what artificial limiting process). The correspondence $U = e^{iH}$ seems much more fundamental. The other point is in connection with the discussion of the values taken by the "quadratic" form associated with a matrix. The author seems unaware of the paper *On the Field of Values of a Square Matrix* by the present reviewer (Proceedings of the National Academy of Sciences, vol. 18 (1932)).

F. D. MURNAGHAN

The Rational Quartic Curve in Space of Three and Four Dimensions. By H. G. Telling. Cambridge, University Press; New York, The Macmillan Company, 1936. viii+78 pp.

This volume, Number 34 in the well known series of Cambridge Tracts in Mathematics and Mathematical Physics, is an introduction to the study of rational curves. The reviewer agrees that the best curve to select as representative of this type is the norm curve in four dimensions with its projections in ordinary space and in the plane. A detailed study of these rational quartics yields a wealth of geometric properties and of related configurations. On the other hand the analytic work involved is based on the algebraic theory of binary forms, and is not especially complicated for the quartic when expressed in the customary symbolic notation.

The present tract is condensed from a Fellowship Essay by the same author, and much of the material is here given as exercises. There are two chapters, the first of 40 pages on the norm quartic curve, and the second of 33 pages on the rational quartic in three dimensions. These are followed by several pages of notes on involutions on the curve. Only a few references are given in the text, but a selected bibliography is included which contains references to the extensive literature of the subject.

This little volume is well written, with excellent choice and arrangement of material. The author has produced a scholarly essay on a subject which richly deserves a place in this important series.

J. I. TRACEY

Idealtheorie. By W. Krull. (Ergebnisse der Mathematik, und ihrer Grenzgebiete, vol 4, No. 3.) Berlin, Julius Springer, 1935. vii+152 pp.

Krull's new book contains a very timely survey of the maze of material accumulated in recent years in the field of abstract ideal theory. Let it also be said to begin with that Krull's own fundamental contributions to the subject give him preeminent qualifications for the task.

The first concept of general ideal theory must be accredited to Dedekind. In his theory of the rings of all integers in fields of algebraic numbers one has the fundamental theorem that every ideal is a unique product of prime ideals. Dedekind also gives, however, consideration to rings in which this fundamental theorem is not true and where it has to be replaced by other decomposition theorems. Another introduction of general ideal theory came through Kronecker's theory of polynomial moduli. This theory was developed particularly by Lasker and Macauley, who showed its close relation with algebraic geometry. The impetus to the modern development came mainly through the work of Emmy Noether who showed how the theory may be based upon simple axiomatic principles. Among the recent contributors one should mention, besides Krull, also van der Waerden for his extensive investigations on polynomial ideals and their geometrical applications.*

Krull limits his book to commutative ideal theory, and also omits essentially the papers connecting ideal theory with number theory. He begins with a short account of the properties of rings and moduli with operators. Then follows the general abstract ideal theory, the properties of prime and primary ideals, and the decomposition theorems. As a natural application follow the theory of polynomial ideals and the elimination theory for polynomials in several variables. The relation between ideal theory and algebraic geometry is considered extensively; let us mention only the theorems of Max Noether, Henzelt, Bezout, and the theory of multiplicities given by van der Waerden.

Among the further subjects covered in the book one may indicate the ideal theory of power series and of infinite algebraic fields. Another chapter deals with the ideal theory by finite algebraic extensions of the ring. This includes the theorems on norms and discriminants, giving generalizations of the results of the ordinary ideal theory of algebraic numbers. One of the most interesting sections gives Krull's own theory of absolute values, the relation of ideal theory to absolute values and metric properties of the ring and to the concept of integral elements.

Krull's book as a whole should not be classified as a textbook of ideal theory. To the beginner it will certainly present great difficulties. However, to the worker in the field it gives a general account which may be highly recommended. The complete bibliography further enhances its value.

OYSTEIN ORE

* Those interested in a further account of the foundation of ideal theory may be referred to my paper: O. Ore, *Abstract ideal theory*, this Bulletin, vol. 39 (1933), pp. 728–745.