THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and seventy-second regular meeting of the Society was held in New York City on Saturday, October 31, 1914, extending through the usual morning and afternoon sessions. The attendance included the following thirty-eight members:

Mr. A. A. Bennett, Professor W. J. Berry, Dr. Emily Coddington, Professor F. N. Cole, Dr. Louise D. Cummings, Dr. H. B. Curtis, Professor L. P. Eisenhart, Professor H. B. Fine, Dr. C. A. Fischer, Professor T. S. Fiske, Professor W. B. Fite, Dr. G. M. Green, Professor C. C. Grove, Professor H. E. Hawkes, Mr. S. A. Joffe, Professor Edward Kasner, Professor C. J. Keyser, Mr. P. H. Linehan, Professor James Maclay, Professor Helen A. Merrill, Dr. R. L. Moore, Professor Richard Morris, Mr. G. W. Mullins, Professor W. F. Osgood, Dr. G. A. Pfeiffer, Dr. H. W. Reddick, Miss C. E. Seely, Professor L. P. Siceloff, Professor Clara E. Smith, Professor P. F. Smith, Professor W. B. Stone, Professor H. D. Thompson, Professor Oswald Veblen, Mr. H. E. Webb, Mr. R. A. Wetzel, Professor H. S. White, Miss E. C. Williams, Professor E. B. Wilson.

Vice-President L. P. Eisenhart occupied the chair. The Council announced the election of the following persons to membership in the Society: Dr. H. R. Kingston, University of Manitoba; Dr. Edward Kircher, Massachusetts Institute of Technology; Mr. Colin MacLennan, Havana Railway, Light and Power Company; Mr. E. E. Moots, Walla Walla, Wash.; Mr. C. N. Reynolds, Jr., Harvard University; Dr. Joseph Rosenbaum, New Haven, Conn.; Dr. Joseph Slepian, Cornell University; Dr. Anna H. Tappan, Iowa State College; Dr. Mabel M. Young, Wellesley College. Four applications for membership in the Society were received.

The Council submitted a list of nominations for officers and other members of the Council, to be placed on the official ballot for the annual election. A committee was appointed to audit the accounts of the Treasurer for the current year. Arrangements were made to adjust the insurance on the property of the Society destroyed by fire on October 10, and to restore the office to working conditions.

The following papers were read at this meeting:

(1) Dr. G. M. GREEN: "On completely integrable systems of homogeneous linear partial differential equations."

(2) Dr. G. A. PFEIFFER: "Contributions to the conformal geometry of analytic arcs."

(3) Dr. C. A. FISCHER: "Conditions for a minimum of an n-fold integral."

(4) Mr. E. C. KEMBLE: "Note on the definition of work."

(5) Professor H. S. WHITE: "Census of the triad systems on 15 letters."

(6) Professor EDWARD KASNER: "A law of reciprocity in the calculus of variations."

(7) Professor K. P. WILLIAMS: "Concerning a certain totally discontinuous function."

(8) Professor T. H. GRONWALL: "Some remarks on conformal representation."

The paper of Mr. Kemble was communicated to the Society and read by Professor Osgood. In the absence of the authors, the papers of Professors Williams and Gronwall were read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. The completely integrable systems of homogeneous linear partial differential equations considered in Dr. Green's paper contain any number m of dependent variables $y_1, y_2, \cdots, y_m$, and any number p of independent variables $u_1, u_2, \cdots, u_p$. Any solution y_k of the system is expressible linearly, with constant coefficients, in terms of a fundamental system of n solutions: $y_k = c_1 y_k^{(1)} + c_2 y_k^{(2)} + \cdots + c_n y_k^{(n)}$; and all derivatives of the dependent variables, of any order, are expressible linearly in terms of n primary derivatives. A certain determinant W, formed from the primary derivatives of a fundamental system of solutions, takes the place in this discussion of the wronskian for a single ordinary homogeneous linear differential equation of the nth order. It is proved that the first derivatives of W with respect to the independent variables are given by the equations

$$W_{u_1} = f_1 W, \qquad W_{u_2} = f_2 W, \qquad \cdots, \qquad W_{u_p} = f_p W,$$

where the f's are functions of the coefficients, and possibly of some of their derivatives, of the given system of differential equations. They satisfy the integrability conditions

$\partial f_s/\partial u_t = \partial f_t/\partial u_s$, so that W may be found by a quadrature from the coefficients of the differential equations.

The chief purpose of the paper is to prove that the system of differential equations may always be reduced, and without any difficulty, to a canonical form, in which the f's are all zero. The coefficients of the canonical form are expressions in the coefficients of the original system of differential equations, and are such that they remain unchanged for any transformation of the dependent variables of the form $y_1 = \lambda_1\bar{y}_1, \cdots, y_m = \lambda_m\bar{y}_m$, where the λ's are arbitrary functions of the independent variables $u_1, \cdots, u_p$. It is important, in Professor Wilczynski's method for investigations in projective differential geometry, to find the above-mentioned seminvariant functions, and the present paper gives a method, always applicable, for calculating them expeditiously.

2. The greater part of Dr. Pfeiffer's paper is devoted to an investigation of the properties of symmetry, i. e., the pairing of points symmetric with respect to an analytic arc, and the associated functional equations. Theorems are given concerning the existence of an analytic arc such that with respect to it one of two given intersecting analytic arcs is the symmetric image of the other. This is "the bisection problem," so called by Kasner. A theorem concerning the "n-sector of two given intersecting analytic arcs" is also obtained.

Closely related results are also given stating necessary and sufficient conditions for the conformal equivalence of certain pairs of intersecting analytic arcs to a rectilinear angle and showing the existence of a unique absolute conformal invariant of a pair of intersecting analytic arcs such that the magnitude of the angle formed by them is commensurable with π. Kasner has already shown that at least one such invariant exists and proved its uniqueness in a few special cases.

3. The necessary conditions for a minimum of a double integral expressed in parametric form have been discussed by Kobb and others. In this paper Dr. Fischer derives the analogous conditions for a minimum of the n-fold integral

$$\int\int \cdots \int F(x_1, x_2, \cdots, x_{n+1}, p_{11}, p_{12}, \cdots, p_{n+1n})du_1\,du_2 \cdots du_n.$$

The analogue of the Lagrange differential equation, and a

variable boundary condition are derived. The variable boundary condition is reduced to a very simple form in the special case where $x_1, x_2, \cdots, x_n$ are the independent variables. Then the analogues of the Legendre and Jacobi conditions are discussed, and sufficient conditions are derived for a permanent sign for the second variation.

4. In Mr. Kemble's paper a difficulty in the usual definition of the work done by a force applied at a point is indicated, and a new definition is suggested. It is proved that the work done as defined is equal to the increase in the kinetic energy of the body to which the force is applied.

5. Special methods from 1852 to 1913 brought to light 12 distinct triad systems on 15 letters. Miss L. D. Cummings added, in 1914, 12 further systems. No attempt had been made to secure an exhaustive list of possible systems. In the present paper Professor White studies first the kind of substitution that can leave a triad system invariant. All such must have as some power a substitution of one or another of seven typical forms. The problem therefore is next to find all systems that are invariant under each of these seven substitutions. In this way only such systems are omitted as may have no group except identity. By this means twenty new systems are found.

6. In a previous paper, Professor Kasner investigated the geometric character of the transversality relations connected with the minimizing of double and simple integrals in space. The main result of the present paper is that with each double integral $\int\int F(x, y, z, p, q)dxdy$ there is associated a simple integral $\int G(x,y,z,y',z')dx$ such that the corresponding transversalities are inverse to each other. Each of the integrand functions F and G determines the other except for a factor involving only x, y, z. Other theorems relate to the case where F and G have the same form, and the case where the two transversalities are orthogonally related.

7. Professor Williams's paper appeared in full in the December BULLETIN.

8. In the present paper, Professor Gronwall proves the following theorems:

I. When the analytic function $z = f(x) = a_0 + a_1x + \cdots + a_nx^n + \cdots$ effects the conformal representation of the circle $|x| < 1$ on a simple (that is, simply connected and nowhere overlapping) region in the z-plane, the area of this region not exceeding A, then, for $|x| \leqq r < 1$,

$$|f(x)| \leqq \sqrt{\frac{A}{\pi} \cdot \log \frac{1}{1-r^2}} \quad \text{and} \quad |f'(x)| \leqq \sqrt{\frac{A}{\pi}} \cdot \frac{1}{1-r^2},$$

and these upper boundaries of $|f(x)|$ and $|f'(x)|$ cannot be replaced by any smaller ones. Less accurate limitations have been given by Koebe and Courant.

II. When $z = f(x) = 1/x + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots$ effects the conformal representation of the circle $|x| < 1$ on a simple region in the z-plane containing the point at infinity, then $|f(x)| < 9/4r$ for $|x| = r < 1$. A less accurate limitation has been given by Fricke.

F. N. COLE,
Secretary.

THE TWENTY-SIXTH REGULAR MEETING OF THE SAN FRANCISCO SECTION.

THE twenty-sixth regular meeting of the San Francisco Section of the Society was held at the University of California on October 24, 1914. Twenty-two persons were present, including the following members of the Society:

Professor R. E. Allardice, Dr. B. A. Bernstein, Professor H. F. Blichfeldt, Professor C. E. Brooks, Dr. Thomas Buck, Professor L. E. Dickson, Professor M. W. Haskell, Professor L. M. Hoskins, Dr. Frank Irwin, Professor D. N. Lehmer, Professor J. H. McDonald, Professor W. A. Manning, Professor H. C. Moreno, Professor C. A. Noble, Professor E. W. Ponzer.

The chairman of the Section, Professor Manning, presided at the opening of the meeting; the chairman-elect, Professor Haskell, then took the chair. The following officers were elected for the ensuing year: chairman, Professor Haskell; secretary, Dr. Buck; programme committee, Professors Manning and Blichfeldt, and Dr. Buck.