THE DECEMBER MEETING OF THE SAN FRANCISCO SECTION.

THE second regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held on Saturday, December 20, 1902, at the University of California. The following sixteen members were present:

Professor R. E. Allardice, Dr. E. M. Blake, Professor H. F. Blichfeldt, Professor G. C. Edwards, Professor R. L. Green, Professor L. M. Hoskins, Dr. D. A. Lehmer, Dr. J. H. McDonald, Professor G. A. Miller, Dr. A. C. Moreno, Dr. C. A. Noble, Dr. T. M. Putnam, Professor Irving Stringham, Dr. S. D. Townley, Mr. A. W. Whitney, Professor E. J. Wilczynski.

A morning and an afternoon session were held, Professor Stringham acting as chairman at both sessions. The officers elected at the May meeting were reëlected for one year, and the by-law relating to the name of the section was amended by replacing "Pacific" by "San Francisco."

The following papers were read at this meeting:

(1) Professor R. E. ALLARDICE: "On a system of similar conics through three points and its transformation group."

(2) Professor H. F. BLICHFELDT: "On a property of conic sections."

(3) Professor L. E. DICKSON: "Generational relations for the abstract group G simply isomorphic with the linear fractional group in the Galois field of order p^n."

(4) Professor L. M. HOSKINS: "A simple method of determining the free nutation of a yielding spheroid."

(5) Dr. D. N. LEHMER: "On the parametric representation of the tetrahedroid surface."

(6) Professor A. O. LEUSCHNER: "Elimination of aberration and parallax in calculating preliminary orbits."

(7) Mr. W. A. MANNING: "The positive primitive substitution groups of class $2p$, p being any prime."

(8) Professor G. A. MILLER: "On the holomorph of a cyclic group."

(9) Dr. C. A. NOBLE: "A problem in relative minima."

(10) Dr. S. D. TOWNLEY: "The probability of collisions amongst the stars."

(11) Professor E. J. WILCZYNSKI: "On a certain congruence associated with a given ruled surface."

Professor Dickson's paper was read by the secretary. In the absence of Professor Leuschner his paper was read by title. All the other papers were read by their authors and most of them aroused considerable discussion. Professor Blichfeldt's paper appears in the present number of the BULLETIN. Abstracts of the other papers are given below.

1. On considering the loci and envelopes connected with a system of similar conics through three fixed points, Professor Allardice found that a number of curves occur which are intimately associated with the circle. In looking for an explanation of this fact, he found that such a system of conics possesses a group of transformations of the following character; Each transformation is equivalent to a quadric inversion ($x = 1/x$, etc.), followed by a rotation about the circumcenter, and then by a second quadric inversion. Any transformation of the group transforms any conic through the given points into a similar conic through the same points; but the transformations are themselves independent of the eccentricity of the conic. The path curves of the group were determined.

3. Professor Moore has given (see Dickson's Linear Groups, page 300) a set of generational relations for the group G considered by Professor Dickson. The object of the present paper is to give a simpler set of such relations, the proof being confined as yet to the twenty-two cases in which $p^n < 49$. Two cases $p = 2$ and $p > 2$ are essentially different.

If $n = 1$ and p is an odd prime, a complete set of generational relations for G is

$$T^2 = S^p = (ST)^3 = I, \quad (S^\tau T S^{2/\tau} T)^2 = I$$

where $\tau = 3, \cdots, \frac{1}{2}(p - 1)$, but only one of the positive residues modulo p of τ, $2/\tau$, $-2/\tau$ is retained.

For $p = 3$ or 5, the only relations are

$$T^2 = I, \quad S^p = I, \quad (ST)^3 = I.$$

Additional relations are:

$$p = 7, \quad (S^3TS^3T)^2 = I.$$

$$p = 11, \quad (S^3TS^8T)^2 = I, \quad (S^4TS^6T)^2 = I.$$

$$p = 13, \quad (S^3TS^5T)^2 = I, \quad (S^4TS^7T)^2 = I.$$

$$p = 17, \quad (S^3TS^{12}T)^2 = I, \quad (S^4TS^9T)^2 = I,$$
$$(S^6TS^6T)^2 = I, \quad (S^7TS^{10}T)^2 = I.$$

$$p = 19, \quad (S^3TS^7T)^2 = I, \quad (S^4TS^{10}T)^2 = I,$$
$$(S^5TS^8T)^2 = I, \quad (S^6TS^{13}T)^2 = I.$$

$$p = 23, \quad (S^3TS^{16}T)^2 = I, \quad (S^4TS^{12}T)^2 = I,$$
$$(S^5TS^5T)^2 = I, \quad (S^6TS^8T)^2 = I, \quad (S^9TS^{13}T)^2 = I.$$

$$p = 29, \quad (S^3TS^{20}T)^2 = I, \quad (S^4TS^{15}T)^2 = I, \quad (S^5TS^{12}T)^2 = I,$$
$$(S^6TS^{10}T)^2 = I, \quad (S^7TS^{21}T)^2 = I, \quad (S^{11}TS^{16}T)^2 = I.$$

For any n and any odd prime p, the set is

$$T^2 = I, \quad (S_1T)^3 = I, \quad S_0 = I, \quad S_\lambda S_\mu = S_{\lambda+\mu} \quad (\lambda, \mu \text{ any marks}),$$
$$(S_\tau TS_{2/\tau}T)^2 = I \qquad (\tau \text{ any mark} \neq 0).$$

These relations are somewhat redundant. For $p^n = 9$, the group of order 360 is generated by T, S_1 and S_j, subject only to the relations

$$T^2 = S_1^3 = S_j^3 = I, \quad S_1S_j = S_jS_1, \quad (S_1T)^3 = I, \quad (S_jTS_jT)^2 = I,$$
$$(S_jS_1TS_j^{-1}S_1T)^2 = I.$$

Its isomorphism with the alternating group on six letters follows from the correspondence $T \sim (12)(34)$, $S_1 \sim (123)$ $S_j \sim (456)$.

For $p = 2$, the set of relations is

$$A^{2^n+1} = I, \quad B^2 = I, \quad (AB)^3 = I \quad (BA^rBA^s)^2 = I,$$

$r = 1, 2, \cdots, 2^n$, the value of S being uniquely determined modulo $2^n + 1$ by r. Among the pairs r, s always occur 1, 2;

2, 1 ; 2^n, $2^n - 1$; $2^n - 1$, 2^n ; and the resulting relations follow from those giving the periods of A, B, and AB. For $n = 1$ or 2, there are no further relations. For $n = 3$, the relations are

$$A^9 = I, \; B^2 = I, \quad (AB)^3 = I, \quad (BA^3BA^5)^2 = I.$$

For $n = 4$, the relations are

$$A^{17} = I, \; B^2 = I, \quad (AB)^3 = I, \quad (BA^3BA^7)^2 = I,$$
$$(BA^4BA^{12})^2 = I, \quad (BA^6BA^9)^2 = I.$$

For $n = 5$, we may restrict r, s to the values 3, 25 ; 4, 10 ; 5, 17 ; 6, 15 ; 7, 11 ; 9, 14 ; 12, 20. The papers giving the proofs have been presented to the American and London Mathematical Societies.

4. The paper by Professor Hoskins discusses the free rotation of a nearly spherical body whose elastic properties and density have spherical symmetry about the center of mass. By taking as axes of reference the lines which at any instant would coincide with the principal axes of inertia if the strain due to rotation were annulled, the equations of angular momentum take simple forms. If A_0, B_0, C_0 are the principal moments of inertia for the unstrained body, and if I is the increment produced in the moment of inertia about the instantaneous axis by the centrifugal forces due to the actual angular velocity, the equations of motion are the same as for a rigid body whose principal moments of inertia are $A_0 + I$, $B_0 + I$ and $C_0 + I$. Since the differences of these quantities are the same as those of A_0, B_0 and C_0, and since I is a small fraction of each of the actual moments of inertia (though not necessarily of their differences), the period of the free nutation is to a close approximation the same as for a rigid body whose principal moments are A_0, B_0 and C_0. The result is general in that it is not restricted to the case in which the axis of rotation deviates but slightly from its mean position, nor to the case in which two of the principal moments of inertia are equal.

5. Dr. Lehmer employed the σ functions of Weierstrass in the parametric representation of the tetrahedroid surface and showed the arrangement of the singular points, the singular

planes, and their interrelations. The 120 lines joining the 16 singular points were shown to be the 120 lines in which the 16 angular planes intersect. Each of these lines meets 12 others in 6 points in involution. The paper has appeared in the January number (volume 25, number 1) of the *American Journal of Mathematics.*

6. Professor Leuschner's paper begins with a discussion of the expediency of taking into account the corrections for aberration and parallax. The author confirms his former conclusions (Beiträge zur Kometenbahnbestimmung) that in certain cases the resulting orbit will be considerably improved by their elimination. He then reviews the existing methods of accomplishing this elimination and points out that no method of eliminating parallax exists which is applicable in all cases, as even the "reduction to the locus fictus" fails for small geocentric latitudes. The author then proposes some very simple and general formulas for eliminating the parallax by means of corrections to the rectangular heliocentric coördinates of the sun. The corrections depend upon the parallax factors and naturally are independent of the geocentric distance of the body. The paper will be published in the *Bulletin of the Lick Observatory.*

7. Mr. Manning obtains the following results. All positive primitive groups of class $2p$, p a prime > 3, contain a primitive subgroup of degree $2p + 1$. None exist unless $2p + 1$ is a prime or a power of 3. For every prime number > 3 satisfying one or the other of these conditions there are just two groups, one of degree $2p + 1$ and the other of degree $2p + 2$. When $2p + 1$ is a prime the group of that degree is semimetacyclic and the group of degree $2p + 2$ is the modular group. When $2p + 1$ is a power of 3 the first group is a subgroup of the holomorph of the abelian group of order 3^β and type $(1, 1, \cdots)$. The group of degree $2p + 2 = 3^\beta + 1$ is the Mathieu group of order $(3^\beta - 1) 3^\beta (3^\beta + 1) / 2$. The paper will be offered to the *Transactions* for publication.

8. It is known that the holomorph k of a cyclic group G is a complete group and that its commutator subgroup is G whenever the order g of G is odd. When g is even and greater than 2 the commutator subgroup of K is the subgroup of G whose order is $g/2$, and K is never complete. The main object

of Professor Miller's paper is to determine additional useful properties of K. In particular the order of every operator of K is determined and also the degree of each substitution when K is represented as a transitive substitution group of degree g. It is pointed out that the properties of the group of isomorphisms of G furnish a very simple proof of Fermat's theorem. Some of the properties of the group of isomorphisms of K when g is even are developed. This paper will be offered to the *Transactions*.

9. On a former occasion (dissertation, Göttingen, 1901), Dr. Noble proved the existence of a function $y = y(x)$ which would render the integral

$$\int_{x_0}^{x_1} f(y, y' ; x)dx$$

a minimum, provided the integrand satisfied the conditions

$$|f(y, y' ; x)| \geqq k > 0, \qquad \left| f(y, y' ; x)\frac{dx}{dy} \right| \geqq k > 0$$

(k arbitrary constant).

The method employed in this existence proof was new — suggested by Hilbert in his Göttingen lectures — in that the minimizing curve was progressively constructed from cluster points, and finally identified with the solution of the Lagrange differential equation by means of the appropriate Weierstrass E-function. The object of the present paper is to extend this existence proof to the case of an integral

$$\int_{x_0}^{x_1} f(y, y', y''; x)dx,$$

under analogous restrictions for the integrand $f(y, y', y''; x)$. Such an integral arises in those problems of relative minima in which it is desired to minimize an integral

$$\int_{x_0}^{x_1} F(y, y', z, z' ; x)dx,$$

by functions $y = y(x)$, $z = z(x)$ which satisfy the conditioning differential equation $z = \phi(y, y' ; x)$. The method is the same as in the existence proof for the simpler case cited above, the theory of the Weierstrass E-function, as developed by Zermelo, serving to identify the resulting minimal curve with the Lagrange solution.

10. A collision between two stars has often been advanced as an explanation of the phenomena of new stars. Dr. Townley has computed the probability of a collision, assuming a uniform distribution of stars in a finite universe of radius of 400 light years. The method adopted for the solution of the problem brings in two series each containing over two million terms. Only roughly approximate values of the sums of these series have thus far been obtained, but it is hoped to obtain more accurate values of them before long. The computation gives an exceedingly small fraction for the probability of a collision; so small indeed as to make the hypothesis of a collision between two stars no longer tenable as a rational explanation of the phenomena of new stars. No reasonable modification of the assumptions upon which the computations are based will produce a probability sufficiently great to explain the observed number of new stars.

11. In a paper recently published in the *Transactions*, Professor Wilczynski has studied a congruence made up of all of the generators of the first kind on the hyperboloids osculating a given ruled surface. This congruence, which there made its appearance in connection with the theory of covariants of a system of linear differential equations, is considered more fully in the present paper. It is found that its focal surfaces are the flecnode surfaces of the original surface. The theory of the developable surfaces contained in the congruence, and the theory of certain other characteristic families of ruled surfaces of the congruence are fully developed. Certain theorems, whose formulation in the previous paper was somewhat inadequate, are completed. Incidentally a simple interpretation is found for the canonical form previously adopted by the author for a system of differential equations of the kind here considered. The paper will be offered to the *Transactions* for publication.

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