characterization is in general equivalent to the following: On the variety there exist r families of curves such that when the tangent r-plane is displaced along a curve of the family its intersection with a neighboring tangent r-plane is the tangent $(r-1)$-plane formed by the tangents of the other $r-1$ curves. Basing on this property one can define on each of these r tangents AA_i, $i=1, \cdots, r$, $r-1$ points A_{ij}, $j\neq i$, having the property that when the s-plane $AA_{i_1} \cdots A_{i_s}$ is displaced along AA_j, $j\neq i_1, \cdots, i_s$, its intersection with a neighboring s-plane is the $(s-1)$-plane $A_{i_1 j} \cdots A_{i_s j}$. The points A_{ij} describe varieties of the same type and are naturally defined as the Laplace transforms of the given variety, there being altogether $r(r-1)$ transforms. Many well known properties of Laplace transforms can be generalized. (Received February 23, 1944.)

160. S. B. Jackson: *Vertices of plane curves.*

A closed curve of class C'', not a circle, has two vertices by the continuity of the curvature. The present paper seeks to characterize geometrically those curves with exactly two vertices. Let a curve be called normalized if it contains no complete circles, and let a simple closed arc of the curve which is never crossed by the curve be called a simple loop. The following facts are established for any normalized curve C with two vertices: (a) C may be divided into two simple arcs; (b) all double points are simple; (c) C contains exactly two simple loops, one containing each vertex; (d) none of the plane regions bounded by C are bounded always in the same sense except those regions bounded by the loops; (e) at any points of tangency the directed tangents coincide. For a curve which is not normalized these results are modified slightly. Two familiar theorems regarding the number of vertices on an oval are generalized to any simple closed curve. The methods employed are entirely elementary, extensive use being made of the invariance of vertices under direct circular transformations. (Received February 4, 1944.)

161. J. E. Wilkins: *The contact of a cubic surface with a ruled surface.*

It is shown that there exist ∞^1 cubic surfaces having contact of order 5 with a nondevelopable ruled surface. If there is any cubic surface having contact of order 6 with a nondevelopable ruled surface, then the surface is itself a cubic surface. In order to obtain these results, there are first derived power series expansions for a nondevelopable ruled surface to terms of the sixth degree. Similar investigations are made for developable surfaces. (Received April 1, 1944.)

Statistics and Probability

162. C. W. Churchman and Benjamin Epstein: *Estimates of error in parallel experiments.*

It is common in many types of tests to have not only a random error from test to test due to a large number of unallocable causes, but it is also possible to have systematic errors present. It is because of this possibility that one tests not only samples of the unknown, but also control samples. The purpose of using control samples is two-fold—(a) to find out whether or not abnormal experimental conditions exist during the test and (b) to establish tentatively a level for the particular test under consideration. It is shown that a statistic can be found which gives the most efficient estimate of the corrections to be applied to the unknown under test for a variety of experimental conditions. It is further shown that this statistic must be a linear function of the random variables x and y where x corresponds to the values assumed by the unknown under test and y corresponds to the values assumed by the control from test to test. (Received March 23, 1944.)

163. P. R. Halmos: *Randon alms.*

Suppose that a pound of gold dust is distributed at random among a countably infinite set of beggars, so that the first beggar gets a random portion of the gold, the second beggar gets a random portion of the remainder, and so on ad infinitum. The purpose of this work is to calculate the actual and the asymptotic distributions of x_n and S_n (where x_n is the amount received by the nth beggar and $S_n = \sum_{i \leqq n} x_i$) and also to study the rate of convergence of the random series $x_1 + x_2 + x_3 + \cdots$, under the assumption that the phrase "random portion," occurring an infinite number of times in the description of the stochastic process, receives the same interpretation each time. The results may be interpreted as properties of a random distribution of a unit mass on the positive integers; they may be used to explain the experimentally observed distribution of sizes of mineral grain particles; and they occur also as distributions of energy in the theory of scattering of neutrons by protons of the same mass. (Received March 24, 1944.)

164. Henry Scheffé and J. W. Tukey: *Contributions to the theory of non-parametric estimation.*

For problems of non-parametric estimation, concerning an unknown cumulative distribution function $F(x)$, three good solutions are available: (i) confidence intervals for the median of F (W. R. Thompson, K. R. Nair), (ii) tolerance limits for F (Wilks), and (iii) confidence limits for F (Wald, Wolfowitz, Kolmogoroff). Heretofore (i) and (ii) have been limited to the case where $F'(x)$ is known to be continuous. By means of a theorem of general application they are extended to the case where F need only be continuous. The only previous result for discontinuous F is that of Kolmogoroff for (iii). The appropriate modifications of (i) and (ii) extending their validity to this case are found. Some uniqueness results limiting the kind of statistics usable in such problems are obtained. Sufficiently complete tables for applying (i) and (iii) have been published, but computations for (ii) have been extremely few and laborious. A simple formula based on the z and χ^2-distributions is found which gives highly accurate approximations in ranges of practical interest. All the results for (ii) also apply to Wald's tolerance intervals for the multivariate case. (Received March 14, 1944.)

Topology

165. E. F. Beckenbach and R. H. Bing: *On generalized convex functions.*

Let $F(x; \alpha, \beta)$ be a two-parameter family of real continuous functions defined in an interval (a, b) such that there is a unique member of the family taking on arbitrary values y_1, y_2 at arbitrary distinct points x_1, x_2 of the interval. A real function $f(x)$ is said to be a sub-$F(x; \alpha, \beta)$ function in (a, b) provided at the midpoint x_0 of each subinterval I of (a, b) we have $f(x_0) \leqq F(x_0; \alpha, \beta)$ for that $F(x; \alpha, \beta)$ which coincides with $f(x)$ at the endpoints of I. The family $F(x; \alpha, \beta)$ is not necessarily topologically equivalent to the set of non-vertical line segments in the strip; hence the study of sub-$F(x; \alpha, \beta)$ functions is not topologically equivalent to the study of convex functions. It is shown among other things that if a sub-$F(x; \alpha, \beta)$ function $f(x)$ is bounded,