of geometry to postulates for number systems in terms of undefined relations (operations) analogous to concepts previously developed by the author, Amer. J. Math. vol. 31. The first of these sets uses the undefined operation "replacement" (transformation) analogous to that employed in Chapter II of the preceding article, p. 373. A set S of elements (α) is combined into dyads $(\alpha\beta)$ of a set T assumed subject to two types of replacement of dyads by elements, symbolized by $R(\alpha\beta)=\gamma$ and $P(\kappa\lambda)=\mu$. These relations are also expressed, $\gamma R(\alpha\beta)$ and $\mu P(\kappa\lambda)$ or $\gamma=\alpha+\beta$ and $\mu=\kappa\times\lambda$. A closer analogy is attained by assuming, instead of $R(\alpha\beta)=\gamma$, that if $\alpha\xi$, $\beta\xi(\xi\alpha, \xi\beta)$ are in T, then γ exists in S such that $\gamma\xi(\xi\gamma)$ in T replaces $\alpha\xi$, $\beta\xi(\xi\alpha, \xi\beta)$; in symbols, $R(\alpha\xi, \beta\xi)=\gamma\xi$ or $\gamma\xi R(\alpha\xi, \beta\xi)$ or $\gamma\xi=\alpha\xi+\beta\xi$; and so on. The relation $\gamma=\alpha+\beta$ then holds if and only if $\gamma\xi=\alpha\xi+\beta\xi(\xi\gamma=\xi\alpha+\xi\beta)$ for any ξ in S. Postulates in terms of $\gamma R(\alpha\beta)$ and $\mu P(\kappa\lambda)$ are also interpreted as analogous to the author's system ${}^2R_2$ (ibid. p. 382). (Received October 19, 1945.)