fined by pairs of functions of two complex variables, the pseudo-conformal group G. This is characterized by the preservation of Kasner's pseudo-angle between a curve and a hypersurface at their common point. Any system of ∞^3 curves which is pseudo-conformally equivalent to a parallel set of lines is said to be bi-isothermal. Any such system consists of ∞^2 isothermal families, each family living on a conformal surface. A characteristic property of bi-isothermal systems is that the pseudo-angle between this system and any parallel pencil of hyperplanes is a biharmonic function. Finally G is characterized within the group of point transformations by the preservation of the totality of all bi-isothermal systems. (Received September 29, 1941.)

481. Walter Prenowitz: *Descriptive geometries as multigroups.*

Let G be the set of points of a descriptive space of arbitrary dimension. In G, define $a+b$ as the set of points between a and b if $a \neq b$, and $a+a$ as a. Then G becomes an abelian multigroup of special type. Convex sets and linear manifolds appear respectively as semigroups (subsets closed under $+$) and subgroups of G. Half-spaces (rays, half-planes, and so on) are cosets, when the latter are properly defined, and some of the elementary properties of half-spaces follow from a coset decomposition theorem. Elementary properties of these three types of figures are derived algebraically and many familiar group theoretic concepts as factor group, homomorphism, congruence relation are used in the development. (Received September 29, 1941.)

482. C. V. Robinson: *Spherical theorems of Helly type and congruence indices of spherical caps.*

The theorems mentioned in an earlier abstract (47-1-67) have been extended to the n-sphere. The principal theorem of Helly type reads: If each $n+k+2$ members of a family of convex subsets of the n-sphere intersect and if one member contains no k-dimensional great hypersphere then there is a point common to all the sets of the family. A study is also made of the congruence indices of spherical caps of various radii with respect to the class of semi-metric spaces. For example, it is found that a cap of spherical radius $\rho < \pi r/2$ of the 2-sphere of radius r will contain isometrically any semi-metric space of more than 6 points if every quadruple of the space is isometrically imbeddable in the cap, that is, the cap has the congruence indices [4, 2]. (Received September 23, 1941.)

Statistics and Probability

483. J. F. Daly: *A problem in estimation.*

Consider a normal population in which each individual is characterized by the variates $y_1, \cdots, y_p, y_{p+1}, y_{p+2}$. Suppose that the latter two are not directly observable, but that for given values of y_{p+1}, y_{p+2} the first set of y's is independently distributed about the "regression line" $y_k = y_{p+1} + k y_{p+2}$ $(k=1, \cdots, p)$ with a common variance σ^2. For each individual, one can thus determine values $\hat{y}_{p+1}, \hat{y}_{p+2}$ from the observed $y_1, \cdots, y_p$, using the method of least squares. Assuming a similar relation between the expected values of $y_1, \cdots, y_{p+2}$ in the original population, these estimates $\hat{y}_{p+1}, \hat{y}_{p+2}$ are of course unbiased. However, if we calculate these $\hat{y}$'s for each individual of a sample of N and substitute them in the Pearson product-moment correlation formula the estimate of the correlation between y_{p+1} and y_{p+2} thus obtained is somewhat biased. The bias depends on the number of observable y's and on the size of the variances and covariances of y_{p+1}, y_{p+2} relative to σ^2. (Received September 2, 1941.)

484. Hilda P. Geiringer: *Some observations on analysis of variance theory.*

The distribution of the test functions used in the analysis of variance has been determined by R. A. Fisher for mutually independent chance variables subject to the same normal law. If it is not possible to determine the distributions of test functions for small samples and sufficiently general populations, the expectations and the variances of the test functions computed under appropriately general assumptions may be used. The expectations of the two quadratic forms known as "variance within" and "among" classes are equal even for non-normal but equal populations (Bernoulli series). Moreover it can be proved that the expectation of their quotient equals one. In case of Lexis and Poisson series, certain inequalities are investigated. These different criteria are completed by the computation of the variances. Also for some other symmetrical test functions general populations may be assumed. The Lexis as well as the Poisson series may then be characterized by equalities. Finally, omitting the restriction to independent chance variables, different kinds of mutual dependence are studied. These investigations which lead to new inequalities among the expectations seem to be related to Fisher's intraclass correlation and to supplement this idea. (Received August 11, 1941.)

485. E. J. Gumbel: *Simple tests for given statistical hypotheses.*

The probability integral transformation reduces the comparison of an observed distribution with a theoretical continuous distribution to the comparison of n observed points (n being the number of observations) with an equidistribution over the interval 0, 1. The well known shortcomings of the χ^2 test, the dependency on the classification and the necessity to combine the contents of the extreme cells can thus be avoided. The comparison of the observed and the theoretical means and measures of dispersion is very simple. The theoretical cumulative frequency of the mth point is the straight line $\bar{y}=m:(n+1)$ where $\bar{y}$, the mean position of the mth point, is plotted as the abscissa and $m(=1, 2, \cdots, n)$, the serial number, as the ordinate. The observed points will be scattered about this straight line. To judge the significance of these deviations, two control curves are constructed: To each point on the straight line correspond points $(\bar{y}-\sigma, m)$ and $(\bar{y}+\sigma, m)$, where σ the standard deviation of the mth point is given by $\sigma(n+2)^{1/2}=(\bar{y}(1-\bar{y}))^{1/2}$. (Received September 27, 1941.)

486. Abraham Wald: *Large sample distribution of the likelihood ratio.*

The large sample distribution of the likelihood ratio has been derived by S. S. Wilks (Annals of Mathematical Statistics, vol. 9 (1938)) in the case of a linear composite hypothesis and under the assumption that the hypothesis to be tested is true. Here a general composite hypothesis is considered and the distribution in question is derived also in case that the hypothesis to be tested is not true. Let $f(x_1, \cdots, x_p, \theta_1, \cdots, \theta_k)$ be the joint probability density function of the variates $x_1, \cdots, x_p$ involving k unknown parameters $\theta_1, \cdots, \theta_k$. Denote by H_ω the hypothesis that the true parameter point $\theta=(\theta_1, \cdots, \theta_k)$ satisfies the equations $\xi_1(\theta)=\cdots=\xi_r(\theta)=0$ $(r\leqq k)$. Denote by λ_n the likelihood ratio statistic for testing H_ω on the basis of n independent observations on $x_1, \cdots, x_p$. It is shown that under certain restrictions on $f(x_1, \cdots, x_p, \theta)$, $\xi_1(\theta), \cdots, \xi_r(\theta)$ one obtains $\lim_{n=\infty}\{P(-2\log\lambda_n<t|\theta)-P[Q_n(\theta)<t|\theta]\}=0$ uniformly in t and θ, where $Q_n(\theta)$ denotes a certain quadratic

form of r normally distributed variates, and $P(z<t|\theta)$ denotes the probability that z is less than t calculated under the assumption that θ is true. (Received August 11, 1941.)

Topology

487. Samuel Eilenberg and Saunders MacLane: *Infinite cycles and homologies.*

An abelian group E is an extension of G by H if G is a subgroup of E and H is the corresponding factor group. With a suitable definition of equivalence and addition the extensions of G by H form a group. The authors consider the qth homology group of an abstract complex with coefficients in a group G obtained using infinite cycles and purely formal boundaries. A direct sum decomposition of this group is given, the first member of which is isomorphic with the group of all homorphic mappings of the qth integral cohomology group (finite chains and cocycles) into G and the second with the group of all abelian group extensions of G by the $(q+1)$th cohomology group. Various consequences of this result are obtained. (Received September 4, 1941.)

488. R. L. Moore: *On continua with dendratomic subsets.*

It is shown that in order that a compact continuum should have dendratomic subsets it is necessary and sufficient that it should not be a web. It follows that every compact continuum which is not a triod has such subsets. In particular every compact irreducible continuum between two points has them. (Received September 5, 1941.)

489. N. E. Steenrod: *Topological methods for the construction of tensor functions.*

The space M' of point tensors (fixed order and weight) over a differentiable manifold M of class r is shown to be a differentiable manifold of class $r-1$ forming a fibre bundle over M in the sense of Whitney. If M'' is a submanifold of M' which is still a fibre bundle over M with fibres F'', methods are given for attempting to ascertain if a tensor function of class $r-1$ exists with values in M''. An approximation theorem reduces the problem to finding a function which is merely continuous. If h is the smallest integer such that the homotopy group $\pi_h(F'')\neq 0$, a function of the required kind may be defined over the h-dimensional skeleton M^h of M. Any such determines a cocycle c^{h+1} in M with coefficients in the groups $\pi_h(F'')$. It is necessary to use here a homology theory based on local coefficient groups connected by local isomorphisms. In order that a tensor of the required kind exist over M^{h+1}, it is necessary and sufficient that $c^{h+1}\backsim 0$. The existence and properties of the characteristic cohomology class are also proved for the more general type of fibre space introduced by Hurewicz and the author. (Received August 5, 1941.)

490. Henry Wallman: *Dimension for general spaces.*

For separable metric spaces there is a body of theorems of deep geometric appeal centering around the concept of dimension. In line with recent trends it seems natural to inquire whether one can extend the domain of application of dimension to spaces of more general character. The three common dimension functions are the Menger-Urysohn dimension d_1, the Čech dimension-in-the-large d_2 and the Lebesgue covering dimension d_3. It turns out that although there is not the slightest difficulty in applying these dimension-functions to spaces of the most general sort, one can hardly