The author's Introduction is not devoid of traces of an aesthetic standpoint. Discussing Carnap's *Logical Syntax of Language* containing two logical languages, one with, and the other without, the multiplicative axiom and the axiom of infinity, he says "I cannot myself regard such a matter as one to be decided by our arbitrary choice. It seems to me that these axioms either do, or do not, have the characteristic of formal truth. I confess, however, that I am unable to give any clear account of what is meant by saying that a proposition is true in virtue of its form."

A one-to-one correspondence can easily be established between these remarks and the well known couplet,

"I do not like thee, Dr. Fell
The reason why I cannot tell."

Of course, there is nothing surprising about this, as the impulse to many mathematical developments lies in similar obscure semi-aesthetic beginnings.

FREDERICK CREEDY

Integralgeometrie, I. By W. Blaschke. Paris, Hermann, 1935. 22 pp.

Integralgeometrie, V. By L. A. Santalo. Paris, Hermann, 1936. 54 pp.

Vorlesungen über Integralgeometrie. Vol. 1. By W. Blaschke. 2d edition. Leipzig and Berlin, Teubner, 1936. 60 pp.

Vorlesungen über Integralgeometrie. Vol. 2. By W. Blaschke. Leipzig and Berlin, Teubner, 1937. 68 pp.

Über eine geometrische Frage von Euclid bis Heute. By W. Blaschke. Leipzig and Berlin, Teubner, 1938. 20 pp.

The first two of these pamphlets are numbers 252 and 357 of the Actualités Scientifiques et Industrielles, and form the first two volumes of a series entitled *Exposés de Géométrie*, published under the direction of Wilhelm Blaschke. The last three pamphlets are volumes 20, 22, and 23 of the Hamburger Mathematische Einzelschriften.

The subject of integral geometry, devised for application to problems in geometric probability (such as Buffon's "needle problem"), has, under the guidance of Blaschke and his students, become an elegant theory of integral invariants, with applications not only to geometric probability, but also to differential geometry, maximum and minimum problems, and geometrical optics.

The first pamphlet is an exposition of the foundations of the subject. A "density" is defined for linear subspaces E_r of euclidean n-space E_n; this density is a differential form in the coördinates of the space S of r-dimensional linear subspaces of E_n. It has the following invariance properties: (1) invariance (Parameterinvarianz) under a change of coördinates in S; (2) invariance (Bewegungsinvarianz) under motions of E_n into itself. *A* "kinematic density" is also defined; it is essentially a density for rectangular coördinate systems C in E_n. In addition to the previous two invariance properties, it is unchanged (Wahlinvarianz) if each coördinate system C is replaced by one rigidly joined to it. This makes it ideal as a density for the positions of a solid body in E_n. These densities are (except for a constant factor) uniquely determined by their invariance properties. Upon integration of them, one obtains an invariant "measure" for linear subspaces of E_n, and an invariant "kinematic measure." Similar measures can be defined in spherical and non-euclidean n-spaces.

The second pamphlet is an original study of the kinematic measure in E_3 with applications. Probably the most important and typical example of the results is the following. Let K be any convex body in E_3; denote its volume by V, the area of its bounding surface S by F, and the integral of the mean curvature of S extended over S by M. Let K_1 be a movable convex body, with corresponding invariants V_1, F_1, M_1. Then the measure of the set of all positions of K_1 cutting K is given by $8\pi^2(V+V_1)+2\pi(M_1F+MF_1)$. The following is an illustration of the manner in which such a result applies to the theory of geometric probability. Let $\overline{K}$ be a convex body inside K. Then the probability that a third convex body K_1 which cuts K also cuts $\overline{K}$ is

$$W = \frac{4\pi(\overline{V} + V_1) + (M_1\overline{F} + \overline{M}F_1)}{4\pi(V + V_1) + (M_1F + MF_1)} \cdot$$

The third pamphlet contains a study of point, straight line, and kinematic measures in the euclidean plane, with applications. Known formulas of Crofton, Poincaré, and Minkowski are proved anew. Some results are proved first for closed convex regions, and later extended to general complexes. An excellent and complete (up to 1936) bibliography is given at the end.

The fourth pamphlet is a continuation of the third, and obtains analogous results for the invariant measures in 3-space, both for closed convex regions and regions bounded by polyhedra. The bibliography is brought up to date.

The last pamphlet is a historical survey of a question in differential geometry in the large, which may be considered as arising from a "theorem" of Euclid. Euclid states that two polyhedra in space are congruent if the faces of one are congruent to the faces of the other. Since any polyhedron can be obtained by identifying pairs of sides in some plane polygonal net, two questions naturally come up: (1) Corresponding to a given plane polygonal net is there always a polyhedron in 3-space? (2) Is this polyhedron unique (that is, are any two such polyhedra congruent under euclidean notions)? Neither of these questions has an affirmative answer without qualification. Cauchy and others have given proofs of the uniqueness theorem (2) under the added hypothesis of convexity of the polyhedron. The existence theorem (1) is not true even under hypotheses on the polygonal net sufficient to insure the closure, orientability, and simply-connectedness of any resulting polyhedron, as well as certain natural metric assumptions on the net. In 1915 Weyl generalized the formulation of these questions as follows: (1) Corresponding to a given two-dimensional Riemannian metric with positive curvature is there always an ovaloid (closed convex surface) in E_3? (2) Is this ovaloid isometrically unique? The uniqueness theorem has been proved by several authors. A proof is given here which holds for polyhedra as well as surfaces with continuous curvature. A proof of the existence theorem has been sketched by Weyl and completed by Lewy. A new possible method of proof, based on the calculus of variations, is sketched by Blaschke.

All five of these pamphlets are well written, and should be of interest both to geometers and students of probability.

S. B. MYERS

Theory of the Integral. By Stanislaw Saks. 2d revised edition. English translation by L. C. Young. Monografje Matematyczne, vol. 7. Warsaw, 1937. 6+347 pp.

This is the third edition of the excellent and eminently useful book by Saks (the first appeared in 1930, in Polish, and the second in 1933, in French; the latter was reviewed in this Bulletin, vol. 40 (1934), pp. 16–18). It is, however, almost a new book, due to numerous changes in exposition and order of the material and important additions of new topics treated. The opening chapter, I (The integral in an abstract space), treats of the modern theory of abstract measure and integration. The basis is