

the equations of compressible flow and then appealing to the work of E. Reissner (*The equations of lifting strip theory for tapered wings*, Curtiss Research Laboratory Report SB-76-S-1) on the flutter of finite aspect ratio it is shown that the compressibility effect can be included in the Prandtl equation by substituting for $\partial C_{L \text{ ino}}(y)/\partial \alpha$ the quantity $(1/(1-\beta^2)^{1/2})\partial C_{L \text{ ino}}(y)/\partial \alpha$. This is done by setting up the problem as a boundary value problem for the appropriate elliptic partial differential equation. (Received July 26, 1943.)

227. Alexander Weinstein: *On the bending of a clamped plate.*

Let $\{p_k(x, y)\}$ denote a complete, but not necessarily orthogonal, sequence of harmonic functions of integrable square in a domain S with the boundary C . Consider the equation $\Delta \Delta w = f$ with the boundary conditions $w = dw/dn = 0$. Let $w_m = GGf + \sum_{i=1}^m a_{mi} Gp_i$, where Gf is the Green's potential of f , and the constants a_{mi} are determined by the equations $\sum_{i=1}^m a_{mi}(p_i, p_k) = -(f, Gp_k)$, $k=1, 2, \dots, m$. It is proved in this paper that w_m converges uniformly to w in $S+C$ and that $\partial w_m/\partial x$, $\partial w_m/\partial y$ converge uniformly to $\partial w/\partial x$, $\partial w/\partial y$ in every domain interior to S . The approximating functions w_m can be computed explicitly in the case when S is a rectangle. The proofs are based on the results of a paper by N. Aronszajn and A. Weinstein, Amer. J. Math. vol. 44 (1942) p. 625. (Received July 10, 1943.)

GEOMETRY

228. Herbert Busemann: *On spaces in which two points determine a geodesic.*

Let R be a finitely compact, convex, and locally strictly externally convex, metric space. A geodesic in R is defined as a continuous curve which is locally isometric with the real axis. (A symmetric variational problem in parametric form will satisfy these conditions, when the extremals are considered as geodesics). There is at least one geodesic through any two distinct points of R . If this geodesic is unique, then R is either simply connected and all geodesics are isometric with a euclidean straight line, or R has a two-sheeted universal covering space and all geodesics of R are congruent to one euclidean circle. (Received July 26, 1943.)

229. John DeCicco: *Kasner's pseudo-angle.*

The theory of functions of a single complex variable is identical with plane conformal geometry. This is not the case in the theory of functions of two or more complex variables. A set of n functions of n complex variables induces a correspondence between the points of a real $2n$ -dimensional space R_{2n} . The infinite group G of these correspondences has been termed the pseudo-conformal group by Kasner. In 1908, Kasner showed that for $n=2$ a transformation of R_4 is pseudo-conformal if and only if it preserves the pseudo-angle between a curve and a three-dimensional variety. This pseudo-angle theorem can be carried over to $2n$ dimensions almost without change. A system of $(2n-1)$ -dimensional hypersurfaces is said to be bi-isothermal if it is pseudo-conformally equivalent to ∞^1 -parallel $(2n-1)$ -dimensional flats. Any system of $(2n-1)$ -dimensional hypersurfaces is bi-isothermal if and only if the pseudo-angle between the given hypersurfaces and any system of parallel lines is a biharmonic

function. Many other results are obtained by using Kasner's pseudo-angle. (Received July 27, 1943.)

230. V. G. Grove: *A general theory of surfaces and conjugate nets.*

This paper studies the projective differential geometry of surfaces and conjugate nets by means of a tensor analysis based on a connection appearing naturally in the theory. All congruences harmonic or conjugate to a surface are found. A simple geometric construction for reciprocal congruences is given independent of the quadrics of Darboux. A simple tensor method of studying conjugate nets is used. Among other applications, a certain interesting involution on each line protruding from the surface is studied. Finally some of the results are specialized to the metric geometry of surfaces in euclidian space of three dimensions. (Received July 23, 1943.)

231. C. C. Hsiung: *An invariant of intersection of two surfaces.*

Let two surfaces S_1, S_2 in ordinary space intersect at an ordinary point O with distinct tangent planes τ_1, τ_2 ; assume that the common tangent is distinct from the asymptotic tangents of the surfaces S_1, S_2 . The author proves the existence of a projective invariant determined by the second order terms of the surfaces S_1, S_2 at the point O , and he gives the invariant a projective as well as a metric characterization. The metric characterization takes the following simple form. Let K_1, K_2 be the total curvatures of the surfaces S_1, S_2 at the point O ; and R_1, R_2 the radii of curvature at O of the curves in which the tangent planes τ_2, τ_1 intersect the surfaces S_1, S_2 respectively. Then $R_2^4 K_2 / R_1^4 K_1$ is the projective invariant under consideration. (Received July 1, 1943.)

232. C. C. Hsiung: *Projective invariants of some particular pairs of space curves.*

It is known that if two plane curves have contact of order $k-1$ at a point P , and if nonhomogeneous projective coordinates are chosen so that the expansions representing these two curves in the neighborhood of P are, respectively, $y = ax^k + \dots$, $y = \alpha x^k + \dots$, $a \neq \alpha$, then the ratio a/α is a projective invariant studied for $k=2$ by H. J. S. Smith (Proc. London Math. Soc. (1) vol. 2 (1867) pp. 196-248) and R. Mehmke (Schlömilchs Zeitschrift für Mathematik und Physik vol. 36 (1891) pp. 56-60, 206-213). Their results were interpreted and extended by C. Segre (Rendiconti dei Lincei (5) vol. 6 (1897) pp. 168-175 and vol. 33 (1924) pp. 325-329). The present paper supplements these investigations by studying pairs of curves in 3-space: (a) tangent at a general point with distinct osculating planes, (b) intersecting at a general point with distinct osculating planes, (c) intersecting at a general point with common osculating plane. The existence of a projective invariant determined by the third order terms is shown for each of these cases; these invariants are characterized geometrically in terms of certain double ratios. (Received June 11, 1943.)

233. C. C. Hsiung: *Theory of intersection of two plane curves.*

This paper is concerned with the study of the projective differential geometry of two plane curves intersecting at an ordinary point. The method used is similar to the

one given by the author in a previous paper (*Projective differential geometry of a pair of plane curves*, to appear in *Duke Math. J.*). Here two projective invariants determined by the fourth order terms in the Taylor expansions of the functions representing the two curves, at the point of intersection, are obtained and characterized geometrically. Further, on the basis of the vanishing or nonvanishing of these two invariants, the author arrives at four different types of canonical representation for the two curves at the point of intersection; the absolute invariants in the expansions of each type are interpreted geometrically in terms of certain double ratios. (Received June 11, 1943.)

234. Edward Kasner: *Motion in a resisting medium.*

Consider the motion of a particle moving in the plane under a generalized field of force and influenced by a resisting medium, the resistance R acting in the direction of the motion and varying as some function of the position (x, y) , the direction y' , and the speed v of the particle. Through a given lineal element E , there pass ∞^1 trajectories. If the osculating parabolas are constructed to these trajectories at E , the locus of the foci varies in shape with the nature of the resistance R . If the focal locus is a circle through the point of E , it is found that R must be of the form $A(x, y, y')v^2 + B(x, y, y')$. In the final part of the paper, this result is extended to space. Construct the osculating spheres at E to the ∞^1 trajectories passing through the lineal element E . If the locus of the centers of these spheres is a straight line, the resistance R is of the form $A(x, y, z, y', z')v^2 + B(x, y, z, y', z')$. Finally it is shown that if a single trajectory is known in the field of gravity with a resisting medium $R(v)$, then the law of resistance $R(v)$ can be completely determined. (Received July 27, 1943.)

235. Edward Kasner and John DeCicco: *A generalized theory of contact transformations.*

The authors present a generalized theory of contact transformations in the plane. Any union-preserving transformation of differential elements of order n into lineal elements is obtained by considering the osculating (up to and including contact of order n) of a given parameterized family of ∞^{n+1} curves to an arbitrary curve. If a union-preserving transformation T is such that any two unions which possess $n \geq 2$ as the order of contact are converted by T into two unions which have at least second order contact, then T must be a contact transformation between lineal elements. As a consequence of this work, the authors find a general theory of evolutes and involutes which contains the osculating circle theory of Huygens and Bernoulli as a special case. (Received July 27, 1943.)

LOGIC AND FOUNDATIONS

236. Theodore Hailperin: *A set of axioms for logic.*

Two well known logical systems claiming adequacy for mathematics are currently studied. These are appellatively described as "Principia Mathematica" and "set-theory." A third and stronger system, called "New Foundations," has been proposed by W. V. Quine. (This system is not to be confused with his *Mathematical logic*, 1940.) Quine's system uses the usual logical primitives for the propositional calculus and the theory of quantifiers, and class membership, but makes no restrictions on the