conclusion that the existence of the simultaneous limit always insures the existence of the iterated limits. This conclusion is, in turn, involved in the proof of the theorem on the interchangeability of order of partial differentiation. (Fortunately, this does not invalidate the proof, but it makes desirable further elucidation of some of the steps.) Finally, it is stated, without proof, that the mere existence of f_{xy} and f_{yx} is sufficient to insure their equality; but counterexamples to this statement are known.

If it is desirable to use the intuitive idea of different modes of approach of the independent variable to its limit, the notion could easily be put on firm ground by giving the Heine sequence definition of a limit and stating its equivalence to the Cauchy definition. That this equivalence is not as simple as appears on the surface is well known, for the proof is impossible without the axiom of choice.

The chapters on infinite series, power series, and Fourier series appealed to the reviewer as very well done. The chapter on Fourier series contains among other topics a simple account of the Dirichlet conditions for convergence, Parseval's equation, the complex form of Fourier series, the asymptotic behavior of the coefficients, and the differentiation and integration of Fourier series.

P. W. KETCHUM

Geometria Descrittiva: Lezioni Redatte per Uso Degli Studenti. By Enea Bortolotti. Padua, Cedam, 1939. 715 pp., 500 figs. (Mimeographed.)

The term descriptive geometry has a wider meaning in Europe than it has with us; this is especially true in Italy where the study of geometry is an important part in the curriculum of all students of mathematics. Moreover, the school programs include work in determinants and matrices, projective and analytic geometry, and the elements of the calculus. Students of mathematics in the universities analogous to our undergraduates, devote all their time to the subject, hence are taking three courses simultaneously. By the time they reach descriptive geometry they will have had sufficient training to allow a teacher or an author to assume an acquaintance with many fundamental concepts.

Textbooks are ordinarily not employed at all; books written on a specific subject are for supplementary reading, usually voluntary, and not controlled. The students are thrown on their own at an early stage, and many of them show a decided precosity. The books are written for the better students who really want to learn about the subject treated.

The book under review is divided into four parts; most of the second can be read without the first, but the later parts make use of both. The first is devoted to descriptive geometry in the narrower sense and covers 243 pages. It starts with h, v orthogonal projections of figures bounded by straight lines and planes. Intuition is used freely, but each theorem is proved rigorously, including those on shades and shadows. Very few hints are given, but occasionally the reader is advised to go through the details of that particular construction. The second chapter of this part treats of perspectivity, both central and parallel, orthogonal and oblique. This is a helpful addition. So many books omit it in the elementary presentations, while more advanced treatments (sometimes in the same book) presuppose it. Now follows anaxometry. The discussion is so condensed that the reviewer frequently wondered whether a reader not having additional help could always follow the reasoning. The theorem of Pohlke is proved and frequently used. A commendable feature is the comparison of the respective advantages of the three methods developed in these three chapters. The closing chapter of this part gives a brief summary of a method used in topography, especially in mapping curves of equal level.

Thus far the subject matter has been almost entirely self-contained. All theorems used have been proved as needed. The second part, curves and surfaces, covers nearly half the volume; it constantly refers to proofs of theorems given in other Italian texts rather than spend time and space on them in the present work. It begins with plane curves, and provides an extensive treatment of plane differential geometry, including a discussion of singularities. It is carefully done, but the processes, applied to a complicated case would become so cumbersome as to be useless. This chapter is followed by a briefer one on space curves and developables. Thus far, no use has been made in this chapter of any of the earlier ones.

Now follows an extensive but condensed discussion of surfaces, from the standpoint of differential geometry; it includes normals, tangent planes, asymptotic lines, geodesics, lines of curvature, conjugate systems, and so on, with special emphasis on ruled surfaces and surfaces of revolution. Many names are used, as discoverers of theorems met, but there is no citation of sources. The mapping of one surface on another is discussed and applied in detail to various kinds of map drawing, in which one property or another is preserved, such as lengths, angles, areas, and so on. Here repeated use is made of the results of the preceding chapters.

A short chapter on graphical calculus and nomography is followed by one called "Complementi," in which details of the theory of linear transformations in one, two, and three dimensions, as well as correlations in two and three dimensions are developed, including the fixed points and a considerable number of particular cases. This is followed by a more detailed study of algebraic curves and surfaces. In the plane, polarity is introduced and used to derive Plücker's numbers, with application to cubic curves. The configuration of the points of inflexion, the constant cross ratio of the tangents from a point on the curve, nodal and cuspidal forms are provided for. Space cubic curves, and both kinds of space quartics are treated briefly.

Even with the omission of many proofs, the development is so rapid and condensed that a reader must be alert and patient to get all the points of the argument. On the other hand, one who has mastered this volume will be in possession of a large part of the knowledge which goes under the generic name of geometry.

VIRGIL SNYDER

Some Integrals, Differential Equations and Series Related to the Modified Bessel Function of the First Kind. By A. H. Heatley. (University of Toronto Studies, Mathematical Series, no. 7.) Toronto, University Press, 1939. 32 pp.

Let $I_n(x)$ denote Bessel's function, and let $T(m, n)$ denote the integral over $0<t<\infty$ of the function $I_n(2at)t^{m-n} \exp(-p^2t^2)$. Differential equations are given for $T(m, n)$, for $T(m, n) \exp(-a^2/p^2)$, and for $T(m, n) \exp(-a^2/2p^2)$; and these lead to explicit formulas for $T(2n+1, n)$ and for $T(n, n)$. Then recursion formulas lead to evaluation of $T(m, n)$ for other pairs of values of m and n.

Power series expansions of $T(m, n)$ and $T(m, n) \exp(-a^2/p^2)$ are obtained. These results of Part I (pp. 7–21) are used in Part II (pp. 21–30) to evaluate an integral used by the author (Physical Reviews, vol. 52 (1937), pp. 235–238) in the Langmuir collector theory.

Part III (pp. 30–32) deals briefly with integrals of $x^{n+m}e^{x}I_n(x)$ and $x^{n+m}e^{-x}I_n(x)$.

R. P. AGNEW

Theory of Probability. By Harold Jeffreys. Oxford, Clarendon Press, 1939. 7+380 pp.

This book of Jeffreys is an outstanding addition to the relatively few substantial treatises of probability in English. It is in line with the author's *Statistical Inference.* At the start, it resembles Keynes' *A Treatise on Probability* in its subjective or psychical approach to probability. But it carries the implications of this approach to a great variety of problems arising in the physical sciences, in biology and in