are outlined. One emphasizes motions; the other takes distance, angle, and congruence as the fundamental notions. Finally, there is a critical discussion of Euclid's *Elements*.

A reading of this book makes one aware of recent progress in mathematics. To Klein, writing in 1908, geometry was coordinate geometry. He makes only bare references to point sets as objects of geometric study and to Hilbert space. He gives only a brief discussion of homeomorphisms and even then only in euclidean 3-space. Geometric manifolds with the complete generality of our modern abstract spaces were then unknown. Metric spaces (in the sense of Hausdorff) had been introduced only in 1906, and topological spaces were not yet clearly defined. The types of geometry that now occupy the center of the stage were only beginning to emerge. Nevertheless, they too fit into the framework erected by Klein.

G. B. PRICE

Advanced Calculus. By Ivan S. Sokolnikoff. New York, McGraw-Hill, 1939. 10+446 pp.

In most American colleges there is an abrupt discontinuity in mathematical education at the end of a first course in calculus. The drop in enrollment due to the completion of required mathematics by hordes of engineering students makes a sudden change in the needs and capabilities of the students. This discontinuity is perhaps more apparent to outsiders than to mathematicians themselves. It is the reason for the widely held popular belief that all useful mathematics ends with the calculus, that Newton made the last discoveries in mathematics, and that mathematics is dead except for a few frills indulged in by a handful of eccentrics. Even the name "Advanced Calculus" encourages this misconception—as though it were just calculus over again, with a few added tricks to be sure, but no new ideas.

It is in a second course in calculus that the number of students with primary interest in mathematics is for the first time an appreciable percentage of the total. The problem of the relative emphasis on method, that is, proofs, versus utility, to meet the respective needs of mathematics majors and students in the applied field, is as perplexing as any problem in the whole of mathematical education. Some sort of compromise is usually necessary. It would be very useful to the teacher if textbooks of good craftmanship were available with a variety of methods of effecting such a compromise. Unfortunately, a number of the existing books in this field are (or at least have the bulk of) treatises, and are not entirely suitable for textbook use.

The reviewer, therefore, welcomes the contribution of a book, entirely suitable for classroom use, from a man who has personal inclinations toward applied mathematics and who has already made a definite contribution in this field with a textbook leaning toward applications, but who now presents a treatment with considerable emphasis on real variable theory.

After an introductory chapter on the concepts of limit and continuity, there are chapters on the notion of derivative, functions of several variables, definite integral, multiple and line integrals, infinite series, power series, improper integrals, and Fourier series. A final chapter in implicit functions is more in the nature of an appendix; the treatment of partial differentiation of implicit functions in an early chapter is only claimed to be heuristic.

By limiting the scope of the book to the central problems of the calculus, the author is able to include a reasonable amount of formal material and at the same time give the student an introduction to "ϵ, δ" arguments. An attempt is made to reduce these arguments to terms understandable by the average student by such devices as prefacing the actual proofs with heuritistic treatments, and by the use of geometric counterexamples. Numerous exercises are also included.

The first chapter is evidently not intended to be taken too seriously; otherwise such a variety of new abstract ideas as Dedekind cuts, ϵ, δ definition of limit, Cauchy criterion for convergence, limit point, inferior and superior limits, uniform continuity, and so on, would overwhelm the average student just out of sophomore calculus. For such students the teacher might want to start with Chapter II and gradually refer back to the first chapter. The proofs in this opening chapter are incomplete, and many of the theorems are merely stated. For example, the proof of the Cauchy criterion for convergence is carried just to the point where one would ordinarily construct a cut; then the author says the remainder of the proof is too difficult and will be omitted. Such a procedure could have been avoided by merely postulating the existence of a least upper bound of any bounded set of real numbers.

Those who believe in being absolutely meticulous of statement in a book of this level will have a number of quarrels with the author. His definition of Riemann integral is inaccurate. Also, although he gives an ϵ, δ definition of limit of a function, he encourages the student to think in terms of the independent variable x approaching x_0 "by any conceivable sequence of steps," without ever making this idea precise. An extension of this vague idea to limits of functions of two independent variables has led the author to the erroneous conclusion that the existence of the simultaneous limit always insures the existence of the iterated limits. This conclusion is, in turn, involved in the proof of the theorem on the interchangeability of order of partial differentiation. (Fortunately, this does not invalidate the proof, but it makes desirable further elucidation of some of the steps.) Finally, it is stated, without proof, that the mere existence of f_{xy} and f_{yx} is sufficient to insure their equality; but counterexamples to this statement are known.

If it is desirable to use the intuitive idea of different modes of approach of the independent variable to its limit, the notion could easily be put on firm ground by giving the Heine sequence definition of a limit and stating its equivalence to the Cauchy definition. That this equivalence is not as simple as appears on the surface is well known, for the proof is impossible without the axiom of choice.

The chapters on infinite series, power series, and Fourier series appealed to the reviewer as very well done. The chapter on Fourier series contains among other topics a simple account of the Dirichlet conditions for convergence, Parseval's equation, the complex form of Fourier series, the asymptotic behavior of the coefficients, and the differentiation and integration of Fourier series.

P. W. KETCHUM

Geometria Descrittiva: Lezioni Redatte per Uso Degli Studenti. By Enea Bortolotti. Padua, Cedam, 1939. 715 pp., 500 figs. (Mimeographed.)

The term descriptive geometry has a wider meaning in Europe than it has with us; this is especially true in Italy where the study of geometry is an important part in the curriculum of all students of mathematics. Moreover, the school programs include work in determinants and matrices, projective and analytic geometry, and the elements of the calculus. Students of mathematics in the universities analogous to our undergraduates, devote all their time to the subject, hence are taking three courses simultaneously. By the time they reach descriptive geometry they will have had sufficient training to allow a teacher or an author to assume an acquaintance with many fundamental concepts.

Textbooks are ordinarily not employed at all; books written on a specific subject are for supplementary reading, usually voluntary, and not controlled. The students are thrown on their own at an early stage, and many of them show a decided precosity. The books are written for the better students who really want to learn about the subject treated.

The book under review is divided into four parts; most of the