BOOK REVIEWS

Stencils for Solving $x^2 \equiv a$ *(mod m)*. By Raphael M. Robinson. Berkeley and Los Angeles, University of California Press, 1940. 14 pp., 272 cards.

These stencils enable one to find directly all the solutions of the given congruence for $m \leqq 3000$. The range of m may be extended by additional computation.

In brief, the theory back of the construction is as follows: to solve the congruence we must find a y such that $a+my$ is a quadratic residue of all numbers E; that is, if m is prime to E, $a/m+y$ and m must have the same quadratic character (mod E) when the quadratic character for $E=16$ is the value of m (mod 8). It is not hard to see that we need consider only $y \leqq m/4$. Each card contains the numbers from 1 to 750, it is headed with a value of E (9, 16 or one of the seven primes from 5 to 23 inclusive), a value of a/m (mod E) and a quadratic character of m (mod E). Holes are punched in the values of y for which $a/m+y$ has the same quadratic character as m (mod E). Then to solve $x^2 \equiv a$ (mod m) one selects the cards appropriate to a and m for the nine values of E, stacks them and looks through the holes.

The stencils are very simple in principle and convert drudgery into fun.

BURTON W. JONES

The Consistency of the Axiom of Choice and of the Generalized Continuum-Hypothesis with the Axioms of Set Theory. By Kurt Gödel. (Annals of Mathematics Studies, no. 3.) Princeton, University Press; London, Humphrey Milford and Oxford University Press, 1940. 66 pp. $1.25.

Abstract. In this study it is proved that the axiom of choice and Cantor's generalized continuum-hypothesis are consistent with the other axioms of set theory if these latter axioms are themselves consistent. The system Σ of axioms here adopted for set theory is essentially that of P. Bernays (Journal of Symbolic Logic, vol. 2, p. 65).

In Σ the primitive notions are: *class*, *set*, and the relation ϵ between class and class, class and set, set and class, or set and set. The first two axioms of Σ specify the relation between classes and sets: Axiom A1. Every set is a class. Axiom A2. Every class, which is a member of some class, is a set. As a consequence of this distinction between class and set, there exists a *universal class* V of all sets in the system Σ. The remaining axioms of Σ are of a form conventional in ordinary set theory, and deal with intersection, complement, power set, etc.

Chapter I of this study includes a statement of the axioms of Σ together with a few preliminary notions, such as *ordered pair* and *single-valued class* of ordered pairs. Chapter II is concerned with the following topics: extension of propositional functions, definition and properties of relations and functions, and properties of operations with classes and sets. Chapter III is devoted to an exposition of ordinal numbers (based on the work of J. von Neumann) in which an ordinal α is the class of all ordinals less than α; for example, $0=$ the null set, $1=\{0\}$, $2=\{0, 1\}$, etc. Chapter IV develops the theory of cardinal numbers. In Chapters V and VI a model Δ is constructed for the system Σ on the assumption that Σ is consistent, for the classes and sets of Δ are merely those classes and sets of Σ characterized as *constructible* (the definition of constructibility being based on certain properties of ordinal numbers). Let L denote the class of all constructible sets in the universal class V. In Chapter VII it is proved that, when the axioms of Σ are applied just to the model Δ, V being the class of all sets in Δ and L being the class of sets constructible within Δ, then $V=L$, i.e., every set in Δ is constructible. Thus, if there exists a model for the system Σ, then there also exists a model for the system Σ augmented by the axiom $V=L$, this latter model consisting of the classes and sets constructible within the given model for Σ. The existence of this latter model shows that, if the system Σ is consistent, then the augmented system Σ' is consistent, where Σ' comprises the axioms of Σ together with the axiom $V=L$. In the last chapter it is proved that the axiom of choice and Cantor's generalized continuum hypothesis are deducible as theorems in the system Σ'.

While a number of subjects are discussed in this study, the material actually developed is largely confined to just those lemmas needed to attain the final result.

C. C. TORRANCE

Metric Differential Geometry of Curves and Surfaces. By Ernest Preston Lane. Chicago, University Press, 1940. 216+8 pp. $3.00.

This book is designed as a text for beginning graduate students; it is not intended to be an exhaustive treatise on the subject. The order and content of the material has been used by the author in his classes for several years. It has therefore been thoroughly subjected to the acid tests of the classroom for teachability and for readability. It should on that account be of much service to the classroom teacher and to the student in his independent reading.

Although the material selected is classic, the point of view in pres-