THE BOULDER COLLOQUIUM

The lectures delivered by Professor R. L. Moore were remarkably well attended, there being a total of 91 persons registered for the series. Since it was thought advisable to dispense with the Friday afternoon session, four lectures of one and one-fourth hours each were given instead of the five one-hour lectures as announced in the program. The subject of the Colloquium, *Point set theory*, being one with which a considerable number of American mathematicians were not familiar, advantage was taken by many of these of this opportunity to gain an insight into some of the underlying concepts of this most fascinating and fundamental branch of mathematics.

Inasmuch as it is expected that the lectures will appear in full in the Society's Colloquium series within approximately twelve months, only a brief sketch of the material covered is here given. A good idea of their organization and content can be obtained from the synopsis furnished by Professor Moore, although some deviations from the synopsis were made. The treatment of the subject, based on a set of axioms, which was given, is to a large extent original with the lecturer, a most noteworthy contribution of this type having been published by him in the Transactions of this Society in 1916. In addition to his own work, however, the material covered embraced that of a number of other mathematicians both in this country and in Europe.

On the basis of a system of seven* axioms stated in terms of the undefined concepts *point* and *region*, it is first shown

* Although nine axioms are stated in the synopsis, it was pointed out by the lecturer that Axioms 0 and 6 are superfluous and that they are included because of their usefulness in studying spaces in which a part, but not all, of the axioms of the system are satisfied. That Axiom 8 also is redundant and is to be dropped from the system was communicated to me by Professor Moore a few days after the lectures were concluded.

that any space satisfying these axioms is homeomorphic with the surface of a sphere or with the euclidean plane according as it is or is not compact. Some of the main features of the treatment up to this point are the facts, 1) that at no point, up until the very last when coordinates are introduced, is it necessary or apparently even of any advantage to introduce or make use of the notion of distance, and, 2) that a very large body of propositions follow from very small sub-systems of the whole system of axioms, such as from Axioms 0 and 1 or from Axioms 1 and 2, for example. A surprisingly large number of very fundamental theorems can be proved on the basis of Axioms 0 and 1 alone; and although a space satisfying these axioms is necessarily metric,* the use of the notion of distance, which is foreign to the system of axioms as a whole, seems to add little if anything to the simplicity of the treatment. These propositions hold true in all spaces satisfying the reduced set of axioms on the basis of which they were proved; and since the class of such spaces obviously is larger (granting the irredundancy of the whole system, of course) than that of all spaces satisfying the whole system, the usefulness of the axiomatic method of procedure is undeniable. This fact was brought out very strikingly by the lecturer in the following manner. It was shown that on the basis of Axioms 1 and 2, it follows that every region is arcwise connected, and that the whole space, if connected at all, is arcwise connected. It was then pointed out that in any locally connected inner limiting set (that is, a set of points which is the common part of a family of open sets—frequently called a G_δ set) in a euclidean space of any number of dimensions, or indeed in much more general space, *region* may be defined in such a way that Axioms 1 and 2 are satisfied. Accordingly, it follows that *any connected and locally connected inner limiting set* (*or G_δ set*)

* This fact, which was unknown to the lecturer at the time of the lectures, was communicated to me by him less than a week after their conclusion.

is arcwise connected. This is a very interesting new result. The axiomatic method thus elucidates its usefulness not only as a very direct and easy approach to the subject, which it certainly is, but also in the approach to new problems through the elimination of extraneous material which adds merely to the confusion of the worker.

The lecturer then pointed out that the whole system of axioms is satisfied in a space whose elements are the elements of any upper semi-continuous collections of mutually exclusive compact continua filling up the surface of a sphere or the plane and none separating the sphere or plane. Accordingly, such a space is equivalent to the sphere or plane respectively. Applications of this extremely useful idea were discussed, as were also applications of upper semi-continuous collections and upper semi-continuous decompositions in the study of the structure of continuous curves and continua in general.

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