BOOK REVIEWS

On growth and form. By D'Arcy W. Thompson. Cambridge University Press; New York, Macmillan, 1942. 1116 pp. $12.50.

It is a rare privilege to write a review of a new edition of a book which has already taken its place as one of the great classics of science. *On growth and form* by D'Arcy Thompson in this new edition is much as it was in the first. Its fundamental ideas have been very little revised, but a wealth of new material has been added, expanding it from 793 to 1116 pages.

To summarize its contents would be a very difficult matter. It is almost an encyclopedia of all the relations that have ever been discussed between mathematics and organic form. Among the subjects treated are: the form of the cell, tissues, concretions produced by living things, shells, horns, and teeth; from the dynamic point of view, growth and the relation between form and mechanical efficiency; and such perennial favorites of the geometrician as the form of the bee's cell and the arrangement of leaves.

For the most part the mathematics used in the book is elementary. Thompson makes no pretentions, but says he is using the tools he has, leaving it to better equipped workmen to carry on the work. Professor Archibald[1] in a review of the first edition deals more competently than I could with the mathematics. The general point of view seems more important than the mathematics itself. In fact the author frequently talks more about mathematics than in its language. He states his purpose in the introductory chapter (p. 14): "to correlate with mathematical statement and physical law certain of the simpler outward phenomena of organic growth and structure or form, while all the while regarding the organism, *ex hypothesi*, as a material and mechanical configuration." And in the Epilogue he concludes (p. 1096) "My task is finished if I have been able to show that a certain mathematical aspect of morphology, to which as yet the morphologist gives little heed, is interwoven with his problems, complementary to his descriptive task, and helpful, nay essential, to his proper study and comprehension of Growth and Form."

Various biologists have placed very different estimates on the value of the book. Professor Sinnott,[2] in his review, has been exceedingly generous and attributes to it a wide and important influence. Professor McClung[3] on the other hand has stated that its influence is slight. In

[1] R. C. Archibald, Bull. Amer. Math. Soc. vol. 24 (1918) p. 403.

[2] E. W. Sinnott, Quarterly Review of Biology vol. 18 (1943) p. 64.

[3] C. E. McClung, Science N. S. vol. 96 (1942) p. 471.

spite of the fact that the book has been very much admired, I am afraid that Professor McClung's estimate is nearer the truth. The ideas of *On growth and form* have played little part in the spectacular advances of biology since the book was first written. For this I think there is a very good reason: the point of view which the book represents is out of fashion, and is indeed the antithesis of the one now in vogue, to which these advances have been due.

When the book was first published in 1917, experimentation in genetics was just beginning to produce its brilliant results and experimental embryology was approaching its most spectacular era. Here were exciting things to attract the young investigator with which the abstract ideas of the mathematician, whose language he all too frequently could not understand, could hardly compete. A rich harvest has been reaped in these fields with very slight if any influence of Thompson's book. Interestingly enough, on the other hand, almost every reviewer has complained that the new edition has not been influenced by this research, which is for the most part hardly mentioned. McClung[3] misses a treatment of cytogenetics, Buchanan[4] a treatment of physiological gradient ideas, and both Sinnott[2] and Hammett[5] list a number of important subjects that one might expect to have been treated.

The failure to take the results of modern biological research into consideration (Sinnott says the revision might just as well have been written twenty-five years ago) is related to the failure to make an important impact upon this research. They are both due to the antithesis of the fundamental ideas. *On growth and form* harks back to an older habit of thought, and may also as Wrinch[6] suggests be the herald of a new era, but it is not an essential part of contemporary biological advance.

There are only three essentially different ways of considering biological form. It may be considered as present from the beginning, existing either as an actual minute replica in the germ, or as an "idea" of some nonphysical formative entity like the entelechy of Aristotle and Driesch; or existing neither actually nor ideally, but merely potentially, in the organization of the germ. In older biological thinking form always pre-existed in some fashion or other. The adult form was in existence from the beginning of the life of the organism as an individual. The "preformationists" of the 18th century—their ideas culminating in Bonnet—believed that the germs of all organisms ever

[4] J. W. Buchanan, Physiological Zoology vol. 16 (1943) p. 135.

[5] F. S. Hammett, Growth vol. 7 (1943) p. 321.

[6] D. Wrinch, Isis vol. 25 (1943) p. 232.

to exist were created at the original creation. Each germ was a minute replica of the adult that was to develop from it. It had merely to expand. There was no real development of form. It had existed from the beginning.

The "epigeneticists," their opponents, believed that organisms are developed from unorganized and unformed matter under the influence of a vital, nonphysical entity. Here the form of the adult preexisted not actually, as in the germ of the preformationist, but as an idea of the entelechy or *vis essentialis*. The unformed material was moulded into shape by this agent—the actual form produced was new, an epigenesis.

With the development of modern techniques it became clear that the epigeneticists were right in that the germ contains no actual replica of the adult. The form develops rather by gradual stages. The notion of a nonphysical formative agent has not appealed to most biologists, however, although Driesch has made a valiant attempt to sustain it. The alternative chosen by most is a modified form of preformationism, in which the germ is considered to contain not a preformed replica of the adult, but an organization which determines the development of the adult form. The nature of this organization may be considered from two points of view: biologically, from the point of view of heredity, as with the trend begun by Weismann and leading to modern genetics, or mathematically and physically as in Thompson's book.

Thompson says (p. 1022) "To look on the hereditary or evolutionary factor as the *guiding principle* in morphology is to give to that science a one-sided and fallacious simplicity" and states his position unequivocally (p. 340): "The efforts to explain 'heredity' by the help of 'genes' and chromosomes, which have grown up in the hands of Morgan and others since this book was first written, stand by themselves in a category which is all their own and constitute a science which is justified of itself. To weigh or criticize these explanations would lie outside my purpose, even were I fitted to attempt the task. . . . I leave this great subject on one side not because I doubt for a moment the facts nor dispute the hypotheses nor decry the importance of one or other; but because we are so much in the dark as to the mysterious field of force in which the chromosomes lie, far from the visible horizon of physical science, that the matter lies (for the present) beyond the range of problems which this book professes to discuss, and the trend of reasoning which it endeavors to maintain."

Thompson's concept of organic form is that it is predetermined by the physical organization of the system in which it develops, and "it is in obedience to the laws of physics that their particles have been moved, moulded and conformed" (p. 10). The nature of the physical organization and of the physical processes by which the form is produced from it is to be arrived at by mathematical analysis and physical interpretation of the adult form (p. 16). "The form of an object is a 'diagram of forces,' in this sense, at least, that from it we can judge of or deduce the forces that are acting or have acted upon it." "In an organism, great or small, it is not merely the nature of the motions of the living substance which we must interpret in terms of force (according to kinetics), but also the conformation of the organism itself, whose permanence or equilibrium is explained by the interaction or balance of forces, as described in statics." His interest is not in the biological analysis of the organization of the germ, from the point of view of either the geneticist or the experimental morphologist. He approaches the problem from the opposite direction.

He thinks of form as a Platonist. In his discussion of the tortoise shell, for example (p. 517), he is obviously talking of an ideal tortoise, more valid than any actual specimen or species, and any variant from which may be considered an "accident." As a matter of fact in the Epilogue (p. 1097) he alines himself with the teaching of Plato and Pythagoras, and again (p. 1094) he says "In natural history Cuvier's 'types' " (which are Plato's ideas under another name) "may not be perfectly chosen nor numerous enough, but types they are; and to seek for stepping stones across the gaps is to seek in vain, forever." On this view his attitude toward embryology (p. 5) and his failure, pointed out by Julian Huxley,[7] to apply his method of transformation of coordinates to stages in the life history of a single organism, are readily explainable. He is not thinking of the development of form as a biological process at all. He takes the adult form as given and analyzes it.

To place Thompson thus among the Platonists and beside the transcendental morphologists of the last century is by no means derogatory. Man has been seeking systematically for answers to his great problems for only 5000 years at most, and on a conservative estimate will be doing so for some 5,000,000 years to come. To attempt to read off with finality any consistently developed line of thought would seem at this stage presumptuous. It needs to be strongly emphasized that none of the hypothetical entities proposed on the basis of experiments in embryology deal with form in the sense that Thompson does. The gradients of Child and the fields of Weiss

[7] J. S. Huxley, *Problems of relative growth*, New York, 1932, p. 105.

serve to organize many significant facts about development, but do not tell us why specific form develops. Child[8] says that the specific form develops because the gradient operates in a substratum of specific protoplasm, but what does that mean? Weiss[9] says "the fact that each cell is bound to react exclusively in accordance with the standards of the species to which it belongs" constitutes "the principle of 'genetic limitation.'" What is the basis of that? In spite of all we have learned about the nature and action of organizers and evocators, Needham admits "that after all the larger part of the mystery remains in that we can as yet form little idea of what constitutes reactivity—competence to react to the morphogenetic inductor."[10] The specificity of protoplasm from the point of view of morphogenesis, and the competence of tissues to respond in a specific way to organizers are associated with the "form" in Thompson's *On growth and form*. The present day experimental embryologist is simply not investigating it at all. The geneticist, with his genom—the fundamental organization of the germ made up of genes arranged on chromosomes—comes nearer to it. It may well be, as Wrinch[6] suggests, that as the chemist learns more of the morphology of huge molecules and their aggregates, and as the biologist learns to apply this knowledge to the structure of the genom, a new place will be found, in our thinking, for the form of organisms as the morphologist, who descends from the line of Plato, thinks of it.

Even before that time arrives, however, this book has a message for the biologist, whose idea of quantitative biology is frequently merely the statistical treatment of data, sometimes even forgetting that "measurements may be as empty of significance as any other kind of descriptive materials. The statistical answers that can be wrung out of such measurements may have very little meaning if the quantities measured depend on a multiplicity of causes. Statistical treatment may indicate that certain results are significant, but what they signify no man may know, or even surmise."[11] Such a man might gain from *On growth and form* an idea of what could be meant by mathematical biology.

For the mathematician who has an interest in the relationship between his abstract ideas and the phenomena of the natural world, the book should be a treasure house. The author attributes to Lobatchevsky the statement (p. 11) that "there is no branch of mathematics

[8] C. M. Child, *Individuality in organisms*, Chicago, 1915, p. 188.

[9] P. Weiss, *Principles of development*, New York, 1939, p. 362.

[10] J. Needham, *Biochemistry and morphogenesis*, Cambridge, 1942, p. xiii.

[11] T. H. Morgan, *Experimental embryology*, New York, 1927, p. 13.

however abstract, which may not some day be applied to phenomena of the real world."

The book is written in an elegant, almost poetic, style that makes it delightful to read, and its many references conveniently placed at the bottom of the page near the related subject matter serve as a lure to further reading.

J. WALTER WILSON

Partial differential equations. By Frederick H. Miller. New York, Wiley; London, Chapman and Hall, 1941. 9+259 pp. $3.00.

According to the author the book is intended to be a text in a first course in partial differential equations. The chapter on ordinary differential equations is intended for review and reference purposes and not as a first course in the subject. The author finds it advisable to include a chapter on direction cosines and partial derivatives, probably because so many exercises in the book are taken from geometry. In the main the book is concerned with the quest for solutions depending on arbitrary constants and arbitrary functions. The examples of Chapter III show very clearly why this viewpoint of the subject is much more complicated in the case of partial differential equations than it is in the case of ordinary equations. In ordinary equations the solution of an nth order equation depends on n arbitrary constants, and conversely, the elimination of n arbitrary constants leads to an equation of nth order. In general, the number of partial derivatives of a given order is higher than the number of independent variables. The elimination of two arbitrary functions may lead to a pair of third order equations in one unknown. Since the first order partial differential equation behaves more like an ordinary equation than do those of higher order, Chapter IV on the linear equation of first order and Chapter V on nonlinear equations of first order are almost entirely devoted to the quest for solutions depending on arbitrary functions and arbitrary constants.

Chapter VI on Fourier series and the boundary value problems in Chapter VII furnish an exception to the above viewpoint. In this work the author is, of course, not seeking solutions depending on arbitrary functions. Chapter VI on Fourier series contains a statement of the expansion theorem for a function continuous except for a finite number of jump discontinuities. Chapter VII on the linear equation of higher order is devoted largely to the consideration of operator methods, undetermined coefficients, and variation of parameter methods for obtaining the particular solution. In case the nth order differential operator can be factored into linear factors a complimentary