there is little doubt that he will find this presentation of the geometric function theory most stimulating, and that he will be eager to pursue the subject farther. And this is one of the highest ends a text book can attain.

O. D. KELLOGG

BRILL'S LECTURES ON ALGEBRAIC CURVES

Vorlesungen über ebene algebraische Kurven und algebraische Funktionen. By Alexander Brill. Braunschweig, Vieweg, 1925. x+340 pp. R. M. 17.50.

The book under review embodies in final form the course of lectures which, for many years, Brill has given at the University of Tübingen. As is well known Brill together with Noether are the twin-stars of German geometricians who did the important pioneer work concerning the geometry on algebraic curves.

One may therefore expect that a treatise on the subject by such a man should contain much that is of value and of fundamental importance for the student of geometry and, in a wider sense, for the mathematician in general.

It is true that the lectures are intended for the beginner, i.e., in American terminology, for the first and possibly second year graduate student. In other words, the student will have a fairly good start in algebraic geometry after he has mastered Brill's lectures.

It is obvious that Brill's purpose does not aim at the comprehensiveness and extensive vistas of Enriques' beautiful *Lezioni* or Severi's penetrating *Vorlesungen.* Brill is anxious to stress the function theoretic aspect of the subject more or less in the Weierstrassian spirit, and expects much from such a systematic treatment of the fundamental ideas of algebraic geometry. Thus, referring to the results of the Italian school and in particular to appendices F and G of Severi's *Vorlesungen,* for which Brill has written such a sympathetic introductory preface, Brill expresses the opinion that an attempt to put Severi's method of proof in algebraic form might turn out to be worth while.

Probably the majority of academic teachers who in the course of years repeatedly lecture on the same topic find it advisable to make progressive changes in the choice of the subject matter and its method of presentation. This is of course as it should be if the teacher keeps track of recent advances, in short, if he is up to date.

Brill states that in former years he put more stress upon the projective point of view, while in later years he returned more and more to the standpoint of the first discoverers in this field, of Descartes, Newton, Cramer, Euler, in so far as the graphical or geometric form relations (gestaltlichen Verhältnisse) are concerned. The reviewer may be prejudiced, but he nevertheless regrets that the projective point of view should have been relegated to second or third place.

What Brill presents is, as one might expect, very penetrating and illuminating. The first part deals with the graph of an algebraic curve, in which continuity of rational and radical functions (Wurzelfunktionen) of one variable, graphical representation of radical functions, the determination of the shape of a curve from its equation, approximation curves for singular and infinitely distant points are discussed. Newton's parallelogram and Cramer's (also De Gua de Malves') analytic triangle are introduced skillfully, and applied to effective illustrative examples.

In the second part we find a rigorous function theoretic treatment of algebraic functions and its geometric interpretation for algebraic curves. Here Brill begins with an excursus on power series and continues with a study of the behavior of a curve in the neighborhood of one of its points. Much use is made of the so called Weierstrassian "Vorbereitungssatz" which in the hands of Brill becomes a powerful instrument of analysis and by which he later proves Noether's fundamental theorem concerning curves through the intersection of two given curves. By these methods is also shown how a reduced polynomial $f(x, y)$ may be resolved explicitly into linear factors concerning y; thus

$$f(x, y) \equiv c(x-a) \cdot \prod_{i=1}^{n} \left\{ y - \frac{1}{(x-a)^{p_i}} Q^{(i)} [(x-a)^{1/\Delta_i}] \right\},$$

in which the Q's are power series. After these algebraic and function theoretic developments the discussion of the branches of a curve offers no particular difficulty. This is illustrated in a number of well chosen examples in conjunction with the auxiliary of the analytic triangle. It may be remarked incidentally, that Brill places the method of the analytic triangle on a rigorous algebraic basis.

The third part deals with projectivity and duality. Among the topics considered under this heading are resultants of polynomials, resultants in irrational form, discriminants, reduced resultants of three ternary forms with a given common point.

The establishment of "reduced resultants" serves as an important example of the theory of elimination, in which the general theory, as ordinarily expounded, is useless. Curiously enough, numerous geometric problems lead to problems of elimination for which the general theory is helpless. A systematic treatment of the theory of elimination with particular reference to the important special cases which may arise would seem to be a very promising task for a specialist in algebra.

Further chapters under part III are concerned with polars, Hessians, duality, projection and linear transformations, triangular coordinates and invariants. The well known graphical determination of perspective projection by means of the center, the invariant axis, the line corresponding to the line at infinity (or the line whose corresponding line is at infinity) furnishes a powerful graphical method for the study of the behavior of a curve at its infinite points. This method is so simple and effective that it should be presented in any elementary treatise on plane algebraic curves.

For the student of algebraic geometry the fourth part is by far the most important. Here Brill gives an account of the geometry on a curve, which occasionally takes the form of a report on the essential results which in the course of many years have been obtained in this important field. The treatment is excellent and to the point. Here the reader feels the master Brill, the pioneer of an important geometric doctrine.

There are only two chapters in the last part, one on birational transformations and one on point groups on a curve. As one may expect, such topics as birational transformations between two curves, correspondences between points on a curve, rational curves, equivalences for superlinear branches and compound singularities are discussed. In the last chapter we find a rigorous proof of Noether's theorem by means of the *Vorbereitungssatz*, the rest-theorem, linear series of point groups, series cut out by system of adjoints, special series, canonical series, non special series, algebraic functions of a class, the theorem of reciprocity, etc.

The treatise ends with a section on algebraic curves in higher spaces, in which Brill touches the important mapping problem, and finally refers to the investigations of Italian mathematicians, who interpret algebraic functions on curves in higher spaces and then conversely utilize the results thus gained for the theory of point series. According to Brill it would be a promising task to cast these fruitful methods of proof in algebraic form as emphasized in his *Vorlesungen.*

There are very few mistakes in the text. I shall point out two which are rather conspicuous. On page 210, figure 81 which illustrates Klein's famous curve

$$x^3y+y^3t+t^3x=0,$$

the vertices of the coordinate triangle, whose sides are the flex-tangents, lie off the curve, which is drawn incorrectly. On page 228, the equianharmonic ratio $\frac{1}{2}+\frac{i}{2}\sqrt{3}$ appearing in the discussion of the equianharmonic cubic is called a cube root of unity.

The reviewer almost regrets to be at the end of his task, because he should like to follow Brill in the higher realms indicated by him and to give an account of a few more such excellent chapters.

But Brill, one of the remaining representatives of an important period of geometric development, has given enough of an algebraic, function theoretic treatment of algebraic geometry to stimulate the student for further reading and research in this direction. In this sense the book will be found very profitable and may be highly recommended.

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