THE FIFTH ANNUAL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

WITH the Annual Meeting held on Wednesday, December 28, 1898, the AMERICAN MATHEMATICAL SOCIETY completed the tenth year of its continuous existence under its present and its former name, having been founded as the New York Mathematical Society on November 24, 1888. Its reorganization as a national society went into effect of date July 1, 1894. Reminiscences of the earlier days and a forecast of the aims and prospects of the widened organization were the theme of Dr. Emory McClintock's presidential address: "The past and future of the Society," delivered at the annual meeting, December 28, 1894, and published in the BULLETIN for January, 1895. The rapid growth of the Society in numbers and influence, in scientific zeal and productivity, is steadily realizing the high aspirations of its supporters and friends. It is safe to say that no scientific body in the country is in a more healthy and vigorous condition. The membership has increased from 135 in May, 1891, to 244 in September, 1894, and to 316 in January, 1899. The number of papers presented in 1898 was 88, against 24 in 1894, and the standard of excellence has been at least maintained. The total attendance of members at the seven meetings of 1898 was 182.

An especially encouraging feature is the large number of members who succeed in attending at least one meeting during the year. Nearly one-third of the members, 104, paid this tribute in 1898 to their interest in the work of the Society. This is certainly a large proportion, considering that the meetings were held only in New York, Chicago and Boston, while the members are busy teachers and professional men scattered over the entire country, the foreign membership also being considerable.

Two factors have contributed powerfully to increase the Society's strength and growth since its reorganization as a national body. One of these is the institution of summer meetings, held usually in connection with the large general scientific gatherings, and thus offering special convenience of time and place. The summer meetings have proved extremely popular and interesting, and together with the colloquia by which they have twice been reinforced, they have done more than any other one influence to unite the workers and teachers of mathematics throughout the country in a common bond of professional interest and sympathy. The second strengthening factor was the founding of the Chicago Section, which loyally represents the interests and disseminates the influence of the Society in the West. The aims of the Section are in perfect harmony with those of the Society. Its meetings, which have thus far been held twice a year, bring together and afford a welcome stimulus to many members to whom the New York meetings would usually be inaccessible. The Section has been a focus of great scientific activity, and its work has reflected great credit on the Society.

The attendance at the Annual Meeting numbered twenty-eight and included the following members of the Society:

Professor E. W. Brown, Dr. A. S. Chessin, Professor F. N. Cole, Professor T. S. Fiske, Mr. G. B. Germann, Professor Harold Jacoby, Professor W. W. Landis, Professor Gustave Legras, Dr. G. A. Miller, Professor Simon Newcomb, Professor G. D. Olds, Mr. J. C. Pfister, Professor M. I. Pupin, Miss Amy Rayson, Professor J. K. Rees, Dr. Virgil Snyder, Professor J. H. Tanner, Miss Mary Underhill, Professor J. H. Van Amringe, Professor J. M. Van Vleck, Professor L. A. Wait, Professor R. S. Woodward.

The President of the Society, Professor Simon Newcomb, occupied the chair during the two sessions, which began at 10.30 a. m. and 2.30 p. m. The Council announced the election of the following persons to membership in the Society: Dr. Charles L. Bouton, Harvard University, Cambridge, Mass.; Dr. Adolph A. Himowich, New York, N. Y.; Dr. Francis S. Macaulay, St. Paul's School, London, England; Professor Buzz M. Walker, State Agricultural and Mechanical College, Miss. Four applications for membership were received. Reports were presented by the Treasurer and the Librarian. These reports will be printed in the List of Members now in preparation. A committee consisting of Professor F. N. Cole, T. S. Fiske, H. S. White and R. S. Woodward, were appointed by the Council to arrange for the summer meeting of the Society.

At the annual election the following officers were chosen:

President,	Professor R. S. Woodward,
First Vice-President,	Professor E. H. Moore,
Second Vice-President,	Professor T. S. Fiske,
Secretary,	Professor F. N. Cole,
Treasurer,	Professor Harold Jacoby,
Librarian,	Professor Pomeroy Ladue.

Committee of Publication,

Professor T. S. FISKE,
Professor F. N. COLE,
Professor ALEXANDER ZIWET.

Members of the Council to serve until December, 1901,

Professor MAXIME BÔCHER,
Professor JAMES PIERPONT,
Professor CHARLOTTE ANGAS SCOTT.

The following papers were presented :

(1) Professor M. I. PUPIN : "On multiple resonance."

(2) Dr. A. S. CHESSIN : "On the development of the perturbative function in terms of the eccentric anomalies."

(3) Dr. A. S. CHESSIN : "On some points of the theory of functions."

(4) Professor E. O. LOVETT : "On the transformation of straight lines into spheres."

(5) Dr. E. J. WILCZYNSKI : "A generalization of Appell's factorial functions."

(6) Dr. VIRGIL SNYDER : "Asymptotic lines on ruled surfaces having two rectilinear generators."

(7) Dr. G. A. MILLER : "On a memoir on the substitution groups whose degree is less than nine."

(8) Dr. W. SCHULZ : "On the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = ke^u$$

and its connection with Dirichlet's principle."

(9) Professor ORMOND STONE : "On the solution of D e launay's canonical system of equations."

In the absence of the authors, Professor Lovett's paper was read by title, Dr. Wilczynski's was presented by Professor Fiske, and Professor Stone's by Professor Woodward. Dr. Wilczynski's paper was offered to the Society through Dr. L. E. Dickson, Dr. Schulz's through Professor J. H. Tanner. Abstracts of those papers not intended for publication in the BULLETIN are given below.

Professor Pupin discussed the forced vibrations of a multiplex under the action of a harmonically varying force.

In a series of papers published in the *Astronomical Journal* (1894, Nos. 326 and 332 ; 1898, Nos. 442 and 452) Dr.

Chessin has given a method by which the computation of the terms in the development of the perturbative function is greatly simplified. The object of the present paper is to indicate a further simplification for the case of the development in terms of the eccentric anomalies as given by Professor Simon Newcomb in his Astronomical Papers, vol. III, part 1. This simplification consists in applying the symbols $\Pi_{n,n'}^{m,m'}$ not to the coefficients $a'A_i$; $a'B_i$; $a'C_i$; $\cdots$ of Professor Newcomb but to these coefficients multiplied by $(1+\varepsilon^2)^{-\mu}(1+\varepsilon'^2)^{-\nu}$, the values of μ and ν being respectively equal to $-i, i$; $-i+1, i+1$; $-i+2, i+2$; $\cdots$. If then we denote by $X_{m,n}^{m,n}$, $Y_{m,n}^{m,n}$, $Z_{m,n}^{m,n}$, $\cdots$ the products of the coefficients $P_{m,n}^{m,n}$, $Q_{m,n}^{m,n}$, $R_{m,n}^{m,n}$, $\cdots$ by $(1+\varepsilon^2)^{-m}(1+\varepsilon'^2)^{-n}$ respectively, we obtain a development of the perturbative function which differs from Professor Newcomb's only in the substitution of the X, Y, Z, $\cdots$ for the P, Q, R, $\cdots$ and which presents this advantage over it: while the computation of the X, Y, Z, $\cdots$ with equal upper and lower indices is not perceptibly different from the computation of the corresponding P, Q, R, $\cdots$; the computation of the terms with different upper and lower indices can be made by considerably simpler formulas than in the case when the P, Q, R, $\cdots$ are used.

From the time that Galois proved that the theory of equations is based upon the theory of substitution groups the determination of all the possible substitution groups of a given degree has been a problem of fundamental importance in algebra. During recent years a large number of substitution groups, usually found by tentative processes, have been published. The first part of Dr. Miller's contemplated memoir is devoted to the theory of the construction of substitution groups. This theory is developed to a sufficient extent to include all the groups whose degree does not exceed nine. The necessary and sufficient condition that a given abstract group may be represented as an intransitive substitution group, not simply isomorphic to one of its transitive constituents, is that it contains at least two self-conjugate subgroups, differing from identity, which have only identity in common. The construction of all the imprimitive groups that contain substitutions besides identity which leave all the systems of imprimitivity unchanged can be reduced to the construction of transitive groups containing given intransitive groups as self-conjugate subgroups and permitting the systems of intransitivity of this self-conjugate subgroup according to known substitution groups. If an imprimitive group is simply isomorphic to the group according to which its systems of imprimitivity are permitted, its abstract group properties are known and hence its forms as a substitution group are evident. The second part of the memoir is devoted to the determination of all the possible substitution groups whose degree is less than nine by means of the theorems developed in the first part. The notation employed exhibits what groups are simply isomorphic and hence may be helpful to the student of abstract groups. Brief historical notes on some of the best known groups are given.

Dr. Schulz introduces the two complex variables

$$z = x + yi, \quad z_0 = x - yi$$

into the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = ke^u$$

and defines

$$u = u_0 + zz_0 u_1 + \cdots + (zz_0)^n u_n + \cdots,$$

where $u_0, u_1, \cdots, u_n, \cdots$ satisfy the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

and are each the sum of two conjugate functions of z and z_0. The principal results are the following: The functions $u_1, u_2, \cdots u_n, \cdots$ (n finite) are absolutely convergent in the same region as u_0, and are uniformly defined by u_0. They tend toward continuous one valued boundary functions with the period 2π, if u_0 is defined by such a boundary function according to Dirichlet's principle. A theorem here introduced represents a distinct advance in the theory of the potential function. The function v_0 conjugate to u_0 also tends toward a continuous limit on the boundary whether u_0 does or does not have a derivative on the boundary. The necessary condition for the convergence of the series for u is that u_0 is defined for a circle of radius equal to or less than a fixed finite quantity. This quantity can be expressed by the boundary values of u_0. If the largest absolute value of the boundary function of u_0 is less than a fixed finite quantity, the function u will be absolutely convergent in the same region of convergence as u_0, and the function u

will be absolutely convergent for a smaller region than u_0. The function u tends to a continuous limit on the boundary of a circle in both cases of convergence and the function u with its boundary function is uniformly defined by the boundary function of u_0. By conformal representation the theorem of convergence can be extended to a simply connected surface with any boundary whatever. Assuming that the boundary is composed of analytic curves with only a finite number of singular points, it is proved that u tends to a continuous limit on the boundary except at singular points where logarithmic discontinuities occur. Finally the boundary functions of u and u_0 correspond to each other uniformly.

In the *Proceedings of the London Mathematical Society* (Vol. 27, p. 385), Professor E. W. Brown gives a simple and interesting solution of Delaunay's canonical system of equations by means of Hamilton's principal function. In Professor Stone's solution Hamilton's function is avoided.

F. N. COLE.

COLUMBIA UNIVERSITY.

THE DECEMBER MEETING OF THE CHICAGO SECTION.

THE fourth regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY was held at the University of Chicago on Thursday and Friday, December 29 and 30, 1898. The attendance and number of papers contributed indicate the interest, in the work of the Society and in the development of mathematical science, of those members who are accustomed to attend the meetings of this Section, and would seem to clearly justify its formation.

The total attendance numbered twenty-six including the following members of the Society:

Professor Henry Benner, Professor E. W. Davis, Dr. Harris Hancock, Professor T. F. Holgate, Mr. H. G. Keppel, Professor Malcolm McNeill, Professor H. Maschke, Professor E. H. Moore, Professor H. B. Newson, Professor James Pierpont, Professor J. B. Shaw, Dr. H. F. Stecker, Professor A. L. P. Wernicke, Professor H. S. White, Mr. Wm. H. Williams, Professor Mary F. Winston, Professor J. W. A. Young.