ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

1. C. R. Adams and A. P. Morse: *Continuous additive functionals on the space (BV) and certain subspaces.*

In a recent paper (Transactions of this Society, vol. 40 (1936), pp. 421–438) Adams introduced for (BV), the space of functions $x(t)$ of bounded variation on $0 \leqq t \leqq 1$, the metric $(x, y) = \int_0^1 |x(t)-y(t)| dt + |T_0^1(x) - T_0^1(y)|$, where in general $T_0^1(z)$ stands for the total variation of $z(t)$ on $0 \leqq t \leqq 1$. Thus metrised, (BV) is not a Banach space; but it is complete, separable, and boundedly compact. On it there exist additive homogeneous functionals which are continuous at each point of a dense set and discontinuous at each point of its dense complement; and there exist additive functionals which are continuous without being uniformly so. The general form of the additive and uniformly continuous functional was found to be $\int_0^1 x(t)\alpha(t)dt$, where $\alpha(t)$ is essentially bounded; conversely, every such integral is such a functional. It is now shown that the general form of the additive and continuous functional on (BV) is (1) $\int_0^1 x(t)dg(t)$ with $g(t)$ continuous; that the general form of the additive and continuous functional on the subspace (CBV) of continuous functions of bounded variation is (1) with $g(t)$ bounded and continuous except at a countable set; and that the general form of the additive and continuous functional on the subspace (AC) of absolutely continuous functions is (1) with $g(t)$ bounded and continuous save at a set of measure zero. In each case, conversely, every such integral is a functional of the kind specified. (Received November 3, 1939.)

2. R. P. Agnew: *On rearrangements of series.*

Let E be the space of permutations $x \equiv \{x_1, x_2, \cdots\}$ of the positive integers in which the metric is that of Fréchet. Let $\sum c(n)$ be a convergent series of real terms for which $\sum |c(n)| = \infty$. It is shown that, for each $x \varepsilon E$ except those of a set of the first category, the inferior and superior limits of the partial sums of the series $\sum c(x_n)$ are respectively $-\infty$ and $+\infty$. In particular, the set of $x \varepsilon E$ for which $\sum c(x_n)$ converges is of the first category. (Received October 23, 1939.)

3. A. A. Albert: *On ordered algebras.*

The note consists of a proof that every ordered algebra is a field. It will appear very soon in this Bulletin. (Received October 16, 1939.)

4. J. V. Atanasoff: *Generalized Taylor expansions.* Preliminary report.

It is easy to see that by employing the idea of functionals one may view a Taylor expansion as a typical biorthogonal development. In this paper a generalization of

Taylor's expansion is exhibited which retains several of the properties of Taylor's expansion including a simple formula for the remainder. It is then shown that all biorthogonal developments may be regarded as special cases of these generalized Taylor expansions. (Received November 4, 1939.)

5. A. A. Aucoin: *Diophantine equations of degree n.*

This paper generalizes the results of an earlier paper by W. V. Parker and the author (National Mathematics Magazine, vol. 13 (1938), pp. 115–117). Solutions are obtained for the equation $f(x_1, \cdots, x_p) = g(y_1, \cdots, y_q)$ where f and g are homogeneous polynomials of degrees n and m respectively, with integral coefficients, and f is such that for a set of integers $a_1, \cdots, a_p$ (not all zero) all the partial derivatives of f of all orders less than $n-1$ vanish for $x_i = a_i$. Two examples of a function f which satisfy the conditions of the theorem are given. One is the determinant $D(x) = |a_{ij}x_{ij}|$ and the other is the product of n linear factors in n unknowns. In the case of the latter function, the equation is also solved by an entirely different method. (Received October 24, 1939.)

6. W. L. Ayres: *A note on the definition of arc-sets.*

In order that a subset A of a Peano space P be an arc-set or A-set either of the following two conditions is necessary and sufficient: (1) Every set separating two points (in the weak *coupure* sense) of A in A separates them in P; (2) A is connected, $\bar{A} \cdot \bar{C}$ is a single point for each component C of $P-A$, and this point y belongs to A if $\bar{A}$ contains two continua S and T such that $S \cdot T = y$. (Received November 3, 1939.)

7. E. T. Bell: *Transformed multiplicative diophantine equations.*

The equations discussed are polynomials of any degree in any number of indeterminates, with integer coefficients, equated to zero, transformable rationally (but not in general birationally) into pure multiplicative equations. *All* integer solutions are found, and a general method of solution, with numerous examples, is given. An extension to systems of diophantine equations is sketched; a simple example is the system $xy+yz+zx = uw+wy+yu$, $x^2+z^2 = u^2+w^2$ in six indeterminates. (Received November 6, 1939.)

8. P. O. Bell: *Projective analogues of the congruence of normals.*

Both the normal and the projective normal at a general point of a non-ruled surface in ordinary space may be defined as cusp axes of certain integral invariants. Moreover, the projective normal, like the normal, is intrinsically connected with the surface, and the curves which correspond to the developables of the projective normal congruence resemble the lines of curvature by forming also a conjugate net. The author shows that these properties, which have been regarded as peculiarly characteristic of a unique substitute for the normal, are shared likewise by every member of an infinite class of such congruences. A method is outlined for the geometric determination of a general congruence of this class and the extremals of the associated integral invariant are interpreted geometrically. Finally certain significant special cases are considered and their associated canonical forms are introduced. (Received October 28, 1939.)

9. Garrett Birkhoff: *An ergodic theorem for arbitrary semi-groups.*

First, convergence with respect to any transitive ordering relation is defined. This specializes to Moore-Smith convergence, and allows one to speak of convergence for the means of the transforms of any element, under any group or semi-group of transformations. This new definition thus enables one to formulate ergodic theorems for arbitrary semi-groups of linear operators, a problem suggested to the author by L. Alaoglu. The main result of the present paper is a universal ergodic theorem for semi-groups of linear isometries and contractions. Surprisingly, the theorem does not apply if the hypothesis of modulus unity is weakened to modulus two. (Received October 21, 1939.)

10. Salomon Bochner: *Finitely additive integral.*

Generalizing a construction recently given for point functions (Annals of Mathematics, (2), vol. 40 (1939), pp. 769–799) we show that a finitely additive positive functional on a partially ordered vector space without any limit requirement can be interpreted as an integral over a suitable Boolean algebra. (Received November 7, 1939.)

11. H. F. Bohnenblust: *Axiomatic characterization of l_p spaces.*

The notion of l_p spaces can be extended to include similar spaces defined over any Boolean algebra with a measure function (cf. H. Freudenthal, Akademie van Wetenschappen, Amsterdam, vol. 39 (1936), p. 641). Similarly a variety of spaces can be defined corresponding to the value p equals infinity. The spaces thus obtained, which include the classical l_p and L_p spaces, are characterized axiomatically as partially ordered Banach spaces in which the norm satisfies the following condition: If a, b and c, d are two pairs of elements such that the inf $(|a|, |b|)$ and the inf $(|c|, |d|)$ are equal to zero and the norms of a and c and the norms of b and d are equal, then the norms of $a+b$ and of $c+d$ are equal. (Received November 4, 1939.)

12. D. G. Bourgin: *Closure of products of functions.*

Let E_s and E_t be sets in n and m dimensional euclidean spaces, respectively. The points of these sets are denoted by s and t. The symbol $T(E)$ denotes a certain class of function spaces in which the domain of the function is E. The sense of the results obtained is: If $[\phi_\mu(s)\psi_\nu(t)]$, $(\mu, \nu=0, 1, \cdots)$, is a sequence of functions in $T(E_s\times E_t)$, then a necessary and sufficient condition for closure is that the sequences $[\phi_\mu(s)]$, $[\psi_\nu(t)]$ have the closure property in $T(E_s)$ and $T(E_t)$ respectively. The complex Hilbert space, the L_p space $(p\geqq 1)$ and the C space are included in $T(E)$. (Received November 3, 1939.)

13. D. G. Bourgin and R. J. Duffin: *The Dirichlet problem for the vibrating string.*

The string equation $y_{xx}-y_{tt}=0$ is investigated subject to Dirichlet conditions on the boundary of a rectangle formed by parallels to the coordinate axes. Considerations of existence and uniqueness of solutions pivot on the character of the parameter γ, where $\gamma=\tau/vS$. Here S is the string length, v is the velocity of wave propagation and τ is the time interval. For instance it is sufficient for the existence of a solution of class C^2 that the data, besides satisfying certain corner conditions, be of class C^{4+k} where γ is an algebraic number of degree k. This condition is not necessary but some such restriction must be introduced for sufficiency assertions. The uniqueness proof is based on a novel use of Fourier transforms and the method has wide application. (Received November 3, 1939.)

14. A. L. Foster: *The structure of regularly ordered natural systems with 3 primes.*

The principal result of this paper is the determination of the structure of all regularly ordered natural systems with 3 primes. The method is an extension of that previously given for 2 primes (Proceedings of the National Academy of Sciences, vol. 24, no. 4). A natural system is a system with single composition $(N, \circ)$ which is closed, associative, commutative, denumerably infinite, and with unique prime (irreducible) factorization. $(N, \circ)$ is regularly ordered if its elements are arranged in a sequence such that $\sigma < \tau \rightarrow \rho_0\sigma < \rho_0\tau$ ($<$ means precedes). (Received November 4, 1939.)

15. O. H. Hamilton: *A fixed point theorem for a cell in a Hilbert space.*

It is shown in this paper that if M is a closed cell in a Hilbert space, then every continuous transformation of M into a subset of itself leaves some point of M invariant. The method employed in this proof can also be used to give a new and simple proof of the well known fixed point theorems for cells in n-dimensional space. (Received November 2, 1939.)

16. A. E. Heins: *The solution of the discrete wave equation.*

Solutions of the discrete wave equation $f(x+1, t)+f(x-1, t)-f(x, t+1)-f(x, t-1)=0$ are considered here for values of $t>0$ (x, t not integers). Symmetric and nonsymmetric, finite and infinite boundary conditions are treated. One need only prescribe one initial condition here, that is, $f(x, t)$ when $[t]=0$, since the derived solution with the aid of the given difference satisfies the second initial condition automatically, namely $f(x, t)$, when $[t]=1$. (Received November 9, 1939.)

17. Einar Hille: *Contributions to the theory of Hermitian series. II. The representation problem.*

A necessary and sufficient condition in order that the Fourier-Hermite series of an analytic function $f(z)$ shall exist and converge to the function in the strip $-\tau<y<\tau$ is that $f(z)$ be holomorphic in the strip and that to every β, $(0<\beta<\tau)$, there shall exist a finite $M(\beta)$ such that $|f(x+iy)| \leqq M(\beta) \exp\left[-(\beta^2-y^2)^{1/2}|x|\right]$ for $-\beta \leqq y \leqq \beta$. In particular, the Hermitian series of an entire function of exponential type can never converge outside of the real axis. (Received October 16, 1939.)

18. P. G. Hoel: *The errors involved in evaluating correlation determinants.*

In evaluating a correlation determinant by the method of Chio, successive reductions introduce errors of rounding which become increasingly important as the order of the determinant increases or the value of the determinant decreases. Through the use of operational methods an upper bound is obtained for the magnitude of these errors. By considering the first order terms in more detail, probability bounds are also obtained for these errors. (Received November 1, 1939.)

19. Mark Kac: *On a problem concerning probability and its connection with the theory of diffusion.*

The paper gives a probabilistic approach to the problem of diffusion. It is shown that the classical solution of the differential equation of diffusion is an asymptotic formula for the statistical problem under consideration. (Received October 23, 1939.)

20. E. P. Lane: *A theorem on surfaces.*

The purpose of this note is to prove the following theorem: If the asymptotic curves on an analytic non-ruled surface S in ordinary space belong to linear complexes, then the asymptotic curves of each family on S are projectively equivalent, not only to each other, but also to all the non-rectilinear asymptotic curves on all the asymptotic ruled surfaces of S which are composed of the tangents of the asymptotic curves of the other family on S, constructed at the points of the various asymptotic curves of the first family on S. (Received October 7, 1939.)

21. J. P. LaSalle: *Linear functions and functionals in linear topological spaces.* Preliminary report.

In this paper a number of Banach's theorems on linear operations are extended (a) to convex linear topological spaces by use of the pseudo metric of von Neumann, and (b) by use of the pseudo norm of Hyers to linear topological spaces which satisfy an additional postulate (the postulate may be weaker than that of convexity). Some theorems other than extensions of Banach's also appear. (Received November 4, 1939.)

22. Rhoda Manning: *On the derivatives of the sections of bounded power series.*

Let $f(z)$ be a power series with sections $s_n(z)$, such that $|f(z)| \leqq 1$ in $|z| < 1$, and $f(0) = 0$. I. Schur and G. Szegö have shown (Sitzungsberichte der preussischen Akademie, 1925, p. 560) that for odd n the radius r_n of the largest circle $|z| \leqq r_n$ in which $|s'_{n+1}(z)| \leqq 1$ is a root of the equation $1-2r-r^2+(-1)^n\{(2n+4)r^{n+1}+(2n+2)r^{n+2}\} = 0$. The author proves that this result holds for all n provided $n \geqq 11$. As a consequence, $r_n \downarrow \rho = 2^{1/2}-1$, n even, $n \geqq 12$. It is also shown that the r_n, $(n = 2, 4, \cdots, 10)$, are algebraic numbers, connected by the following order relations (for simplicity n is written for r_n): $1<2<3<4<5<6<7<9<8<11<13< \cdots <\rho< \cdots <14<12<10$. Hence for even n, $n \geqq 10$, the sections $s'_{n+1}(z)$ remain bounded "longer" than $f'(z)$ itself. For large n, asymptotic expressions for r_n are given. (Received October 30, 1939.)

23. W. T. Martin: *Analytic functions and multiple Fourier integrals.*

The paper considers the class of entire functions $f(z_1, \cdots, z_n)$ satisfying relations of the form $\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} |f(x_1+iy_1, \cdots, x_n+iy_n)|^2 dx_1 \cdots dx_n < Ae^{a(|y_1|+ \cdots +|y_n|)}$ for all finite values of $y_1, \cdots, y_n$. It is proved that this class is identical with the class of functions having Fourier transforms which vanish outside a certain bounded region and that growth-function of f, namely $h(\lambda) = \lim_{\rho\to\infty}(1/2\rho) \log \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} |f(x_1 + i\lambda_1\rho, \cdots, x_n+i\lambda_n\rho)|^2 dx_1 \cdots dx_n$, is equal to the supporting function $k(\lambda)$ of the convex body K which is the intersection of all convex bodies in whose exterior the Fourier transform of f vanishes. These results are related to those of Plancherel and Pólya (Commentarii Mathematici Helvitici, vol. 9 (1936–1937), pp. 224–248; vol. 10 (1937–1938), pp. 110–163). Analogous results are also obtained for functions in an octant $I_m[z_k] > 0$, $(k = 1, \cdots, n)$. (Received November 2, 1939.)

24. A. B. Mewborn: *Abstract local geometry of paths.* Preliminary report.

The local geometry of a space of paths in a topological space with postulated Banach coordinates is considered from the standpoint of the representations of these paths in distinguished coordinate systems analogous to normal coordinates. It is shown that allowable transformations of coordinates can be defined in a space with this structure, and yield a linear connection form. It is proposed to study the affine, projective and descriptive properties of this space by means of this connection. (Received November 4, 1939.)

25. A. D. Michal: *Differentials in abstract abelian topological groups.*

Let G_1 and G_2 be abstract abelian groups (written additively) with continuous operations in a Hausdorff topology. A differential is defined for functions on a Hausdorff neighborhood in G_1 to G_2. The fundamental theorems on group sum and difference, and composition of functions are proved. A differential is additive and continuous in the increment and is a "first order approximation" in a prescribed manner to the group increment of the function. A differential is proved unique for all values of the increment in an existing abelian topological subgroup of G_1. The uniqueness of a differential for all values of the increment in G_1 is proved under an additional postulate on the topological groups. This additional postulate is automatically satisfied whenever G_1 and G_2 are linear topological spaces. (Received November 4, 1939.)

26. F. J. Murray: *Nullifying functions.*

A measurable function, defined on the unit interval, is termed nullifying, if it takes a set of measure one into a set of measure zero. In this note a function $f(x)$ is given with the property that $f(x)+\rho x$ is nullifying for every value of the parameter ρ. (Received November 4, 1939.)

27. W. H. Myers: *Finite linear groups with abelian subgroups of variety 3.*

H. F. Blichfeldt (*Finite Collineation Groups*, University of Chicago Press, 1917, p. 96) proves that no finite group of linear transformations which is primitive can contain a transformation (other than a similarity transformation) which in canonical form has its multipliers when located on the unit circle lying not more than 60° on either side of some one of them; and (page 101) no primitive linear group can contain an abelian subgroup of variety 3 and order $k\phi$ where $k \geqq 30$. In this paper, by extending the first-mentioned theorem somewhat, it is proved that if G is a finite group of linear transformations in n variables which contains an abelian subgroup K of variety 3 and order $k\phi$, where $k>16$ and $k\neq 20$ or 25, then G cannot be a primitive group. (Received November 2, 1939.)

28. G. B. Price: *Convex operators.*

The process of forming the convex extension of a given set may be thought of as a transformation which carries the set into a second set, its convex extension. This transformation is accomplished by means of a certain operator, whose construction and properties are known, and which may be called the ordinary convex operator. This paper describes, in the first place, the construction of other operators with properties similar to those of the convex operator. The ordinary convex operator can be applied to sets in a vector space. This paper introduces not only new convex operators for vector spaces, but also convex operators for a wide variety of more general spaces. In the second place, this study investigates the properties of the class of convex operators. Certain operations can be performed on members of the class, and there are numerous interesting subclasses. Finally, the paper examines the properties of sets which are obtained by transforming a given set by one of the convex operators. A later paper will develop the applications of these convex operators in the theory of integration. (Received October 31, 1939.)

29. E. J. Purcell: *A multiple null-correspondence and a space Cremona involution of order* $2n-1$.

Consider a curve C of order n having $(n-1)$ points on a straight line d, and a curve C' of order m having $(m-1)$ points on a second line d' (m, n any integers). A general point P determines a ray r intersecting C once and d once. P also determines a ray r' intersecting C' once and d' once. Rays r and r' determine a plane which is the null-plane of P in a $(1, mn, m+n)$ correspondence. This paper discusses this correspondence and also a Cremona involution of order $2n-1$ which arises when the curves are in special position. This paper will appear soon in this Bulletin. (Received October 31, 1939.)

30. Tibor Radó and P. V. Reichelderfer: *On cyclic transitivity.*

This paper is a continuation and extension of a previous paper by T. Radó (see abstract 45-5-223). Let $\mathfrak{R}$ be a binary relation defined in a set 1 of elements; write $a_1\mathfrak{R}a_2$ to express the fact that the elements a_1 and a_2 of 1 are in the $\mathfrak{R}$-relation. The following concept is fundamental. The binary relation $\mathfrak{R}$ is said to be *cyclicly transitive* if, for every finite cyclicly ordered set of distinct elements $a_1, a_2, \cdots, a_n$ of 1 satisfying $a_1\mathfrak{R}a_2\mathfrak{R}\cdots\mathfrak{R}a_n\mathfrak{R}a_1$, it is true that $a_i\mathfrak{R}a_j$ for every choice of the subscripts i and j. In the first part of the paper there is developed an abstract theory which parallels closely the cyclic element theory for Peano spaces originated by G. T. Whyburn. In the second part of the paper this abstract theory is applied to the special case of Peano spaces. (Received October 9, 1939.)

31. W. C. Randels: *On the absolute summability of Fourier series.* III.

It is proved that, if a function $f(x)$ is such that, in the neighborhood of every point x, it is identical with a function whose Fourier series is absolutely summable $|(C, 1)|$, then the Fourier series of $f(x)$ is absolutely summable $|(C, 1)|$. This is analogous to a theorem of Wiener for absolute convergence. (Received October 20, 1939.)

32. G. E. Raynor: *On Serret's integral formula.*

The paper is concerned chiefly with a method leading to the evaluation of various definite integrals arising in gamma function theory. The principal tool employed is a formula due to Alfred Serret which appears in his classic paper, *Note sur quelques formules de calcul intégral*, Journal de Mathématiques Pures et Appliquées, vol. 8 (1843), p. 7. (Received October 11, 1939.)

33. Fred Robertson: *The general differential operator.*

The object of this paper is the unification of theories of general integration and differentiation by the use of a second order partial differential equation. The discussion includes the derivation of the differential equation, the relation of its solutions to generalized integration and differentiation and the development of the concept of the logarithmic operator. The theory is then applied to the solution of an integral equation with a logarithmic kernel. (Received November 2, 1939.)

34. D. T. Sigley: *Sets of conjugate subgroups in direct product groups.*

The distribution of the subgroups of a group G which is the direct product of finite abstract groups is studied, with applications to the problem of the enumeration of the subgroups of G. The particular cases which are considered are those in which the constituents of the direct products are abelian, hamiltonian, or non-abelian groups whose orders are the product of two distinct prime numbers. The problem of the construction of an abstract group in which the number of sets of conjugate subgroups is equal to the number of units in the order of the group is initiated. (Received November 3, 1939.)

35. E. R. Smith: *Generating functions for certain confluent hypergeometric functions.*

Generating functions corresponding to certain rational values of the subscripts of the Whittaker functions $W_{k,m}$, where $k-m-1/2$ is not a negative integer, are found by elementary methods. The function is first expressed as a curvilinear integral taken over a closed path enclosing the positive real axis. The integral is evaluated for a number of important sets of functions. It is also shown that the generating functions for integral values of k and m satisfy a partial differential equation of the second order. (Received November 9, 1939.)

36. R. H. Tripp: *Stresses in an orthotropic elastic layer.*

In an elastic isotropic layer of finite depth h supported on a rigid base, the stresses and displacements may be determined from a function $\phi(x, y)$ satisfying $\nabla^4\phi(x, y)=0$ interior to the region and known conditions on the boundary. In an orthotropic finite layer, where the elastic constants are known in orthogonal directions, the stress function satisfies $\nabla^4\phi(x, ky)=0$ where k is a constant. The stresses and displacements throughout the layer are investigated by examining the properties of integrals of the form $\int_0^\infty f(\alpha, y) \cos \alpha x/h \, d\alpha$. (Received November 2, 1939.)

37. E. P. Vance: *On locally non-separating and locally non-alternating transformations.*

The transformation $T(A)=B$ is said to be locally non-separating at a point p provided for any point p and neighborhood U_p of p there exists a neighborhood V_p lying in U_p such that if u and v are any two points of $V_p - T^{-1}(T(p))$, then $u+v$ lies in a connected subset of $U_p - T^{-1}(T(p))$. T will be called locally non-separating on A if it is locally non-separating at all points of A. A similar definition is made for a locally non-alternating transformation. Necessary and sufficient conditions are proved for both transformations. It follows that if T is locally non-separating and for no point x of B does $T^{-1}(x)$ contain an open set, then T is completely non-separating (and therefore non-separating). Other necessary conditions are given for both transformations and the transformations are applied to special spaces. The main result is that if T is locally non-separating on A, then B must be Peanian, regardless of A. (Received October 13, 1939.)

38. Morgan Ward: *Residuated distributive lattices.*

A residuated lattice $\mathfrak{S}$ with ascending chain condition is called a P-lattice if $a \supset b$ in $\mathfrak{S}$ only if there exists an element c such that $ac=b$. $\mathfrak{S}$ is called an S-lattice if every irreducible is primary and if for any two elements a, b the residuals $a:b$ and $b:a$ are coprime. Both S-lattices and P-lattices are known to be distributive, but in a distributive residuated lattice with chain condition, the properties of being a P-lattice or S-lattice are completely independent of one another. In this paper, the decompositions of elements into products and cross-cuts of irreducibles and primaries are determined, and shown to be unique. Both types of lattice may be characterized by simple properties of their primes and primaries. (Received November 6, 1939.)

39. W. F. Whitmore: *Convergence theorems for functions of two complex variables.*

Lindelöf has stated a theorem to the following effect: Let $f(z)$ be an analytic function of one complex variable, defined and bounded in the upper half plane. If $f(z)$ converges to a definite limit for approach to infinity along the negative real axis, then it also converges to this limit inside the angular domain $0<\eta<\arg z<\pi$. A similar result is here established for functions of two complex variables, with the aid of the notions of distinguished surface and functions of extended class introduced by Bergmann (Mathematische Annalen, vol. 109 (1934), pp. 324–348, and Composito Mathematica, vol. 6 (1939), pp. 305–335). A theorem is first obtained for the case of the bicylinder $|z_1|<r$, $|z_2|<1$. Using this as a domain of comparison, the result is extended to a more general class of domains of the form $E[z_1=th(z_2, \lambda), |z_2|<1, 0\leqq t<1, 0\leqq\lambda\leqq 2\pi, h(z_2, 0)=h(z_2, 2\pi)]$, where the function $h(z_2, \lambda)$ satisfies certain conditions of regularity. (Received October 20, 1939.)

40. Max Wyman: *Curl differential equations in abstract spaces.*

Let E be a Banach space, and let $F(x, \xi_1, \xi_2)$, $G(\xi_1, \xi_2)$ be bilinear functions of ξ_1, ξ_2 on E^3 to E, and E^2 to E respectively. Differential equations of the form (1) $\phi(x, \xi_1; \xi_2)-\phi(x, \xi_2; \xi_1)=F(x, \xi_1, \xi_2)$, (2) $\phi(x, \xi_1; \xi_2)-\phi(x, \xi_2; \xi_1)=G(\phi(x, \xi_1), \phi(x, \xi_2))$ arise in abstract transformation group theory, and it is of interest to know under what conditions these equations possess solutions. It is possible to prove the following three theorems. Theorem 1: A necessary and sufficient condition for (1) to have a solution $\phi(x, \xi)$ linear in ξ, and such that $\phi(x, \xi_1; \xi_2; \xi_3)$ exists continuous in x for $\|x\|<K$, is that (a) $F(x, \xi_1, \xi_2)=-F(x, \xi_2, \xi_1)$ for $\|x\|<K$, (b) $F(x, \xi_1, \xi_2; \xi_3)$ exists continuous in x for $\|x\|<K$, (c) $F(x, \xi_1, \xi_2; \xi_3)+F(x, \xi_2, \xi_3; \xi_1)+F(x, \xi_3, \xi_1; \xi_2)=0$ for $\|x\|<K$. Theorem 2: If $G(\xi_1, \xi_2)=-G(\xi_2, \xi_1)$, then equation (2) always possesses a solution $\phi(x, \xi)$, which possesses a continuous second Fréchet differential for all x in E. Theorem 3: If $G(G(\mu, \nu), \lambda)+G(G(\nu, \lambda), \mu)+G(G(\lambda, \mu), \nu)=0$, and if it is required that $\phi(x, x)=g(x; x)$ where $g(x)$ is a given function, then there exists a unique solution of (2) which will meet this condition. (Received November 4, 1939.)

41. Max Wyman: *Projective non-holonomic tensor analysis.*

In considering a projective tensor analysis theory for a Hausdorff space H, with coordinates in a Banach space E, Professor A. D. Michal introduces the normed ring R of linear transformations on E to E (see A. D. Michal, *General projective differential geometry*, Proceedings of the National Academy of Sciences, vol. 23 (1937)). He postulates an inner product in E, and a contraction in R. In exactly the same way one can obtain a projective non-holonomic tensor analysis theory, if in addition to H and E another Banach space E_1 is considered. The defining equations of parallel displacement along the curve $x=x(t)$ are taken to be $dV/dt+K(x, V, dx/dt)=\alpha(t)V$, where $K(x, V, \xi)$ is a non-holonomic linear connection, and $\alpha(t)$ is a numerical scalar field. From this it can easily be shown that the most general change of linear connection which preserves parallelism is of the form $K'(x, V, \xi)=K(x, V, \xi)+\phi(x, \xi)V$ where

$\phi(x, \xi)$ is linear in ξ and scalar valued. It is possible to define what is meant by a projective non-holonomic flat space, and to give necessary and sufficient conditions for a space to be projectively flat. (Received November 4, 1939.)

42. Max Zorn: *Topological studies in the theory of analytic functions. II. Potential functions.* Preliminary report.

This paper points out simple topological properties of potential functions in the plane. In particular, it furnishes a characterization of conjugate pairs. (Received November 6, 1939.)

43. R. P. Agnew: *On Tauberian theorems for double series.*

Let s_{mn} and σ_{mn} denote respectively the sequence of partial sums and the C_1 transform of a double series $\sum u_{mn}$ in which m and n take values $1, 2, 3, \cdots$. The question whether $\sigma_{mn} \rightarrow s$ and the Tauberian condition $mnu_{mn} < K$ imply $s_{mn} \rightarrow s$ was raised and left unanswered by Knopp in his recent paper *Limitierungs-Umkehrsätze für Doppelfolgen*, Mathematische Zeitschrift, vol. 45 (1939), pp. 573–589; p. 581. It is shown that neither the condition $mnu_{mn} < K$ nor any one of several stronger conditions will serve. The situation is the same for many other methods of summability, including the Cesàro methods of all positive orders and the Abel power series method. Questions involving iterated limits of s_{mn} and σ_{mn} are also answered. (Received December 1, 1939.)

44. A. A. Albert: *On p-adic fields and cyclic algebras.*

Cyclic fields over a p-adic field are considered and it is shown that there are only a finite number of such fields of a given degree. The number is explicitly determined in the case where the ramification order is prime to p. It is also shown that in this case every abelian non-cyclic field is the direct product of two cyclic fields, one of which is completely ramified. The results are used to prove that there are only a finite number of algebraic extensions of a given degree over a p-adic field. The paper concludes with a discussion of the question of the existence of a cyclic field of degree n over the rational field such that its composite with the algebraic number centrum K of the division algebra D of degree n over K splits D. (Received November 13, 1939.)

45. H. W. Alexander: *On the edge of regression of a general surface.*

In this paper the analogue for a general surface of the edge of regression of a developable surface is considered, and its properties, both local and in the large, are developed. The edge of regression is a curve E along which the mean curvature is infinite. Any plane normal to E through a point P of E cuts the surface in a curve having an algebraic cusp at P. The curve E is also the envelope of both sets of asymptotic lines. An index j_R is defined for a region R of an orientable surface, and is used to define the index of an umbilic. Finally, the Euler characteristic of a region entirely bounded by the edge of regression is expressed in terms of the indices of the umbilics in the interior. (Received November 24, 1939.)

46. Stefan Bergman: *On the approximation of functions satisfying a linear partial differential equation.*

In this paper a method is developed for the calculation of a sequence of approximations V_n of a function U given by its values $\tilde{U}$ on the boundary $\mathfrak{C}$ of a convex domain $\mathfrak{B}$ and satisfying in $\mathfrak{B}$ the equation $L(U) \equiv U_{z\bar{z}} + a_1 U_z + a_2 U_{\bar{z}} + a_3 U = 0$,

$z=x+iy$, $\bar{z}=x-iy$, $U_z=\partial U/\partial x-i\partial U/\partial y$, $U_{\bar{z}}=\partial U/\partial x+i\partial U/\partial y$, a_k being an analytic function of z, $\bar{z}$ for $k=1, 2, 3$. A method is presented for the calculation of a sequence of particular solutions W_k, $(k=1, 2, \cdots)$, of $L(W_k)=0$, which are complete for certain classes of functions $F(z, \bar{z})$: $L(F)=0$, $z \varepsilon \mathfrak{G}$, $\mathfrak{G}$ being an arbitrary convex domain whose boundary curve satisfies certain conditions. In the case of the first boundary problem $V_n=\sum_{k=1}^{n} A_k^{(n)} W_k$ is calculated with the assumption that $A_k^{(n)}$ are defined by the condition $\int_{\mathfrak{G}}(\tilde{U}-V_n)^2 ds=\min$. Analogous processes can be employed for the solution of certain characteristic value problems and in the theory of equations of hyperbolic type. (Received November 17, 1939.)

47. R. P. Boas: *A completeness theorem.*

Let $G(x) \varepsilon L^2(-\pi, \pi)$. It is shown that the set of functions $\{e^{2nix}, G(x)e^{(2n+1)ix}\}$ is complete in $L^2(-\pi, \pi)$ if and only if $G(x+\pi)+G(x)\neq 0$, $(-\pi<x<0)$, except perhaps on a set of measure zero. By the use of a well known theorem of Paley and Wiener (*Fourier Transforms in the Complex Domain*, American Mathematical Society Colloquium Publications, vol. 19, 1934, p. 13), this result can be transformed into a uniqueness theorem for entire functions of exponential type π which belong to L^2 on the real axis. An interesting special case is that if $f(z)$ is such a function, q is a positive integer, and $f(2n)=f^{(q)}(2n)=0$, $(n=0, \pm 1, \pm 2, \cdots)$, then $f(z)\equiv 0$. (Received November 13, 1939.)

48. R. P. Boas: *Some uniqueness theorems for entire functions.*

Let $\phi(z)$ be regular in the rectangle $|x|<L$, $|y|<\pi$, so that $\phi(z)=\sum_{\nu=0}^{\infty} a_\nu z^\nu$, $|z|<\min(L, \pi)$. If $f(z)$ is an entire function of exponential type, the series $\sum_{\nu=0}^{\infty} a_\nu f^{(\nu)}(z)$ is either convergent or summable by the method of Mittag-Leffler, and may be denoted by $\phi(D)f(z)$, where D stands for d/dz. Consider the class of functions $f(z)$, of exponential type, satisfying $f(iy)=O(e^{k|y|})$, $|y|\to\infty$, $k<\pi$; and $f(x)=O(e^{l|x|})$, $|x|\to\infty$, $l<L$. A necessary and sufficient condition that any $f(z)$ of this class should vanish identically if $f(2n)=\phi(D)f(2n)=0$, $(n=0, \pm 1, \pm 2, \cdots)$, is that $\phi(z+i\pi/2)-\phi(z-i\pi/2)\neq 0$, $|x|<L$, $|y|<\pi$. An interesting special case is that if $f(z)$ belongs to the class, with $l<(\pi/2)\cot \pi(1-1/r)/2$, then $f(z)\equiv 0$ if r is an odd integer and $f(2n)=f^{(r)}(2n)=0$, $(n=0, \pm 1, \pm 2, \cdots)$ (compare the preceding abstract). (Received November 13, 1939.)

49. D. G. Bourgin: *The Dirichlet problem for the damped wave equation.*

This paper treats the Dirichlet data problem for $y_{xx}-y_{tt}+Ky^2=0$. The fundamental contour is a rectangle or parallelogram with sides equally inclined to the characteristic directions. The existence and uniqueness properties of solutions are closely interrelated with results in elementary number theory. The theory is richer than that of the undamped equation because of the presence of two basic parameters, $\alpha=T/m$ and $\beta=(Km)^2$, where $T\pi$, $m\pi$ and K are the two side lengths and the damping coefficient respectively. In particular, there are now various degrees of non-uniqueness of solutions. Generalizations are indicated to other contours and equations. (Received November 21, 1939.)

50. A. T. Brauer: *On the density of the sum of sets of positive integers. II.*

This paper is the continuation of the author's paper in the Annals of Mathematics, (2), vol. 39 (1938), pp. 322–340. Let $\alpha_1 \leqq \alpha_2 \leqq \cdots \leqq \alpha_n$ be the densities of the sets $A_1, A_2, \cdots, A_n$ of positive integers, and $\gamma<1$ the density of the Schnirelmann sum $S=A_1+A_2+\cdots+A_n$. If the famous conjecture that $\gamma\geqq\alpha_1+\alpha_2+\cdots+\alpha_n$ were correct, then one would obtain in particular that $\alpha_1+\alpha_2+\cdots+\alpha_m<m/n$ for all n and m with $n\geqq m$. For $m=1$, this is the theorem of Khintchine and for $n=m=2$ the theorem of Schnirelmann. In this paper it is proved that the inequality is correct in all events, if n and m satisfy the condition $1/(n-1)+1/(n-2)+\cdots+1/(n-m+1)\leqq m/n$. Moreover the inequalities of I. Schur (Sitzungsberichte der preussischen Akademie der Wissenschaften, 1936, pp. 269–297) for $\alpha_1+\alpha_2+\cdots+\alpha_n$ and $\gamma/(\alpha_1+\alpha_2+\cdots+\alpha_m)$ are essentially improved. (Received November 20, 1939.)

51. Richard Brauer and H. S. M. Coxeter: *A generalization of theorems of Schönhardt and Mehmke on polytopes.*

Let G be an irreducible finite group of orthogonal transformations in an n-dimensional space, and let $V_1, V_2, \cdots, V_k$ be the distinct h-dimensional linear subspaces which can be obtained from one such subspace (for instance V_1) by the transformations of G. Then the arithmetic mean of the orthogonal projections of any vector $\mathbf{z}$ on $V_1, V_2, \cdots, V_k$ is equal to $(h/n)\mathbf{z}$. I. Schur's formulas for the coefficients of such a group are used to prove this, as well as a more general theorem in which the projections are not restricted to be orthogonal. (Received November 18, 1939.)

52. J. W. Calkin: *Semi-bounded forms and self-adjoint boundary value problems.*

This paper develops further the author's theory of abstract boundary conditions (Transactions of this Society, vol. 45 (1939), pp. 369–442), with special reference to boundary value problems with which a semi-bounded bilinear symmetric form in Hilbert space is associated. (Received November 16, 1939.)

53. Leonard Carlitz: *A set of polynomials.*

For $m=a_0+a_1p^n+\cdots+a_kp^{nk}$, $(0\leqq a_i<p^n)$, we define $G_m(t)=\psi_0^{a_0}(t)\psi_1^{a_1}(t)\cdots\psi_k^{a_k}(t)$, where $\psi_k(t)$ is the "linear" polynomial discussed by the author (Duke Mathematical Journal, vol. 1 (1935), p. 137). In this paper we use $G_m(t)$ to derive certain results concerning the arithmetic of polynomials with coefficients in the $GF(p^n)$. (Received November 21, 1939.)

54. Leonard Carlitz: *On certain sums involving polynomials in a Galois field.*

This note is concerned with certain sums extended over the set of polynomials of fixed degree. Some of the results were proved earlier, but all are derived here in a more direct manner. In particular the sums $\sum M^{p^{nk}-1}$ and $\sum M^{1-p^{nk}}$ are evaluated. (Received November 21, 1939.)

55. R. F. Clippinger: *On matrix products of positive powers of given matrices.* Preliminary report.

In the special case considered two second order matrices are given. The problem is equivalent to that of finding the region filled by solutions of $dY/dt=Y\left|\rho\log A+\mu\log B\right|$ through the unit matrix of the second order, where ρ and μ are arbitrary positive functions of t. It is shown that if the determinants of A and B are one, this region is that filled by $A^\alpha B^\beta A^\gamma$. This theorem serves as a lemma to show that the region in the special case considered is that filled by $A^\alpha B^\beta A^\gamma$. (Received November 21, 1939.)

56. L. W. Cohen: *On topological completeness.*

The complete space S^* constructed for the space S in the author's paper, *On imbedding a space in a complete space*, Duke Mathematical Journal, vol. 5 (1939), is shown to be complete in the sense of A. Weil. It is also shown that Weil's uniform space is a special case of the space S of that paper. (Received January 5, 1940.)

57. A. H. Copeland: *Transformations on probability sequences.*

In 1919 von Mises developed a theory of probability based on two axioms and four types of transformations. The first axiom concerned the definition of probabilities as limits of success ratios. Roughly the second stated that no selection made in advance could increase one's likelihood of winning a game of chance. This defined the range of applicability of one transformation—namely, that of selection. von Mises' system possesses a strong intuitional appeal but logical objections have been raised against the second axiom. To avoid these objections a number of authors have devised alternative systems in many of which there has been a tendency to include only transformations by selection. This tendency is unfortunate since all four transformations are necessary in solving probability problems. Moreover, Greville has recently indicated certain classical problems which cannot be handled by von Mises' system. Although Greville's transformations are capable of handling these problems, a restricted label space is employed and the selections must be separately treated. It is the purpose of this paper to generalize Greville's theory so that the label space is an abstract space and the selections as well as the other three transformations are special cases of the transformations used. (Received November 21, 1939.)

58. J. J. De Cicco: *The horn angle in the geometry of element-series.*

This is another extension of the horn angle problem which was developed by Kasner (*Conformal geometry*, Proceedings of the Fifth International Congress of Mathematicians, vol. 2 (1912), p. 81; and *Equilong invariants and convergence proofs*, this Bulletin, vol. 23 (1917), pp. 341–347). The deviation between two series S_1 and S_2 contained in a field F is the distance between the centers of their tangent turbines constructed at a common element E. If this is zero, the two series are said to form a horn angle (of first order). A transformation between two fields is said to be equideviate if it preserves the deviation between any two series. Under the group of equideviate transformations, a horn angle possesses the unique invariant $M_{12}=(\Gamma_2-\Gamma_1)^2/(d\Gamma_2/dU_2-d\Gamma_1/dU_1)$, where $d\Gamma_i/dU_i$ is the variation of the geodesic curvature Γ_i with respect to the angle U_i between any two lineal elements of the series S_i at the common element E of the horn angle. Two horn angles of two different fields may be mapped into one another by an equideviate transformation if and only if they have the same measure M_{12}. (Received November 13, 1939.)

59. J. J. De Cicco: *The magnilong near-Laguerre transformations.*

In a previous paper, *The analogue of the Moebius group of circular transformations in the Kasner plane*, this Bulletin, vol. 45 (1939), pp. 936–943, the author discussed the point transformations with respect to the maximum number of vertical parabolas preserved. This discussion is isomorphic to that of the line transformations with respect to the maximum number of circles preserved. A nonmagnilong transformation carries at most $2\,\infty^2$ circles into circles. On the other hand, a magnilong transformation (not a Laguerre transformation) carries at most ∞^1 circles into circles. In this paper, the author obtains the set of all magnilong near-Laguerre transformations (those, which convert exactly ∞^1 circles into circles). These are of the form L_2TL_1, where L_1 and L_2 are Laguerre transformations and T is any one of the three transformations e^z, $\log z$, z^n, $(z=x+jy, j^2=0)$. The family preserved is a linear pencil of circles in the sense of higher circle geometry. (Received November 13, 1939.)

60. A. H. Diamond: *Postulates for the theory of strict implication.*

In 1932 Lewis gave a set of postulates for the theory of strict implication. Among the primitive symbols listed by Lewis is the symbol $=$. Lewis, however, gives an alleged definition of this symbol as follows. Definition: $p=q. \;=\; .p \prec q \cdot q \prec p$. Huntington in his American Academy paper of 1937 gives a set of postulates for Lewis's theory of strict implication in which he takes as primitive symbols all those of Lewis's list except $=$, and defines $=$ legitimately in terms of the others. In giving these postulates, however, Huntington departs from Lewis's method of stating the postulates as primitive sentences of a language S_1 and states them instead in the so-called word-language. It is the purpose of this paper to give a set of postulates for strict implication following Lewis's method of stating the postulates as the primitive sentences of a language S_1 and using as primitive symbols all those listed by Lewis including the symbol $=$. A postulate system is thus obtained which is free of Lewis's circularity in the use of the symbol $=$ and, moreover, a system which affords greater freedom of interpretation than does Huntington's system. (Received November 21, 1939.)

61. R. P. Dilworth: *Ideals in Birkhoff lattices.*

Let $\mathfrak{S}$ be an arbitrary lattice. A sublattice $\mathfrak{a}$ is an ideal if $x \supset a$, $x \varepsilon \mathfrak{S}$, $a \varepsilon \mathfrak{a}$ implies $x \varepsilon \mathfrak{a}$. The ideals of $\mathfrak{S}$ form a lattice $\mathfrak{L}$ containing $\mathfrak{S}$ as a sublattice of principal ideals. $\mathfrak{S}$ is said to be a Birkhoff lattice if $\mathfrak{a}$ covers $\mathfrak{a} \cap \mathfrak{b}$ implies that $\mathfrak{a} \cup \mathfrak{b}$ covers $\mathfrak{b}$ in $\mathfrak{L}$. This clearly reduces to the usual definition if $\mathfrak{S}$ is archimedean. By means of the ideals much of the arithmetical theory of archimedean Birkhoff lattices is extended to general Birkhoff lattices. In particular, the Kurosch-Ore decomposition theorem for modular lattices in its most general form is shown to follow from a simple exchange property of independent bases. (Received November 22, 1939.)

62. J. L. Doob: *On a certain type of family of chance variables.*

Let t vary in any simply ordered set, and let $\{x_t\}$ be a family of chance variables. The chance variables are said to have the property $\mathcal{E}$ if, whenever $t_1 < t_2 < \cdots < t_n < t_{n+1}$, the conditional expectation of $x_{t_{n+1}}$ for given values of $x_{t_1}, \cdots, x_{t_n}$ is equal to x_{t_n}: $E[x_{t_1}, \cdots, x_{t_n}; x_{t_{n+1}}] = x_{t_n}$, with probability 1. Let $\cdots, x_{-1}, x_0, x_1, \cdots$ be a sequence of chance variables with the property $\mathcal{E}$. Then (i) the x_j for $j \leqq n$ are uniformly integrable (any n); (ii) $E|x_j|$ (the expectation of $|x_j|$) is monotone non-decreasing; (iii) $\lim_{j\to-\infty} x_j$ exists and is finite, with probability 1; (iv) if $\lim_{j\to+\infty} E|x_j| < \infty$, $\lim_{j\to+\infty} x_j$ exists and is finite with probability 1. These theorems and other related ones can be applied to a series of independent chance variables $\sum_1^\infty y_j$ with $Ey_1 = Ey_2 = \cdots = 0$, since the partial sums of such a series have the property $\mathcal{E}$. Let $\{x_t\}$, $(-\infty < t < \infty)$, be a family of chance variables with the property $\mathcal{E}$. It is shown that the random function x_t can be considered (in a sense made precise in the paper) as a function of t which is continuous except perhaps on a denumerable set, and which for all t has the property that $\lim_{h \downarrow 0} x(t+h)$ exists if $E|x_t \log |x_t|| < \infty$. (Received November 20, 1939.)

63. R. J. Duffin and J. J. Eachus: *Sets invariant under a polynomial operator.*

An investigation is made of certain properties of curves and domains which are transformed into themselves under a polynomial operator. For example, the boundary and interior of the unit circle are invariant under the polynomial z^n. In general, the closed curves that arise may be quite complicated, non-analytic to say the least. The use of nonlinear q-difference equations is found to facilitate the study of analytic invariant curves. (Received November 21, 1939.)

64. Samuel Eilenberg: *An invariance theorem for subsets of S^n.*

Let A and B be two homeomorphic subsets of the n-sphere S^n. If the number of components of S^n-A is finite, then it is equal to the number of components of S^n-B. The proof uses Vietoris cycles and duality theorems. (Received November 20, 1939.)

65. Samuel Eilenberg: *Fixed points for periodic transformations.*

Let R be a commutative ring with a unit element and p a prime number such that $px\neq 1$ for all $x \in R$. Given a separable, metric, finite dimensional space X in which every cycle with coefficients in R bounds, every periodic transformation of X with the period p has a fixed point. In particular R can be taken to be the ring of integers reduced modulo q where q is a multiple of p. The theorem does not hold if R is the ring of integers reduced modulo q not a multiple of p. The theorem generalizes a fixed point theorem of P. A. Smith (Annals of Mathematics, (2), vol. 35 (1934), pp. 572–578). (Received November 20, 1939.)

66. Samuel Eilenberg: *On the homotopy type of S^n.*

A necessary and sufficient condition is given in order that an infinite polyhedron have the homotopy type of the n-sphere S^n. The condition is expressed in terms of homotopy groups and infinite cohomology groups. As an application the author proves that if S^{r-n-1}, $(n\neq 1)$, is topologically imbedded into S^r then S^r-S^{r-n-1} has the homotopy type of S^n if and only if the fundamental group of S^r-S^{r-n-1} vanishes. A condition is also given for the case $n=1$. (Received November 20, 1939.)

67. Samuel Eilenberg and E. W. Miller: *On 0-dimensional upper-semi-continuous collections.*

A collection Φ of closed subsets of a compact metric space X is said to be upper-semi-continuous if $F\cdot\lim\inf F_i\neq 0$ implies $\lim\inf F_i\subset F$ for every sequence $F, F_1, F_2, \cdots$ of sets of Φ. The class Φ can be considered as a separable metric space. The following theorem is proved: Let X be a unicoherent Peano continuum and let Φ be 0-dimensional. If none of the sets $F \in \Phi$ cuts X between two points x_1 and x_2, then the set $\sum F$, for $F \in \Phi$, does not cut X between x_1 and x_2. (Received November 20, 1939.)

68. Benjamin Epstein: *Growth properties of analytic functions of two complex variables.* II. Preliminary report.

Using methods developed previously (abstract 45-9-311), the author is able to deduce a number of other growth theorems for functions regular in domains $\mathfrak{M}^4$: $E[\Re z_k\geq 0,\ k=1, 2]$. Roughly stated the results show that a function $f(z_1, z_2)$ regular in $\mathfrak{M}^4$ cannot sink too low in value on certain non-analytic (three-dimensional) hypersurfaces without compensating with high growth properties in other hypersurfaces. As an example of the results the following theorem is stated: Given $f(z_1, z_2)$ regular in $\overline{\mathfrak{M}}^4$ satisfying the conditions: (a) $\log |f(\pm iy_1, \pm iy_2)| \leqq -\mathfrak{H}_1|y_1|^{\alpha_1}-\mathfrak{H}_2|y_2|^{\alpha_2}$; (b) $\log |f(r_1e^{i\phi_1}, \pm iy_2)| \leqq -\mathfrak{H}_3r_1^{\beta_1}-\mathfrak{H}_4|y_2|^{\beta_2}$; (c) $\log |f(\pm iy_1, r_2e^{i\phi_2})| \leqq -\mathfrak{H}_5r_2^{\gamma_1} -\mathfrak{H}_6|y_1|^{\gamma_2}$; (d) $\log |f(r_1e^{i\phi_1}, r_2e^{i\phi_2})| \leqq M_1r_1^{\delta_1}+M_2r_2^{\delta_2}$, where $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2, \delta_1, \delta_2$ may be determined, with $\mathfrak{H}_i$, $(i=1, \cdots, 6)$, M_1, M_2 as fixed positive constants, and where the inequalities are satisfied on certain hypersurfaces, then the function is identically zero in $\mathfrak{M}^4$. $\overline{\mathfrak{M}}^4$ is swept out by a class of hypersurfaces and the constants $\alpha_i, \beta_i, \gamma_i, \delta_i$, $(i=1, 2)$, are related in a specific fashion to the class used. The results are also extended to considerations of integrals taken over zero-lines. (Received November 20, 1939.)

69. H. L. Garabedian: *A new formula for the Bernoulli numbers.*

A unique method is given for deriving what appears to be a new formula for the Bernoulli numbers: $B_{k+1}=((-1)^{k+1}(k+1)/(2^{k+1}-1))\sum_{i=0}^{k}(\Delta^i a_0/2^{i+1})$, $a_n=(1+n)^k$, $(k=0, 1, 2, \cdots)$. This formula gives all of the Bernoulli numbers, including the zero ones, together with the algebraic signs which are sometimes attributed to them. The derivation involves a theorem established recently by the author (this Bulletin, vol. 45 (1939), pp. 592–596). (Received November 18, 1939.)

70. G. N. Garrison: *Quasi-groups.*

A subset of the elements of a finite quasi-group S (a multiplicative system in which $ax=b$ and $ya=b$ are uniquely solvable for x and y) is called an invariant complex if for every pair of elements a, b in S there exists a corresponding element c in S such that $(Ha)(Hb)=Hc$. It follows that the elements of S fall into disjoint classes of the form Ha and that these form a quotient quasi-group which is a homomorphism of S. The principal result is then that every homomorphism of S is given by the quotient quasi-group of a suitably chosen invariant complex. Invariant complexes with special properties $a \subset Ha$ or $(Ha)(Hb)=H(ab)$ are also discussed. (Received November 18, 1939.)

71. Abe Gelbart: *On the growth of a function of two complex variables given by its power series expansion on certain hypersurfaces.*

In this paper the author deals with the growth of a function $f(z_1, z_2)$ given by its power series expansion $\sum_{mn=0}^{\infty} a_{mn}z_1^m z_2^n$ on a sum-manifold of surfaces which are topologically different from the type previously considered (abstract 45-11-418). With the use of the integral formula as developed by Bergman (Matematicheskii Sbornik, vol.1 (43) (1936)) a lower bound for max $[M(G^2, f)]$ is obtained, where $M(G^2, f)$ is the maximum-modulus of f on G^2, G^2 being a part of the intersections of the hypersurfaces $i_1^3=E[z_2=Ar^s e^{i\lambda_1},\ 0\leqq \lambda_1 \leqq 2\pi]$, $i_{k+1}^3=E[z_1=Ar^s e^{i\lambda_{k+1}}+c_k z_2,\ 0\leqq\lambda_{k+1}\leqq 2\pi,\ k=1,\ 2]$, which form the boundary of a domain $\mathfrak{M}^4$; f is analytic in $\overline{\mathfrak{M}}^4$. Further, a surface H^2 is considered that lies in the hypersurface $S_{\lambda_1=0}^{2\pi} I_1^2(\lambda_1, r)$, which is that part of the boundary of $\mathfrak{M}^4$ that belongs to i_1^3. Bergman has associated with such surfaces two numbers α_1, α_2 (Mathematische Annalen, vol. 109 (1936), p. 347). A lower limit for $M(H^2, f)$ is obtained depending only on the coefficients a_{mn} and α_1, α_2. Analogous results can be obtained for more general types of hypersurfaces. (Received November 27, 1939.)

72. P. W. Gilbert: *Two-to-one transformations on linear graphs.* Preliminary report.

Given a linear graph G, denote by $E(G)$ the negative of the Euler characteristic of G, and by T an exactly two-to-one continuous transformation defined over G. A necessary condition for the existence of T is that $E(G)$ be even. This condition is not sufficient, as is shown by an example, for the case $E(G)=2$, where T does not exist. However, for the case where the maximal cyclic elements of G are simple closed curves, the above necessary condition is also sufficient. In the general case, a method is described for reducing G to a subset H, such that T can be defined over G if and only if it can be defined over H. The cases $E(G)=0$ and 2 are completely treated. (Received November 22, 1939.)

73. André Gleyzal: *A general theorem on the structure of linear orders.*

The paper defines a number of operations on simply ordered sets and, more generally, on order types. With a given order type α, there are associated certain order types denoted by α^E, α^I, α^T, and so on. These order types may be characterized by means of closure and minimal properties. It turns out that if A is any order, and α any order type, A is either of type α^T or may be formed by substituting orders of type α^T for elements of an order no proper segment of which has property α. As a particular application of this theorem, it is shown that every order of cardinal $\aleph_\lambda$ contains either the normal order ω_λ, or its reverse, or contains an order every proper segment of which has $\aleph_\lambda$ elements. (Received November 20, 1939.)

74. Harriet M. Griffin: *The abelian quasi-group.*

This paper is concerned with the study of the abelian quasi-group when certain conditions with respect to coset expansions are imposed by stated associative laws. Detailed study is made of the minimal quasi-group of units, and definite laws for the order of elements are developed. Certain problems about structure and quotient quasi-groups for this case are answered. (Received November 21, 1939.)

75. O. G. Harrold (National Research Fellow): *Continua of finite degree and certain product sets.*

In this paper the class of continua of finite degree, which has been studied by G. T. Whyburn (American Journal of Mathematics, vol. 55 (1933), pp. 11–16) and the author (abstract 45-9-321), is identified with each of the classes of continua defined by the following properties: (1) M is a locally connected continuum such that to each other pair of closed, disjoint sets A, B in M there exists a finite collection of disjoint, perfect sets $N^1, N^2, \cdots, N^k$ such that any continuum in M intersecting both A and B contains some N^i; (2) if K, K^i, $(i=1, 2, \cdots)$, are nondegenerate continua in the continuum M with $\lim K^i=K$, there exists an integer n for which $\prod_n^\infty K^i$ is uncountable. (Received November 16, 1939.)

76. O. G. Harrold (National Research Fellow): *Exactly $(k, 1)$ transformations on linear graphs.*

This paper initiates a study of exactly $(k, 1)$ continuous transformations defined on a linear graph A. The principal results are: for $k=2$, $B=f(A)$ is a graph, and f is locally interior on A at all but possibly a finite number of points; for $k>2$, $B=f(A)$ need not be a graph but is a stably regular curve, and f is locally interior on A except possibly for a closed set of dimension zero. For $k=2$ a formula is given which limits decidedly the types of graphs on which an exactly (2, 1) transformation can be defined. It is shown that an exactly (2, 1) mapping on a simple closed curve gives a simple closed curve and that the transformation f is topologically equivalent to either (a) $w=z^2$ on $|z|=1$, or, (b) $w=z^2$ on $|z|=1$ for $I(z)\geqq 0$ and $\bar{w}=z^2$ on $|z|=1$ for $I(z)\leqq 0$. Examples are given which show there is no such simple behavior for $k>2$. (Received November 16, 1939.)

77. O. G. Harrold (National Research Fellow): *The dimensionality of continuous transforms of an n-cell.*

A classical result in dimension theory due to W. Hurewicz states that the dimensionality of a compact metric space is raised at most $k-1$ units by a continuous $(k, 1)$ transformation. Since it is possible to map a 1-cell continuously onto a 2-cell by a (3, 1) mapping, the question arises as to whether or not this can be accomplished by a continuous (2, 1) transformation. In answer to this we show that the dimensionality of an n-cell is raised at most $k-2$ units by a continuous $(k, 1)$ mapping, $k>1$. This result is the best obtainable for an n-cell. (Received November 16, 1939.)

78. M. H. Heins: *On the conformal mapping of a multiply-connected region into itself.*

The work of Ritt and Aumann-Carathéodory on the conformal mapping of a multiply-connected region into itself suggests the following problem: what is the behavior of the iterates of a function $w(z)$ analytic and single-valued in a multiply-connected region G_z with at least three boundary points such that $w(G_z)\subset G_z$? Excluding the case where there exists a point $z_0\subset G_z$ such that $z_0=w(z_0)$, and positing that G_z be a region of finite connectivity $p>1$ bounded by p Jordan curves, one may show that either the iterates $w_n(z)$ of $w(z)$ converge continuously to a single point of the boundary of G_z, or else $w(z)$ is necessarily a one-to-one conformal map of G_z onto itself. The proof is based on Julia's lemma and on the behavior in the neighborhood of $|x|=1$ of a function which maps $|x|<1$ onto G_z^∞, the universal covering surface of G_z. (Received November 21, 1939.)

79. M. S. Hendrickson: *On certain properties of arbitrary functions.*

The present paper is related to the article of Henry Blumberg (Acta Mathematica, vol. 65, pp. 263–282), whose principal theorem characterizes, in a certain significant sense, the internal structure of the general real function in terms of associated measurable functions of restricted character. The present paper presents various companion theorems to this theorem of Blumberg, which are based on generalized types of continuity, related to, but somewhat different from, those treated in the literature. The theory of this generalized continuity is developed in certain directions. (Received November 20, 1939.)

80. A. D. Hestenes: *On the solutions of a certain functional equation* $\phi_c(x)-c\log\phi_c(x)=x-c\log x+1$. Preliminary report.

In a paper shortly to appear in the Annales de l'Institute Henri Poincaré, G. D. Birkhoff has discussed a class of linear functional equations (termed by him ϕ-difference equations) which seem to be of comparable importance with linear and q-difference equations. Roughly speaking these equations of Birkhoff are the linear equations in which the operations $x'=x+1$ and $x'=qx$ in these classical cases are replaced by $x'=\phi_c(x)$ where $\phi_c(x)$ is defined by the relation $\phi_c(x)-c\log\phi_c(x)=x-c\log x+1$. In his paper Birkhoff has treated only the homogeneous ϕ-difference equation of the first order. The author proposes, in this paper, to discuss the values of the underlying function of Birkhoff type $\phi_c(x)=x+1+c/x+c_2/x^2+\cdots$ in the complex plane. (Received November 22, 1939.)

81. A. D. Hestenes: *A solution of the nonhomogeneous ϕ-difference equation of the first order.* Preliminary report.

This paper presents solutions of the nonhomogeneous ϕ-difference equation $y(\phi_c(x))-a(x)y(x)=b(x)$ in which $\phi_c(x)$ is a function of Birkhoff type. These solutions are shown to be analytic in certain regions of the complex plane and are asymptotically represented there by a formal series solution $S(x)$. (Received November 22, 1939.)

82. A. D. Hestenes: *On certain matrix ϕ-difference equations.* Preliminary report.

The author extends the methods used in solving the Birkhoff homogeneous and nonhomogeneous ϕ-difference equations to a special nth order case in which the roots of a certain characteristic equation are distinct and different from zero in the homogeneous case $Y(\phi_c(x))=A(x)Y(x)$ and different from zero and one in the nonhomogeneous case $Y(\phi_c(x))-A(x)Y(x)=B(x)$. The results are parallel to those of the classical linear difference equation. In a subsequent paper the author hopes to extend these methods to the most general linear ϕ-difference equations. (Received November 22, 1939.)

83. M. R. Hestenes: *A theorem on quadratic forms and its application in the calculus of variations.*

In the present paper it is shown that if $Q(\xi, \eta)$ is a quadratic form in $2n$ variables $\xi_1, \cdots, \xi_n, \eta_1, \cdots, \eta_n$ such that $Q(\xi, \eta)>0$ whenever the $n\times 2$ dimensional matrix (ξ_i, η_i) has rank one, then there exists an n-rowed skew-symmetric matrix (S_{ik}) such that the form $Q(\xi, \eta)+S_{ik}\xi_i\eta_k$ is positive definite. The method used is an extension of the method used by McShane (abstract 45-5-209) for the case $n=3$. With the help of this result it is shown that if $z_i(x, y)$ $((x, y)$ on A; $i=1, \cdots, n)$ is an admissible surface S for the calculus of variations integral $\int\int f(x, y, z, p, q)dx\,dy$, where $p_i=\partial z_i/\partial x$, $q_i=\partial z_i/\partial y$, such that the Legendre form $Q(\xi, \eta)$ for the integral f is positive on S whenever the matrix (ξ_i, η_i) has rank one, then there is an invariant integral $\int\int g(x, y, z, p, q)dx\,dy$ such that the Legendre quadratic form for $f+g$ is positive definite on S. The results described here will be submitted for publication with those of McShane (abstract 45-5-209). (Received November 24, 1939.)

84. Dunham Jackson: *Note on certain orthogonal polynomials.*

A well known property of the kernel associated with an ordinary system of orthogonal polynomials, as set forth for example in Szegö's Colloquium volume (vol. 23, p. 39), is extended to orthogonal trigonometric sums, and more generally to other sets of orthogonal polynomials on plane curves. (Received November 18, 1939.)

85. R. D. James: *A problem analogous to the problem of prime pairs.*

It has long been conjectured that there is an infinite number of primes p such that $p-2$ is also a prime. At present no proof is known although numerical evidence indicates that the statement is true. In this paper it is shown that there is an infinite number of primes p such that $p-k$ is the sum of two primes. Here k is any odd integer, positive or negative. By taking $k=p'+2$, where p' is any odd prime, it follows that there is an infinite number of primes p such that $p-2$ is the sum of three primes, one of which is arbitrary. The method of proof is to establish the asymptotic formula $2A(N)=N^2(\log N)^{-3}S(k)+O(N^2 \log\log N (\log N)^{-4})$, where $A(N)$ is the number of solutions in primes p_1, p_2, p_3 not exceeding N of the equation $p_1-k=p_2+p_3$, and where $S(k)\geqq c>0$. The proof is a modification of that given by Vinogradov for Goldbach's problem (Comptes Rendus de l'Académie des Sciences de l'URSS, vol. 15 (1937), pp. 169–172). (Received November 20, 1939.)

86. Walter Jennings: *Some implications of the continuum hypothesis.*

A theorem is derived which subsumes various results due to Sierpiński and Lusin and has as corollaries a number of apparently new implications of the continuum hypothesis. (Received November 18, 1939.)

87. B. W. Jones: *Related genera of quadratic forms.* Preliminary report.

This paper shows that with every genus g of quadratic forms and every prime number p there is associated, by a set of transformations, a genus or set of genera G, having the property that the numbers represented by the forms of G are $1/p$ times the multiples of p represented by the forms of g. At least for positive forms, Siegel's expression for the number of representations of any number by the forms of g is a simple function of the corresponding expressions for G. One use of this process is to be able to eliminate, by successive steps of this kind, all the factors common to the number represented and the determinant of the form. Another use is the establishment of relationships between the number of classes of g and the number of classes of G. (Received November 20, 1939.)

88. F. B. Jones: *Almost cyclic elements and simple links of a continuous curve.*

The principal object of this paper is to define two classes of subsets of a non-locally compact continuous curve: (1) the *cyclic nucleae*, which are closely related to basic sets, and (2) the *almost cyclic elements*, which are closely related to cyclic elements and which *are* cyclic elements if the continuous curve is locally compact. A few of the properties of these sets are investigated to the extent that the reader can supply others by analogy to known propositions concerning cyclic elements. It is also pointed out that if H is a nondegenerate almost cyclic element of a locally connected complete Moore space in which the Jordan curve theorem holds true, then considered as a space, H is a locally connected complete Moore space in which the Jordan curve theorem holds true but which contains no cut point. (Received November 20, 1939.)

89. F. B. Jones: *Certain consequences of the Jordan curve theorem.*

If the Jordan curve theorem holds true in a locally connected Moore space M, it is well known that M need be neither a subset of a plane nor even metric. However, M has many properties of a locally connected inner-limiting subset of the plane. Besides the properties of the space itself, the properties of its subcontinua are investigated. Particular emphasis has been placed on separation theorems. (Received November 20, 1939.)

90. Mark Kac: *On the partial sums of the exponential series.*

The formula $(1/\pi^{1/2})\int_{-\infty}^{\omega}e^{-u^2}du=\lim_{x\to\infty}e^{-x}\sum x^k/k!$, where the summation extends over all $k<x+\omega(2x)^{1/2}$, is valid for every real ω. The case $\omega=0$ was proved by

Ramanujan. A connection with a number theoretical problem is indicated. (Received November 16, 1939.)

91. E. R. van Kampen: *On uniformly almost periodic multiplicative and additive functions.*

An additive (multiplicative) function $f(n)$, $(n=1, 2, \cdots)$, is a function which may be represented in the form $\sum_p (\prod_p) f_p$, where p runs through all primes and $f_p(n)=f(p^k)$ if $p^k | n$ and $p^{k+1} \nmid n$. The following theorem is proved: An additive (real-valued multiplicative) function f is uniformly almost periodic if and only if the above representation of f as an infinite sum (product) is uniformly convergent and each term (factor) is u.a.p. The sufficiency of this condition is of course evident. In case of a general multiplicative function the question whether or not the above conditions are necessary depends on whether or not they are necessary in the case of a u.a.p. function f which is additive (mod 2π). (Received November 22, 1939.)

92. Edward Kasner and J. J. De Cicco: *The conformal near-Moebius transformations.*

In a previous paper, *Characterization of the Moebius group of circular transformations*, Proceedings of the National Academy of Sciences, vol. 25 (1939), pp. 209–213, the point transformations of the plane with reference to the maximum number of circles preserved were discussed. A nonconformal point transformation converts at most $2\infty^2$ circles into circles. A conformal transformation not of the Moebius type carries at most $2\infty^1$ circles into circles (excluding the minimal lines). In this paper, the authors determine the set of all conformal near-Moebius transformations (those which preserve exactly $2\infty^1$ circles). These are of the form M_2TM_1, where M_1 and M_2 are Moebius transformations and T is any one of the three transformations e^z, $\log z$, z^n. The two families preserved are two orthogonal linear pencils of circles. The conformal near-collineation problem, solved by Kasner in the paper *The problem of partial geodesic representation*, Transactions of this Society, vol. 7 (1906), pp. 200–206, is a special case of this. Any conformal near-collineation is of the form S_1TS_2, where S_1 and S_2 are similitudes. The family preserved is a pencil of straight lines. (Received November 13, 1939.)

93. Edward Kasner and J. J. De Cicco: *Transformation theory of integrable double-series of lineal elements.*

In this paper, the authors completely solve a problem in the theory of differential elements of three-space. A collection of ∞^2 lineal elements of space is said to form a double-series. If ∞^1 unions can be found whose lineal elements coincide exactly with those of a given double-series S, then S is called an integrable double-series. Thus an integrable double-series S is the configuration obtained by constructing at each point of a surface a single tangent direction. The fundamental result is that the entire group of lineal element transformations which carry every integrable double-series into an integrable double-series is exactly the group of extended point transformations. This paper will be published in full in this Bulletin. (Received November 20, 1939.)

94. J. L. Kelley: *On the hyperspaces of a given space.* Preliminary report.

A study is made of the sets $S(A)$ and $C(A)$ consisting respectively of all closed and all closed and connected subsets of a metric separable space A. Particular attention is given the classification of properties of A in terms of the hyperspaces. For example, if for $x \varepsilon A$ we designate by C_x the collection of elements in $C(A)$ which contain x, then if x is a local separating point, C_x is found to have interior points. If A is Peanean, this is a necessary and sufficient condition that x be a local separating point. Theorems on local separating points are obtained from corresponding theorems on separating points. (Received November 21, 1939.)

95. C. G. Latimer: *On a certain equation.*

Let a, b, c, d be integers such that $abcd \neq 0$, not all have the same sign, each is without a square factor greater than 1 and a, b, c have no common prime factor. The integers a, b, c determine an integer D such that *if* $\theta = (abcd)^{1/2}$ *is rational* $ax^2+by^2+cz^2+dw^2=0$ *has a solution in integers, not all zero, if and only if* $D=-1$. *If* θ *is irrational, such a solution exists if and only if no prime factor of* D *is the product of two distinct prime ideals in the field* $F(\theta)$. The equation is reduced to the form $\zeta^2 = abcdw^2$, where ζ is an element in a quaternion algebra $\mathfrak{A}$ with the fundamental number D. A theorem on the quadratic subfields of $\mathfrak{A}$ and the theorem that $\mathfrak{A}$ is a non-division algebra if and only if $D=-1$ is then applied. The result seems simpler than former results, all of which require considerable separation of cases. (See Mordell, this Bulletin, vol. 38 (1932), p. 277; Archibald, this Bulletin, vol. 37 (1931), p. 607, where numerous other references are given.) (Received November 20, 1939.)

96. Norman Levinson: *A proof of Hardy's theorem on the zeros of the zeta function.*

Let $Z(t) = \Xi(t)e^{\pi t/4}/(t^2+1/4)$ where $\Xi(t)$ is the entire function having as its zeros the non-trivial zero of $\zeta(s)$. Then a certain Mellin transform of $e^{-\pi t/4}Z(t)$ is $\phi(x) = 2\sum e^{-n^2x} - \pi^{1/2}/x^{1/2}$. The fact that $\phi(x) \to \infty$ as $x \to 2\pi i$ demonstrates at once that the integral of $|Z(t)|$ over $(0, \infty)$ diverges. The fact that $\phi(x)$ is bounded as $x \to \pi i$ shows that the integral of $e^{-\delta t}Z(t)$ over $(0, \infty)$ is bounded as $\delta \to 0$. But these two facts imply that $Z(t)$ must have an infinity of zeros. (Received November 20, 1939.)

97. F. A. Lewis: *Generators of permutation groups simply isomorphic with $LF(2, p)$.*

From the results of this paper the generators of any group of the class may be readily obtained. (Received November 20, 1939.)

98. A. N. Lowan: *Note on upper bounds of the derivatives of certain transcendental functions.*

Starting with certain integral representations of the functions $J_0(z)$ and e^{-x^2}, upper bounds of the absolute values of the kth derivatives of these functions are obtained. Specifically, $|(d^k/dz^k)J_0(z)| < I_0(r \sin \theta)$ and $|(d^k/dx^k)e^{-x^2}| < 2^k\{k/2\}!$, where $\{k/2\}$ designates the integer which is either equal to $k/2$ if k is even or the integer immediately smaller than $k/2$ if k is odd. Similarly, starting with the integral representation of the first derivative of arc tan x, an upper bound of the modulus of the kth derivative of the latter function is found to be $x^{-k}(k-1)!$. The knowledge of these upper bounds is of paramount importance in determining the numbers of terms required in the Taylor expansions of the functions under consideration. The computation of tables of these functions by the Project for the Computation of Mathematical Tables (Work Projects Administration, New York City) is now in progress. (Received November 10, 1939.)

99. R. G. Lubben: *Separabilities of higher orders and related properties.*

A point set M is strongly α-separable provided that there exists N such that $\overline{N} \supset M \supset N$ and either N is countable or its power is less than α; if this is true for α the power of M, M is semi-separable. M is almost perfectly α-compact in itself provided that if $M \supset N$, α is not greater than the power of N, and δ is less than the power of N, then M contains a limit point of N of an order relative to N at least as great as δ. If $\alpha > \aleph_0$ is a regular cardinal, the following are equivalent properties of point sets in a space satisfying Hausdorff's Axioms A and B: (1) each point set of power α contains a limit point of itself; (2) each point set either is strongly α-separable, or it contains a limit point of itself of an order not less than α; (3) each point set is almost perfectly α-compact in itself; (4) if M has a regular power which is not less than α, then either M is semi-separable or it is similar to the set of all its complete limit points which belong to it. (Received November 13, 1939.)

100. H. F. MacNeish: *A set of postulates for a finite geometry represented by the Pappus configuration.*

Nine postulates are stated both geometrically and abstractly which define a geometry with three points on a line and three lines through a point. Postulates 6 and 8 are dual, and Postulates 7 and 9 are dual. The duals of Postulates 1, 2, 3, 4, 5 are proved as theorems. Therefore, in view of the fact that the definitions of the set have duality, the entire finite geometry has duality. A euclidean parallel postulate holds. Duality leads to the idea of parallel points. The Pappus theorem is proved as a proposition. The Pappus configuration of nine points and nine lines is proved to satisfy the nine postulates and includes all elements of the finite geometry. The complete independence proof for the postulates is given. Triangles are classified according as lines joining the vertices, and intersections of the sides exist in the finite geometry. A geometry of triangles is given leading to two special Desargues' theorems. The geometry of quadrilaterals is also considered in this paper. (Received November 16, 1939.)

101. H. F. MacNeish: *A sufficient condition for integrability.*

The paper contains a generalization of the condition that $F^n(x)$ be integrable to the case that $\prod_1^r F_i^{n_i}(x)$ be integrable. The condition is that one of the functions say $F_j^{n_j}$ is such that $F_j^{n_j} = k(\prod F_i)(\sum (n_i+1)F_i'/F_i)$, $(i=1, 2, \cdots, r;\ i \neq j)$. It follows that $\int \prod_1^r F_i^{n_i}(x)dx = k\prod F_i^{n_i+1}(x) + c$, $(i=1, 2, \cdots, r;\ i \neq j)$. By this process many functions may be integrated easily, which are not included in the well known categories of functions integrable by standard methods, and which might be considered as not integrable in a finite number of terms. (Received November 21, 1939.)

102. J. D. Mancill: *On the Carathéodory condition for unilateral variations.*

The two formulations and proofs of the Carathéodory condition in the calculus of variations given by Graves (American Journal of Mathematics, vol. 52 (1930), pp. 13–19) do not necessarily apply to the case when the minimizing curve may have arcs in common with the boundary of the region of admissible variations. The purpose of this note is to show how his first formulation and proof can be modified so as to be applicable to unilateral (one-sided) variations in the plane. (Received November 20, 1939.)

103. J. D. Mancill: *On the Jacobi condition for unilateral variations.*

The proof of the Jacobi condition in the calculus of variations by means of the Kneser envelope theorem does not necessarily apply to all extremal arcs of the minimizing curve in the case when the minimizing curve may have arcs in common with the boundary of the region of admissible curves. So far as the author knows no use has yet been made of the second variation in this connection. In the present paper additional properties of the envelope of a one-parameter family of extremals in the plane are derived, by means of which, with the aid of the Kneser theorem, a Jacobi condition is proved for the case of unilateral (one-sided) variations in the plane. Incidentally, the results of this paper are of historical interest since they eliminate the restriction, in the case of the geometric formulation of the Jacobi condition, that the envelope have a regressive branch at its point of contact with the minimizing curve. (Received November 20, 1939.)

104. Karl Menger: *On Cauchy's theorem in the real plane.*

If $p(x, y) = q(x, y) = f(x+y)$ where f is a continuous nowhere differentiable function on one real variable, then $\int pdx+qdy$ is independent of the path though neither p nor q has any partial derivative. It is sufficient for the independence that $\lim (\Delta q/\Delta x - \Delta p/\Delta y) = 0$ holds for certain rectangular nets of points. This condition is satisfied not only in the classical case that $\partial p/\partial y$ and $\partial q/\partial x$ exist and are equal and continuous everywhere, but also if existence, equality and certain continuity properties of $\partial p/\partial y$ and $\partial q/\partial x$ are guaranteed on a denumerable set only; it is satisfied even for some nowhere differentiable functions. By introducing subdivided nets we get a condition that is both sufficient and necessary in order that the integral be independent of the path. (Received November 13, 1939.)

105. A. N. Milgram: *Iterations of mappings of a set on its square.*

Let M be an abstract set, f and g two mappings of M on itself such that if $x \varepsilon M$, $y \varepsilon M$ there exists $t \varepsilon M$ for which $x = f(t)$, $y = g(t)$. Then for $x_1, \cdots, x_k$ any elements of M, and $n_1 \leqq n_2 \leqq \cdots \leqq n_k$, $m_1 < m_2 < \cdots < m_k$ two sets of positive integers, the equations $x_1 = f^{n_1}g^{m_1}(t), \cdots, x_k = f^{n_k}g^{m_k}(t)$ will always have a solution, where f^n means f iterated n times. If M is the interval $(0, 1)$ and f and g are continuous, the above gives a continuous mapping of $(0, 1)$ on the k-dimensional unit cube. If M is compact space, and f and g are continuous, then even each denumerable set of such equations has a solution. Thus if M is $(0, 1)$, taking $x_k = f^{n_k}g^{m_k}(t)/k$, one obtains a continuous mapping of $(0, 1)$ on the fundamental cube in Hilbert space. (Received November 17, 1939.)

106. Harlan C. Miller: *A theorem concerning closed and compact point sets which lie in connected domains.*

In this paper it is proved that in a connected space satisfying Axioms 0, 1, and 2 of R. L. Moore's *Foundations of Point Set Theory* every closed and compact point set is a subset of a compact continuum. (Received November 28, 1939.)

107. Deane Montgomery and Leo Zippin: *A Hilbert axiom for topological transformation groups of space.*

The paper shows that Hilbert's Axiom III in his *Grundlagen der Geometrie* (Mathematische Annalen, vol. 56 (1902)) can be replaced by its more intuitive and simpler "two-point" form. This materially weakened axiom suffices for Hilbert's development of plane geometries and also for the extension of this program to three-space, recently given by the authors (abstract 45-11-402). (Received November 27, 1939.)

108. R. L. Moore: *Concerning accessibility.*

In this paper it is shown that in a locally peripherally compact space (F. B. Jones, Transactions of this Society, vol. 42 (1937), p. 53) satisfying Axioms 0, 1 and 2 of the author's *Foundations of Point Set Theory* (American Mathematical Society Colloquium Publications, vol. 13, 1932), every continuum with a compact boundary is compactly connected; and if the boundary of the domain D is a compact point set β, then (1) if no component of D is closed, the components of D form a semi-contracting collection, (2) if D is connected, $\bar{D}$ contains a compact continuum containing β. (Received November 18, 1939.)

109. R. L. Moore: *Concerning domains whose boundaries are compact.*

In this paper it is shown that, in a space satisfying Axioms 0–5 of the author's *Foundations of Point Set Theory*, (1) no domain with a compact boundary has uncountably many components, (2) if N is a continuum whose boundary is a subset of a compact continuous curve with no continuum of condensation, then N is itself a continuous curve. (Received November 22, 1939.)

110. I. M. Niven: *Integers of quadratic fields as sums of squares.*

L. J. Mordell (Mathematische Zeitschrift, vol. 35 (1932)) gives necessary and sufficient conditions that a positive integral binary quadratic form be expressible as a sum of squares of integral linear forms. Since every integer of a quadratic algebraic field $R(m^{1/2})$ is expressible in an infinitude of ways as a quadratic form in 1, $m^{1/2}$, it is possible to give necessary and sufficient conditions that such an integer be a sum of squares of integers of the field. It is shown that every integer of the form $a+2b\theta$, where $\theta^2=-m$, m being a positive integer, is expressible as a sum of three squares of integers of the field $R(\theta)$; in case m is congruent to three modulo four, every integer of the field is so expressible. Necessary and sufficient conditions that a Gaussian integer be expressible as a sum of two squares of Gaussian integers are established. Similar results are also obtained for real quadratic integers. (Received November 20, 1939.)

111. Rufus Oldenburger: *Symbolic elements in dynamics.*

In the present paper relations are obtained between the properties of periodicity, minimalness, and recurrence of a symbolic trajectory T and these properties for the rays based on T. Since symbolic elements are essentially rays, similar considerations hold for symbolic trajectory and elements based on this trajectory. It is on the use of these elements that much of the theory of trajectories in dynamics is based. It is proved, in particular, that if all elements based on a symbolic trajectory T are recurrent, T is recurrent, while conversely, if $T=T^{-1}$ and T is recurrent, all elements based on T are recurrent. It is further proved that the property of minimalness of a symbolic trajectory T as defined in terms of elements based on T is equivalent to minimalness as defined in terms of rays based on T and that periodicity of all elements or all rays based on T is equivalent to the periodicity of T. If a trajectory T is recurrent and the recurrency functions of the elements based on T are bounded, these properties are possessed by each limit trajectory of T. (Received November 20, 1939.)

112. W. A. Patterson: *Inverse problems of the calculus of variations for multiple integrals.*

For a partial differential equation of the form $F \equiv A_{\alpha\beta}(x_1, \cdots, x_n, z, p_1, \cdots, p_n)p_{\alpha\beta} + B(x_1, \cdots, x_n, z, p_1, \cdots, p_n) = 0$, $p_i = \partial z/\partial x_i$, $p_{ij} = \partial^2 z/\partial x_i \partial x_j$, $(i, j, \alpha, \beta = 1, 2, \cdots, n)$, where α and β are umbrals and the functions $A_{ij} = A_{ji}$ and B are analytic but otherwise arbitrary, a function $M(x_1, \cdots, x_n, z, p_1, \cdots, p_n) \neq 0$ and such that the equation $M \cdot F = 0$ has a self-adjoint equation of variation is called a multiplier. When $F = 0$ admits a multiplier, the integral hypersurfaces of $F = 0$ are the extremals of an associated variation problem of the form $\int f(x_1, \cdots, x_n, z, p_1, \cdots, p_n)dx_1dx_2 \cdots dx_n = \min$. In the present paper the generality of the multiplier M and the form of the integrand function f in the most general associated variation problem is determined for certain important classes of partial differential equations of the form $F = 0$. (Received November 21, 1939.)

113. R. S. Phillips: *On linear transformations.*

The purpose of the paper is to give an integral characterization of linear limited and completely continuous transformations both on the common Banach spaces to an arbitrary Banach space and vice versa. The author first studies the abstractly valued function spaces and integrals needed in these characterizations. As an application of this work, he shows that completely continuous transformations on the spaces L^p, l^p, c, C, $(1 \leqq p \leqq \infty)$, to an arbitrary Banach space are approximable in the norm by degenerate transformations. In this, use is made of the theorem characterizing compact sets in L^p, $(1 \leqq p \leqq \infty)$, which is also proved. A final section is devoted to the extension of linear limited transformations. (Received November 16, 1939.)

114. G. B. Price: *On the theory of integration.*

This paper treats the integrals of functions defined on an abstract space with values in a Banach space. Furthermore, the measure function itself is abstract, that is, a transformation defined over the Banach space rather than a number. The convex operators recently introduced by the author are used to obtain a generalization of the upper and lower integrals. Sufficient conditions that a function be integrable are given. The methods yield a unified treatment of a large class of the more common integrals, a class which includes the Riemann integral, the Lebesgue integral, various Stieltjes integrals, various line integrals, and the integral of a function of a complex variable along a curve in the complex plane. (Received November 20, 1939.)

115. Eric Reissner and H. A. Wood: *On boundary value problems of bi-potential theory for an infinite sector.*

The solution of the equation $\nabla^2\nabla^2 w = 0$ is derived for the case that the values of the function and its normal derivative are prescribed along the boundaries of a sector (in polar coordinates (r, θ) along $\theta = \alpha$ and $\theta = -\alpha$). The method consists in superimposing an infinite number of elementary solutions such as $r^n \cos n\theta$, $r^n \sin n\theta$, $r^{n+2} \cos n\theta$, $r^{n+2} \sin n\theta$ which leads to a solution in the form of a Mellin integral. The properties of this solution are discussed. (Received November 16, 1939.)

116. R. M. Robinson: *On the mean values of an analytic function.*

The main result obtained is that if a function $f(z)$ is regular for $|z| < 1$, and if the mean of $|f(z)|$ on $|z| = r$ is less than or equal to 1 for each $r < 1$, then (for any integer $p > 1$) the mean of $|f(z)|^p$ on $|z| = r$ is less than or equal to 1 for $r \leqq p^{-1/2}$, but not in general for larger values of r. (Received November 28, 1939.)

117. Barkley Rosser: *On the computation of logarithms to a large number of decimal places.*

To find log N to a large number of decimal places, it is desirable to find an M very near N such that log M is known to the requisite number of places. Then, if $N=M(1+\epsilon)$, ϵ will be very near zero, and the series for log $(1+\epsilon)$ will converge rapidly. In this paper are tables which, for any N, enable one to find an M of the form $2^a3^b5^c7^d$ (where a, b, c, and d are integers) with $|\epsilon| \leqq .000007$. Then log M is known to 250 decimal places. Table 1 enables one to find an M_1 of the desired form such that $N=M_1(1+\epsilon_1)$ and $|\epsilon_1| \leqq .0024$. Table 2 gives (in order of magnitude) certain values of $2^a3^b5^c7^d$ which are near unity. By looking in Table 2 for a value near $1+\epsilon_1$, one can find an M_2 such that $1+\epsilon_1=M_2(1+\epsilon_2)$ and $\epsilon_2 \leqq .000094$. Table 3 is similar to Table 2 except that values nearer unity are given. By looking in Table 3, one can find an M_3 such that $1+\epsilon_2=M_3(1+\epsilon_3)$ and $|\epsilon_3| \leqq .000007$. (Received November 22, 1939.)

118. A. C. Schaeffer and Gabor Szegö: *Polynomials whose real part is bounded on a Jordan curve in the complex plane.*

Let C be a closed Jordan curve in the complex z-plane consisting of a finite number of analytic arcs forming angles not equal to 0 or 2π with each other. Let $f(z)$ belong to the class of polynomials of degree n or less whose real part is bounded by 1 on C. It is shown that at any point z_0 on C the order of magnitude of max $|f'(z)|$ as n becomes infinite is n^α where $\alpha\pi$ is the exterior angle of C at z_0. For the same class, restricted by the additional condition that $\Im f(z)$ vanishes at a fixed interior point of C, the maximum of $|\Im f(z)|$ over C is also discussed; the rate of growth of this maximum as n becomes infinite is log n. The method used is based on conformal mapping, and especially on the theorems of Osgood-Taylor concerning the behavior of the map function near the boundary. The results of the present paper are well known in the special case in which C is the unit circle. (Received November 20, 1939.)

119. I. J. Schoenberg: *On metric arcs of vanishing Menger curvature.*

A topological proof is given of the so-called n-lattice theorem for metric arcs (Menger, Mathematische Annalen, vol. 103 (1930), p. 487). In addition, Menger's conditions (loc. cit., pp. 488–492) on a metric arc, under which identically vanishing curvature implies the congruence of the arc with the euclidean segment, are replaced by weaker conditions in terms of Ptolemy's classical inequality in elementary geometry. The paper will appear in the Annals of Mathematics. (Received November 21, 1939.)

120. G. E. Schweigert: *A local property arising from certain interior transformations.*

Let $T(A)=B$ be an interior transformation where A is compact and metric and B is a Peano continuum. If U is an open set in A such that $T(U)$ is connected and if x is any point of U, there exists a point y of B and a connected set K in U such that y is distinct from $T(x)$ and K intersects both $T^{-1}T(x)$ and the inverse of y. A certain form of this result will combine with other necessary conditions to characterize particular non-alternating interior transformations. (Received November 19, 1939.)

121. W. T. Scott: *Interpolation by continued fractions.*

In this paper it is shown that interpolation by continued fractions is always possible, even when the well known Thiele interpolation formula fails. The method given here leads to the general corresponding type continued fraction (Leighton and Scott, this Bulletin, vol. 45 (1939), pp. 596–605). A generalization of the Thiele remainder formula for continued fractions is obtained. (Received November 20, 1939.)

122. C. E. Sealander: *Some third order irregular boundary value problems.*

In a previous paper (abstract 44-1-4), the author discussed expansions in series of characteristic functions of the differential system $u'''(x)+p(x)u'(x)+[\rho^3+q(x)]u(x)=0$, $u(0)=u'(0)=u(\pi)=0$, in which the functions $p(x)$ and $q(x)$ were restricted to have special forms. The results of that paper have been extended to the case where these functions are only required to have derivatives of all orders on an interval I of which $x=0$ is an interior point. It is shown that a function $f(x)$ will have an expansion in a series of characteristic functions of this system which converges uniformly to $f(x)$ in every interval $0\leqq x\leqq a<\pi$ interior to I, provided that $f(x)$ has a continuous second derivative in $(0, \pi)$ and that the series $f(x)-w_1(x)/\rho^3+w_2(x)/\rho^6-\cdots$, where the $w_n(x)$ are defined by recurrence relations involving $f(x)$, $p(x)$, and $q(x)$ converges uniformly in I and its second derived series converges at $x=0$. Regions of convergence of the series for $f(x)$ when x is complex are determined. The analogous problem for the system $u'''(x)+\lambda x^\nu u(x)=0$, $u(-\alpha)=u'(-\alpha)=u(\beta)=0$, $(\alpha, \beta>0)$, λ being a complex parameter and ν a real positive integer, in which the coefficient of the parameter may change sign, is then shown to reduce to the above problem by a series of transformations of variable. (Received November 27, 1939.)

123. Seymour Sherman: *Some new properties of transfinite ordinals.*

It is well known that if A is an ordinal, then A is uniquely representable in the form $\sum_{i=0}^{n-1}\omega^{\alpha_i}a_i$, where $A>\alpha_0>\cdots>\alpha_{n-1}$ and $\omega>a_0\cdots a_n>0$. This sum is the Cantor normal form of A. A necessary and sufficient condition that x be a left factor of A is that either $x\leqq\omega^{\alpha_{n-1}}$ or $x=\omega^{\alpha_j}r_j+\sum_{i=j+1}^{n-1}\omega^{\alpha_i}a_i$, $r_jp_j=a_j$, $0\leqq j\leqq n-1$. A necessary and sufficient condition that x be a right factor of A is that either $x=\sum_{i=0}^{j-1}\omega^{-\alpha_j+\alpha_i}+\omega^{\alpha_j}r_j$, $r_jp_j=a_j$, $0\leqq j\leqq n-1$, or $x=\sum_{i=0}^{n-1}\omega^{-\beta+\alpha_i}a_i$, $\beta\leqq\alpha_{n-1}$. These theorems are used in the further characterization of the sets of right and left factors of ordinals. In addition it is proved that $(\alpha+\beta)\gamma\leqq\alpha\gamma+\beta\gamma$ for α, β, and γ ordinals. Necessary and sufficient conditions that the equality hold are given. (Received November 20, 1939.)

124. M. F. Smiley: *A note on the Jacobi condition for extremaloids.*

Recently McShane (Duke Mathematical Journal, vol. 5 (1939), pp. 184–206) gave a treatment of the Jacobi condition for extremals which permitted a simultaneous discussion of this condition for the parametric and nonparametric problems. It is the purpose of this note to formulate the Jacobi condition for extremaloids in a similar fashion. In order to obtain flexibility at the corners we find it necessary to alter McShane's (Bliss') definition of normality of solutions of the Jacobi equations. No attempt is made to discuss questions of tensor invariance. (Received November 21, 1939.)

125. H. W. Smith: *The oscillation of solutions of the differential boundary value problems of the fourth order.*

The general ordinary linear homogeneous differential equation of the fourth order, whose coefficients are continuous functions of x and λ and are of class C' with respect to x, is reduced to the system: $y_1'=p(x,\lambda)y_2$, $y_2'=q(x,\lambda)y_3$, $y_3'=r(x,\lambda)y_4$, $y_4'=s(x,\lambda)y_1$. The p, q, r are essentially positive and it is assumed that s does not change sign on

$a \leqq x \leqq b$. Then if p, q, r, s are monotonic functions of λ on $\Lambda_1 < \lambda < \Lambda_2$, having finite upper bounds while $|p\ q\ r\ s| \to 0$ as $\lambda \to \Lambda_1$ and having positive lower bounds while $|p\ q\ r\ s| \to \infty$ as $\lambda \to \Lambda_2$, it is shown that if $s < 0$ there exists an infinite sequence of values of λ for each of which the system has a solution satisfying the conditions $y_1(a) = y_1'(a) = y_1(b) = 0$ and one other condition (not implying $y_1'(b) = 0$) which may be imposed at either a or b. The solution corresponding to λ_k has k zeros on $a < x < b$. If $s > 0$, there exists a similar sequence for which the system has solutions satisfying the conditions $y_1(a) = y_1'(a) = y_1(b) = y_1'(b) = 0$, the solution corresponding to λ_k has k zeros on $a < x < b$. (Received November 16, 1939.)

126. D. C. Spencer: *On mean one-valent functions.*

Suppose $f(z)$ is regular for $|z| < 1$, and that $f(z)$ maps the unit circle on a Riemann domain W. Let $W(R)$ denote the area (regions covered multiply being counted multiply) of that portion of W which lies inside the circle $|w| \leqq R$. Then, if $W(R) \leqq p\pi R^2$ for all $R > 0$, where p is a positive number (not necessarily integral), $f(z)$ is said to be mean p-valent. In this paper the family of mean one-valent functions (which includes schlicht functions) is studied. If $f(z) = z + a_2 z^2 + \cdots$ is mean one-valent and d is the distance of the boundary of its W from $w = 0$, then in particular the following three results are true: (i) $|a_2| \leqq 2$; (ii) $d \geqq 1/4$; and (iii) $|z|/(1+|z|)^2 \leqq |f(z)| \leqq |z|/(1-|z|)^2$. Equality can occur in any one of the inequalities only if f is the Koebe function. The results are classical for schlicht functions. (Received November 16, 1939.)

127. Otto Szász: *On strong summability of Fourier series.*

A series $\sum_{n=0}^{\infty} u_n$ is said to be strongly summable $(C, 1)$ with index k and sum s if $(n+1)^{-1}\sum_{j=0}^{n} |s_j - s|^k \to 0$ as $n \to \infty$, where $k > 0$ and $s_n = \sum_{j=0}^{n} u_j$. In 1913 Hardy and Littlewood obtained a sufficient condition for strong summability with index 2 of the Fourier series of an integrable function $f(t)$ at $t = x$. Recently Fejér gave two new proofs of this result for the case in which $F(t)$ is continuous at $t = x$. In the present paper several new proofs are given for the general Hardy-Littlewood theorem and for the special case treated by Fejér. Moreover, necessary and sufficient conditions for the strong summability of Fourier series are obtained. The methods are elementary. A typical result is this: Let $A_n \equiv \int\int \psi(t)\psi(u) \cot t/2 \cot u/2\ Q_n(u, t) dt\, du$, $(0 \leqq u \leqq t,\ u + t < r)$, where $f(t)\ \varepsilon\ L$, $\psi(t) = [f(x+t) + f(x-t)]/2$, $Q_n(u, t)$ denotes $\sum_{j=0}^{n} (n+1-j)^2 \sin jt \sin ju$. Then $A_n = o(n^3)$ is necessary and sufficient for strong summability of index 2 of the Fourier series of $f(t)$ at $t = x$. (Received November 21, 1939.)

128. A. W. Tucker: *The algebraic structure of complexes.* II. Preliminary report.

Let X, Y, Z be complexes in a general algebraic sense (see abstract 45-11-437 and a forthcoming note in the Proceedings of the National Academy of Sciences). Then a suitably chosen "tensor," contravariant in X and Y and covariant in Z, determines: (1) a product cycle on XYZ^*, (2) a chain-mapping of Z in XY or a cochain-mapping of XY in Z, (3) a chain-mapping of Y^*Z in X or a cochain-mapping of X in Y^*Z, (4) a product cocycle on X^*Y^*Z. Items (2) and (3) may also be given interpretation as "cup" or "cap" products (Whitney, Annals of Mathematics, (2), vol. 39 (1938), pp. 397–432). In like fashion a general "tensor" determines a variety of mappings, graphs, and products. If two "tensors" of the same type give results which are equivalent, or homologous, in any one respect, they agree also in all other respects. This is the essence of the invariance properties of the "tensors." (Received November 24, 1939.)

129. L. I. Wade: *Certain quantities transcendental over the field $\Phi(x)$, where $\Phi=GF(p^n)$.*

Let $\psi(t)=\sum_{j=0}^{\infty}(-1)^j t^{p^{nj}}/F_j$ so that $\psi(\xi E)=0$ for all polynomials E in x with coefficients in the finite field Φ and ξ is fixed (L. Carlitz, Duke Mathematical Journal, vol. 1 (1935), p. 137). It is proved that ξ is transcendental with respect to the field $\Phi(x)$. More generally $\psi(\alpha)$ is transcendental, where $\alpha\neq 0$ is algebraic over $\Phi(x)$. From this it follows that $\lambda(\alpha)$ is also transcendental, where $\psi(\lambda(t))=t$. (Received November 21, 1939.)

130. H. S. Wall: *Continued fractions and totally monotone sequences*

The object of this paper is to characterize completely totally monotone sequences in terms of continued fractions. It is shown that the sequence $c_0, c_1, c_2, \cdots$ has the property $\Delta^m c_n \geqq 0$, $(m, n=0, 1, 2, \cdots)$, if and only if the power series $c_0-c_1x+c_2x^2-\cdots$ has a corresponding continued fraction of the form $c_0/1+g_1x/1+g_2(1-g_1)x/1+g_3(1-g_2)x/1+\cdots$, which may or may not terminate, where $0<g_n<1$, $(n\geqq 1)$, in case the continued fraction is non-terminating, while the last partial quotient may be irregular in the case of the terminating continued fraction. (Received November 21, 1939.)

131. A. D. Wallace: *Relatively non-alternating transformations.*

Let $T(A)=B$ be a continuous transformation defined on the metric continuum A and let G be a collection of closed subsets of B; the transformation T is said to be non-alternating relative to G (or G n-a) if for no pair (b, Y), with b a point of B and Y an element of G, does there exist a separation $A-T^{-1}(Y)=A_1+A_2$ such that $A_1\cdot T^{-1}(b)\neq 0\neq A_2\cdot T^{-1}(b)$. This generalizes a definition due to G. T. Whyburn (American Journal of Mathematics, vol. 56 (1934), pp. 294–302) since a non-alternating transformation may be defined as non-alternating relative to the set of all points of B. In this paper the author carries over many known theorems concerning non-alternating transformations to G n-a transformations. It is interesting to note that T is monotone if and only if it is non-alternating relative to the collection of all closed subsets of B. (Received November 20, 1939.)

132. M. E. Wescott: *Sets of Newton polynomials analogous to Laguerre's polynomials.*

Let $x_0<x_1<x_2<\cdots$ be an infinite set of discrete variates with interval $h=x_{i+1}-x_i$, $(i\geqq 0, x_0\equiv 0)$. Set $a=(1+h^2)^{1/h^2}$. The expansion of $h^{-n}a^{x+nh}\Delta_h^n[x^{(n)}a^{-x}]$, where $x^{(n)}=x(x-h)\cdots[x-(n-1)h]$, yields, for $n=0, 1, 2, \cdots$, a set of Newton polynomials $\{\mathcal{L}_n(x)\}$ analogous to Laguerre's polynomials. The set is orthogonal on the discrete range $[0, \infty]$ with weight function a^{-x}, normalizes to $[(n!)^2a^{(n+1)h}]/K$, $K=(a^h-1)/h$, possesses a recurrence relation, and satisfies a second order difference equation. As $h\rightarrow 0$, the set reduces to the continuous Laguerre polynomials $\{L_n(x)\}$, and all properties of the discrete set become their counterparts in the continuous set. For $h=1$, the set reduces to a set $\{l_n(x)\}$, which is orthogonal with weight function 2^{-x} on the range of all positive integers and zero. This set normalizes to $(n!)^2 2^{n+1}$. The coefficients of the Newton polynomial $l_n(x)$ are precisely those of the Laguerre polynomial $L_n(x)$. The polynomials $\{l_n(x)\}$ may be used to approximate an empirical function $\{y_x\}$, under a least square criterion, through the relation $y=\sum_{k=0}^{n}c_k l_k(x)$. Here $c_k=\sum_{r=0}^{k}\{(-1)^r[C_{k,r}]^2(k-r)!M_r\}/(k!)^2 2^{k+1}$, where $M_r=\sum_{x=0}^{\infty}2^{-x}x^{(r)}y_x$ is the rth observed moment. M_r converges for empirical data because an index p always exists such that $y_x\equiv 0$ for all $x\geqq p$. (Received November 28, 1939.)

133. Norbert Wiener: *A canonical series for symmetric functions in statistical mechanics.*

A symmetric function of a set of particles can under certain conditions be expanded in a series of which the nth term is an n-tuple sum over the particles. This method is used to express symmetric functions of the positions of a set of molecules of a monatomic gas at time t in terms of the positions at time 0. The distribution function of the particles is deduced. (Received November 22, 1939.)

134. R. L. Wilder: *Local connectedness and generalized manifolds.*

Although existing definitions of generalized n-manifolds (g. n-m.) usually imply that the manifold is locally connected in all dimensions, this local connectedness is of the "ϵ-δ" type and not the type which provides for each point arbitrarily small neighborhoods U such that $p^i(U)=0$ for all i. (In the case of local 0-connectedness the two types are well known to be equivalent when possessed by a space at all points, but this is not the case for local i-connectedness in either the homology or homotopy sense when $i>0$.) One result of this is that a g. 3-m. can be constructed with points which cannot be ϵ-separated by 2-spheres nor, indeed, by 2-manifolds. The following stronger type of definition will eliminate this feature: An M_0 is a pair of points. Assuming an M_{n-1} defined an M_n is a compact metric continuum whose "small" r-cycles $(r<n)$ bound on M_n, and such that if $p \in M_n$, $\epsilon>0$, there exists a "sphere-like" M_{n-1} such that $M_n - M_{n-1} = A+B$ separate, $p \in A \subset S(p, \epsilon)$, and the basis cycle of M_{n-1} bounds on $\bar{A}$ and $\bar{B}$. (Received November 21, 1939.)

135. Alexander Wundheiler: *Are complex numbers vectors?*

Against the widespread belief, the complex numbers are not isomorphic with vectors, as the complex multiplication does not correspond to any invariant operation on vectors. But if a distinguished direction α is introduced in the (metric) vector-plane, the complex product u of the numbers $x=x^1+ix^2$, $y=y^1+iy^2$ becomes equivalent to an invariantive operation performed on the three vectors $\overset{h}{u} = \eta^{h}_{\cdot klm} \overset{k}{x} \overset{l}{y} \overset{m}{a}$, where $\overset{h}{u} = \delta^{h}_{\cdot k} g_{lm} + g \cdot g \cdot g^{hj} \epsilon_{jk} \epsilon_{lm}$, $(h, j, k, l, m=1, 2)$. (g_{lm} is the metric tensor, $g=|g_{lm}|$, $\delta^{k}_{\cdot k}$ is the Kronecker delta, $\epsilon_{jk}=+1, 0, -1$ as $j<k$, $j=k$, $j>k$.) $a=1$ gives the familiar product. Thus, a complex number is a vector in a metric plane with a distinguished direction. Analogous results apply to quaternions. (Received November 20, 1939.)

136. J. W. T. Youngs: *A remark on cyclic transitivity.*

Given a set 1 of elements x, a class Γ of subsets E of 1 generates a binary relation $a\ R(\Gamma)\ b$ defined as follows: for every $x \neq a, b$ there is a set $E \in \Gamma$ such that $a+b \subset E \subset 1-x$. If the class Γ satisfies certain requirements, the relation $R(\Gamma)$ is reflexive, symmetric and cyclicly transitive, and an abstract theory of cyclic elements can be developed in terms of $R(\Gamma)$. (See Radó, abstract 45-5-223; Radó and Reichelderfer, abstract 46-1-30.) The object of this note is to investigate the results of requiring that Γ have two properties: P_1: 0, 1, $x \in \Gamma$; P_2: if $E_1, E_2 \in \Gamma$, $E_1 \cdot E_2 \neq 0$ and $a+b \subset E_1+E_2$, then there exists a set $E \in \Gamma$ such that $a+b \subset E \subset E_1+E_2$. It is to be noticed that in a Peano space arcs have P_2. (Received November 21, 1939.)

137. Max Zorn: *Alternative rings with nilpotent elements.*

This paper contains a proof for the analogue of Engel's theorem in alternative systems, which satisfy the ascending chain condition for subrings. It discusses the relations to the theory of the radical and completes the theory of linear alternative algebras in this respect. (Received November 20, 1939.)