Mathematische Grundlagen der Quantenmechanik. By Johann von Neumann. (Die Grundlehren der Mathematischen Wissenschaften, Band 38.) Berlin, Springer, 1932. 262 pp.

The contents of this book are well described by its title if this is taken to imply, not an exposition of the mathematical technique of working quantum mechanical problems, but a thorough and logical development and discussion of the mathematical axioms on which quantum mechanics is founded. Its purpose is essentially the same as that of Dirac's well known book; its procedure, however, is markedly different. While Dirac presents his reasoning with admirable simplicity and allows himself to be guided at every step by physical intuition—refusing at several places to be burdened by the impediment of mathematical rigor—von Neumann goes at his problem equipped with the nicest of modern mathematical tools and analyzes it to the satisfaction of those whose demands for logical completeness are most exacting. A treatment of this sort is likely to be tiresome for the reader, but it must be said with emphasis that von Neumann has excellently avoided this danger, for his style is luminous and vivid, and has sufficient intrinsic momentum to carry the reader through the more intricate points of the analysis.

The proof of the equivalence of the theories of Heisenberg and Schrödinger is conducted in a manner more fundamental than is customary in physical textbooks on the same subjects. It is carried over into Hilbert space where the elements of the two theories become alike, and where the entire structure of the so-called transformation theory takes a pretty and simple form. As the author points out, this method circumvents numerous analytical flaws of the better known elementary theories and avoids, in particular, the use of the self-contradictory δ-function introduced by Dirac. To acquaint the reader with the properties of abstract Hilbert space, a fairly lengthy chapter on this subject is inserted.

Physical applications are usually omitted, except for two instances which are treated with interesting and illuminating detail: Heisenberg's principle of related indeterminacies (vulgarly denoted in English by the misleading term "uncertainty principle", which was apparently invented to baffle philosophers) and Dirac's theory of radiation.

There are two important respects in which this book is far superior to the treatments of quantum mechanics which have come to the reviewer's notice. The first is a serious endeavor to expose the status of causality. Von Neumann takes the view that causality consists in the possibility of inferring the results of future measurements from the results of past observations. On this basis he shows that the statistical implications of quantum mechanical axioms refute causality. In this connection proper attention is drawn to the twofold possibility of changes in the state of physical systems: first the "regular" continuous change of ψ-functions governed by Schrödinger's time equation; second the abrupt statistical change caused by measurements. The latter is discussed with interesting earnestness—not glossed over because of its difficulties of comprehension—and finally reduced, it appears, to the insolvable mystery of the correlation between physical stimulus and sensory consciousness.

The second point of unusual merit in von Neumann's treatment is that it goes to the trouble of discussing quantum mechanical axioms in connection with statistical assemblies of systems. Clearly, if theorems like that of related indeterminacies are to have meaning in terms of measurements at all, they must be applied to large collections of similar systems. The material presented in relation to this matter is of course largely that published in the author's earlier papers and includes his treatment of thermodynamics. On the whole, the book is probably the most complete and rigorous discussion of the subject indicated by its title.

Naturally, no book is free from minor defects, and no review is complete without their exposition. We feel that von Neumann's book could have been improved by adding an alphabetical index. It also contains no foot-notes. All notes are collected in the appendix, and it happens occasionally that the reader, already disconcerted by having to turn many pages in order to find a reference given in the text, is referred again to a previous note to secure an insignificant bit of information.

HENRY MARGENAU

Wave Mechanics. Elementary Theory. By J. Frenkel. Oxford, Clarendon Press, 1932. viii+278 pp. $5.00.

This is one of the international series of monographs on physics edited by Fowler and Kapitza. Planned originally as a translation of the author's relatively brief *Einführung in die Wellenmechanik*, the text has been so expanded in revision that the completed work is expected to consist of three volumes, of which this is the first. The second is already in press, and the third is planned for next year.

The present volume is intended to give a general survey of the whole subject, using only elementary mathematics. The approach is from the physical rather than from the mathematical point of view, and the author succeeds in covering a vast amount of material. Starting with a brief historical survey of the corpuscular and wave theories of light and a glance at the relativity theory, he discusses the extension of the dual particle-and-wave concept to matter, and leads up to the wave mechanics through Heisenberg's uncertainty relation. A chapter on the wave mechanics of the motion of a particle in a field of force, in which considerable emphasis is laid on the properties of potential barriers, is followed by one on the wave mechanics of a system of particles. Next comes a chapter on statistical mechanics, in which the Einstein-Bose and Fermi-Dirac statistics are contrasted with the classical theory, and the book closes with a chapter on the application of the quantum statistics to the electron theory of metals, heat motion, and radiation. In this last chapter, the author adds to the familiar electron, proton, and photon a new particle, the phonon.

The author's comments and discussion, rendered in a most vivid style, are very illuminating and form the most valuable feature of the book. On the other hand, the book lacks the beautifully logical development of the subject exhibited in de Broglie's *Théorie de la Quantification*, and the reader, unless he already possesses an expert knowledge of the subject, is likely to emerge a trifle bewildered by the complexity and range of the material covered.

LEIGH PAGE