

JAMES JOSEPH SYLVESTER.

Address delivered at a memorial meeting at the Johns Hopkins University, May 2, 1897,

BY DR. FABIAN FRANKLIN.

We have come together to do honor to the memory of the great man whose work in initiating and for seven years conducting the mathematical department of this institution, will always remain one of the proudest traditions of the John Hopkins University. To me, as one who was long his pupil, and who owes so much to his inspiration, has been assigned the task of saying something about the work and the genius of Sylvester, and especially about the influence which he exerted, while in Baltimore, upon the study of mathematics here and upon the advancement of mathematical research in America.

Since his death, there has appeared in the English journal *Nature*, and has been reprinted in the Johns Hopkins Circulars, a review of his life and work by Major McMahon; and in 1889, when that work was well-nigh ended, Sylvester's great compeer and friend, Professor Cayley, contributed to the columns of the same journal a sketch of Sylvester's labors. One of his Baltimore pupils, too, Professor Halsted of the University of Texas, has given in *Science* an account of his life and achievements. It is therefore the less necessary to undertake here to give anything in the nature of an enumeration of even his most signal contributions to mathematics.

His influence upon the development of mathematical science rests chiefly, of course, upon his work in the theory of invariants. Apart from Sir William Rowan Hamilton's invention and development of quaternions, this theory is the one great contribution made by British thought to the progress of pure mathematics in the present century, or indeed since the days of the contemporaries of Newton. From about the middle of the eighteenth century until near the middle of the nineteenth, English mathematics was in a condition of something like torpor. The second half of the eighteenth century was one of the most brilliant periods in the history of mathematics; but the magnificent achievements of Euler, Lagrange, Laplace awakened no response on the other side of the narrow seas. It seems almost incredible that the complacent conservatism of Cambridge went so far that even the notation of mathematical analysis

as used on the Continent was untaught there until about 1820. Babbage tells us, in his "Passages from the Life of a Philosopher," how he, together with Herschel, Peacock and a few others, founded in 1812 the "Analytical Society" for promoting (as Babbage humorously expressed it) "the Principles of D-ism in opposition to the Dot-age of the University." It is from the translation by these three men (in 1816), of Lacroix's *Treatise on the Differential and Integral Calculus*, together with the publication by them, four years later, of two volumes of illustrative examples, that the first impulse toward a revival of mathematics in England is usually dated. Nothing could show more thoroughly the insular and retrograde condition of English mathematics in the early part of this century. The sticking to Newton's fluxions and dots, and the barring out of Leibnitz's differentials and d's, may be set down as a consequence of the great Newton-Leibnitz controversy; but, whatever the cause, so complete a separation from the great current of European thought implies stagnation deep-seated and not easily to be removed. And accordingly, it proved to be the case that in the magnificent extension of the bounds of mathematics which was effected by the continental mathematicians during the first four decades of the present century, England had no share. It is almost literally correct to say that the history of mathematics for about a hundred years might be written without serious defect with English mathematics left entirely out of account.

That the like statement cannot be made in regard to the past fifty years is due preëminently to the genius and labors of three men: Hamilton, Cayley and Sylvester. Hamilton was a high and solitary genius, who constructed and developed, unaided, a great mathematical method. Great as was this work, it lay so entirely apart from the general line of research that it did not, in his own time at least, awaken widespread activity on the part of others either at home or abroad. On the other hand, the theory of invariants had a history of what may be called the normal type. Its origin is to be found in Boole's discoveries of isolated instances of invariance; these led Cayley to institute a systematic investigation of this remarkable and significant phenomenon, and Cayley's researches awakened the ardent interest of Sylvester. Under the hands of these two great masters, a new and important province was rapidly added to the domain of algebra. Not only did the English mathematicians join in the work, but Hermite in France, Aronhold and Clebsch in Germany, Brioschi in Italy, and other continen-

tal mathematicians seized upon the new ideas, and the theory of invariants was for three decades one of the leading objects of mathematical research throughout Europe. It is impossible to apportion between Cayley and Sylvester the honor of the series of brilliant discoveries which marked the early years of the theory of invariants. Their names are linked together as the creators of a new and beautiful development of algebra, the ideas of which have profoundly influenced the progress also of geometry and of analysis generally. "The Theory of Invariants," says MacMahon, "sprang into existence under the strong hand of Cayley, but that it emerged finally a complete work of art, for the admiration of future generations of mathematicians, was largely owing to the flashes of inspiration with which Sylvester's intellect illuminated it." It is pleasant to know that the triumphs of neither were marred by any dispute as to personal claims or by anything even approaching jealousy. On the contrary, these two men of genius, antipodes of each other in temperament and habits of work, were alike in the constancy of their mutual friendship, regard and admiration.

I have dwelt thus long on Sylvester's connection with the creation of the theory of invariants because it is by that chiefly that he left his trace upon the history of mathematics in its large outlines. But his genius is quite as strikingly shown in researches of a more isolated character. Ten years before the date of his work in invariants, he wrote in quick succession several remarkable memoirs on algebraic subjects, especially on Sturm's functions and on elimination. His researches in the theory of partitions of numbers are among the most original and remarkable of his works. He made important and striking additions to the theory of matrices. In the theory of numbers, he was especially interested in ternary cubic forms. The question of the distribution of prime numbers had a great fascination for him; and he succeeded, while in Baltimore, in making an impression upon this recondite problem in that he contracted the limits found by Tchebycheff for the number of primes contained within a given range. His work seldom touched on geometry, but his "Theory of Residuation" in connection with cubic curves is a beautiful structure, to which he made some remarkable additions while in Baltimore. I am not, however, attempting to give a survey of his work; suffice it to add that, in addition to the subjects named, he made contributions to astronomy, to dynamics and to the theory of link-motion, besides other special subjects.

One of the most striking of Sylvester's achievements was his demonstration and extension of Newton's unproved rule concerning the number of imaginary roots of an algebraic equation. Newton had left no trace of the process of thought by which he had arrived at his rule, nor had he given any indication of the basis on which it rests. All attempts of later mathematicians to establish it had proved futile. It was characteristic of Sylvester to set himself the task of filling up this lacuna in mathematics. The things that attracted and fascinated him were of two kinds, which may be called opposite to each other. On the one hand, he revelled in any new and prolific method; the feeling of creation, of abounding productiveness, was to him as the breath of his nostrils. It was largely this that made the Theory of Invariants so congenial to him. To see a whole new world, full of unexpected and harmonious relations, expanding before him, was to fill him with an absorbing and exuberant enthusiasm. In the case of invariants, it may be said that his joy in this sense of creation was not even confined to the discovery of theorems; the algebraic forms themselves were to him as living beings, and the processes, invented largely by himself, for causing these creatures of the mathematical intellect to generate their kind, were to him a source of genuine delight.

Alongside of this love of prolific creation, another intellectual bent, on the surface at least of quite the opposite character, was equally strongly marked in Sylvester. Any crucial problem, especially one that was associated with the name of one of the great masters, if once it attracted Sylvester's attention, fastened itself upon his mind with a grip that seemed never to slacken its tenacity. It kept coming up, again and again, for years, and, as long as it remained unsolved, seemed to become periodically a source of unrest and discomfort to his mind. He had not the serenity which belonged to many other great mathematicians, and notably to Cayley, and which, in a great measure, permitted them to choose among the possible subjects of thought such as they deemed most profitable to pursue. With Sylvester, such tranquil and deliberate choice was entirely out of the question. His temperament was essentially poetic, and it would have been as impossible for him to concentrate the powers of his mind on one subject when the current of his thought was setting toward another as it would have been for Burns to decide in cold blood to write a poem like "Highland Mary" or "The Daisy" when the inspiration of "Tam O'Shanter" was upon him.

It was the mention of Sylvester's demonstration of Newton's rule that suggested these reflections. We, who knew him well in later years, can find no difficulty in understanding the hold this problem had upon him. It was the good fortune of his early hearers in this university to be present when he came into the lecture-room, flushed with the achievement of a somewhat similar task. A certain fundamental theorem in the theory of invariants, which had formed the basis of an important section of Cayley's work, had never been completely demonstrated. The lack of this demonstration had always been, to Sylvester's mind, a most serious blemish in the structure. He had, however, he told us, years ago given up the attempt to find the proof as hopeless. But, upon coming fresh to the subject in connection with his Baltimore lectures, he again grappled with the problem, and, by a fortunate inspiration, succeeded in solving it. It was with a thrill of sympathetic pleasure that his young hearers thus found themselves in some measure associated with an intellectual feat by which had been overcome a difficulty that had successfully resisted assault for a quarter of a century. Nor was this the only instance in which we had an opportunity of observing the tenacious hold upon his intellect of any problem that had come to assume in his mind the aspect of a challenge to the powers of mathematicians.

I have said that Sylvester's powers were set in motion by two opposite kinds of stimulus; that of abundantly rewarding results, and that of the stubborn resistance of concentrated difficulty. In both these kinds of endeavor, he achieved many and signal triumphs. That intermediate kind of effort which slowly and patiently builds up and improves and perfects one's own work, and which gives minute and prolonged study to the work of others, he did not command in any notable degree. He seemed incapable of reading mathematics in a purely receptive way. Apparently a subject either fired in his brain a train of active and restless thought, or it could not retain his attention at all. To a man of such a temperament, it would have been peculiarly helpful to live in an atmosphere in which his human associations would have supplied the stimulus which he could not find in mere reading. The great modern work in the theory of functions and in allied disciplines, he never became acquainted with. No one who witnessed the flaming up of his energies when, at the age of 62, in Baltimore, he felt himself, for the first time, among a band of enthusiastic young workers pursuing pure mathematics for its own

sake, can doubt what the effect would have been if, in the prime of his powers, he had been surrounded by the influences which prevail in Berlin or in Göttingen. It may be confidently taken for granted that he would have done splendid work in those domains of analysis which have furnished the laurels of the great mathematicians of Germany and France in the second half of the present century.

Cambridge, his natural intellectual home, would have been far less helpful, since it was examinations and not research which gave tone to the mathematical life there. But Cambridge would of course have been immeasurably better than the situations in which he actually found himself for forty years after his winning of the Second Wranglership. From a career at Cambridge, to the great loss of that University, of himself, and of mathematics, he was debarred by the religious tests then obtaining in the old English universities. Professor Halsted, in his account of Sylvester's work already referred to, points out how the vicissitudes of his career were reflected in the richness or the meagreness of his mathematical production from period to period.

The life and work of Sylvester illustrate in a striking way the futility of the dispute as to the relative importance of native qualities and of external circumstances in determining the achievements of great men. If any man was ever an original genius, with consuming ardor for one intellectual pursuit, with love and devotion to it burning in youth and undiminished in age, Sylvester was such a man. If any province of thought is open to every worker in it, to work in as he pleases, uninfluenced by the doings of those who happen to be in his neighborhood, in his university, in his country, one would say that mathematics is that province. Yet no one could know Sylvester without feeling that, great and original as was his genius, environment must, in his case, exercise an extraordinary influence on its activity. He was sensitive, passionate, fiery; the glowing language in which he habitually indulged in the midst of his mathematical memoirs was but a reflection of his ardent and excitable temper. Such a man must needs be keenly subject to depression and exaltation, to fits of apathy and ardor, according to the nature of his surroundings and experiences. Those who knew him cannot fail to be convinced that, eminent as were his actual achievements, they do not afford a true measure of his mathematical powers, in comparison with those of his great contemporaries. For he was at once less advantageously circumstanced than they, and in an exceptional degree subject to the influence of his surroundings.

Of his work as a teacher, I can speak only upon the basis of his activity in this university. The one thing which constantly marked his lectures was enthusiastic love of the thing he was doing. He had in the fullest possible degree, to use the French phrase, the defect of this quality; for as he almost always spoke with enthusiastic ardor, so it was almost never possible for him to speak on matters incapable of evoking this ardor. In other words, the substance of his lectures had to consist largely of his own work, and, as a rule, of work hot from the forge. The consequence was that a continuous and systematic presentation of any extensive body of doctrine already completed was not to be expected from him. Any unsolved difficulty, any suggested extension, such as would have been passed by with a mention by other lecturers, became inevitably with him the occasion of a digression which was sure to consume many weeks, if indeed it did not take him away from the original object permanently. Nearly all of the important memoirs which he published while in Baltimore arose in this way. We, who attended his lectures, may be said to have seen these memoirs in the making. He would give us on the Friday the outcome of his grapplings with the enemy since the Tuesday lecture. Rarely can it have fallen to the lot of any class to follow so completely the workings of the mind of the master.

Not only were we thus privileged to see "the very pulse of the machine," to learn the spring and motive of the successive steps that led to his results, but we were set aglow by the delight and admiration which, with perfect naïveté, and with that luxuriance of language peculiar to him, Sylvester lavished upon these results. That in this enthusiastic admiration he sometimes lacked the sense of proportion cannot be denied. A result announced at one lecture and hailed with loud acclaim as a marvel of beauty was by no means sure of not being found before the next lecture to have been erroneous; but the Esther that supplanted this Vashti was quite certain to be found still more supremely beautiful. The fundamental thing, however, was not this occasional extravagance, but the deep and abiding feeling for truth and beauty which underlay it. No young man of generous mind could stand before that superb gray head and hear those expositions of high and dear-bought truths, testifying to a passionate devotion undimmed by years or by arduous labor, without carrying away that which ever after must give to the pursuit of truth a new and deeper significance in his mind.

As is well known, Sylvester had an extraordinary faculty

for the coinage of words, which, indeed, was merely a part of his remarkably keen sense for language in general. In this matter of the coining of words, he doubtless went to extremes, as he did in other things; but there can be no question of the great service he rendered to the new science of invariants by the creation of a whole vocabulary which rendered possible the crystallization of thought in what would otherwise have been a comparatively amorphous mass. There are doubtless other departments of mathematics which would be made more manageable by the skillful application of just such a name-creating faculty. Any mathematical conception with which Sylvester had much to do had to be equipped with a name. He justly felt that the absence of it impeded thought, and he could not be comfortable in this state of things. His hearers will not forget how, after getting along for some time with the notation $\varphi(n)$, by which mathematicians had been content, from the time of Legendre, to designate the number of numbers less than a given number and prime to it, he came into the lecture-room one afternoon, and began in his most emphatic manner, thus: "Gentlemen, I am about to introduce to you a name that has been struggling for birth for a century!" I may mention here an instance of his delicate sense for words—and indeed for things—which occurred during a walk I was taking with him. We were speaking of Mitchell, then a fellow in mathematics here, and I said that he impressed me as bearing a resemblance to Abraham Lincoln. He seemed struck with the idea, and, after a moment's silence, said "Yes, there is a certain not inelegant stiffness about him which reminds one of Lincoln." Where Sylvester got his impression of Lincoln I do not know; but surely it would have been difficult to hit off the outward effect of the man in words more accurately chosen.

Another direction which his talent for expression and his love of the niceties of language took was that of versification. He made some excellent translations from Horace and from German poets, besides writing a number of pieces of original verse. The tours de force in the way of rhyming, which he performed while in Baltimore, were designed to illustrate the theories of versification of which he gives indications in his little book called "The Laws of Verse." The reading of the Rosalind poem at the Peabody Institute was the occasion of an amusing exhibition of absence of mind. The poem consisted of no less than 400 lines, all rhyming with the name Rosalind (the long and short sound of the *i* both being allowed). The audience quite filled the

hall, and expected to find much interest or amusement in listening to this unique experiment in verse. But Professor Sylvester had found it necessary to write a large number of explanatory footnotes, and he announced that in order not to interrupt the poem he would read the footnotes in a body first. Nearly every footnote suggested some additional extempore remark, and the reader was so interested in each one that he was not in the least aware of the flight of time, or of the amusement of the audience. When he had dispatched the last of the notes, he looked up at the clock, and was horrified to find that he had kept the audience an hour and a-half before beginning to read the poem they had come to hear. The astonishment on his face was answered by a burst of good-humored laughter from the audience; and then, after begging all his hearers to feel at perfect liberty to leave if they had engagements, he read the *Rosalind* poem.

Sylvester was quick-tempered and impatient, but generous, charitable and tender-hearted. He was always extremely appreciative of the work of others and gave the warmest recognition to any talent or ability displayed by his pupils. He was capable of flying into a passion on slight provocation, but he did not harbor resentment, and was always glad to forget the cause of quarrel at the earliest opportunity. I have it on extremely good authority that, in his intercourse with Professor Cayley, toward whom he maintained a life-long and devoted friendship, and his admiration of whom might be said to amount to reverence, little episodes of this kind were not absent. Some fancied injury would lead Sylvester to write Cayley an angry letter; Cayley, who was as serene and tranquil as Sylvester was passionate and excitable, would quietly leave the letter unanswered. In a few days another letter was sure to come from Sylvester, written as though nothing whatever had happened. The mention of Cayley leads me to recall an incident of the farewell reception given to Cayley in Hopkins Hall at the close of his residence here, which affords another illustration of Sylvester's felicity of expression. The platform was abundantly decorated with flowers, and Cayley, who was extremely shy and retiring, looked very uncomfortable in his conspicuous position upon it while Sylvester was speaking. Referring to Cayley's modesty, Sylvester suddenly turned toward him and said, "There he sits, like a victim decked with flowers!"

Sylvester did not, I believe, like to speak about religion. He was born a Jew, and was buried in the Jewish cemetery at Dalston. I am sure that he would not have subscribed

to any formulated creed, but he was a man of truly reverent mind and a sincere theist. It was notable that, in speaking of his own designs for the future, he quite habitually used the phrase "please God," with an accent that showed it was not a mere form of words. Once, when I asked him what was his estimate of Clifford, he said, with great earnestness: "Clifford is a very great genius; I only wish he would stick to mathematics instead of talking atheism."

Of Sylvester's influence upon this university, not only through his teaching, through the foundation of the *American Journal of Mathematics*, and through the constant stimulation of mathematical interest here by his incessant productiveness, but also through the infection of his enthusiasm, which was felt in every department of the university, it would be impossible to speak too strongly. His aggressive and singular personality seemed to act the part of a ferment which spread itself through the entire body of the university. In its prosperity and progress and fame he took the deepest interest, and his attachment to it was not weakened when he returned to his native land at the call of the University of Oxford.

Professor Sylvester's residence at the Johns Hopkins University constitutes an episode quite unique in the history of mathematics and education. Up to the time when he came to America, the study of the higher pure mathematics may be said, with almost literal truth, to have been non-existent in our country. He came, a man who had almost filled out what is usually spoken of as the allotted span of life, and at once inspired zeal and activity in a field which had been left almost uncultivated among us. The earliest outward effect of his ardor was the foundation of the *American Journal of Mathematics*, the first mathematical journal of any importance ever published in America, and almost the first journal devoted to any scientific specialty. It may truly be looked upon as the father of that army of scientific journals which have since overspread the country and testified to the growth of the higher learning among us. The prestige of his name and the fertility of his work could not do otherwise than excite emulation in other American centres of learning. While there doubtless would, in any case, have been progress in this direction, it must be set down as preëminently the result of Sylvester's presence in Baltimore that mathematical science in America has received the remarkable impetus which the last twenty years have shown. American names are no longer absent from the record of mathematical progress. We have not yet pro-

duced one of the heroes of mathematics ; but there are now among us a dozen universities in each of which something, be it much or little, is being added to that splendid monument of human thought which bears the record of conquests made by so many of the intellectual giants of our race.

Among these giants Sylvester has without question the right to be reckoned. In the history of mathematics, his place will not be with the very greatest; but his work, brilliant and memorable as it was, affords no true measure of his intellectual greatness. Those who came within the sphere of his personality, could not but feel that, through the force of circumstances combined with the peculiarities of his poetic temperament, his performance, splendid as it was, has not adequately reflected his magnificent powers. Those of us who were connected with him cherish his memory as that of a sympathetic friend and generous critic. And in this university, as long as it shall exist, he will be remembered as the man whose genius illuminated its early years, and whose devotion and ardor furnished the most inspiring of all the elements which went to make those years so memorable and so fruitful.

HYPERBOLEA AND THE SOLUTION OF EQUATIONS.

BY MR. C. H. HINTON.

In the following pages, after a few remarks on the system of mathematics in vogue in Hyperbolea, I wish to show that a consideration of the methods of the Hyperboleans leads to a graphical representation of quantities by which, given an appropriate train of mechanism, not only the real, but also the imaginary roots of an equation can be mechanically found.

Hyperbolea is a land in which distance is measured by the function $\sqrt{x^2 - y^2}$. This, with its attendant consequences, sufficiently defines the locality.

Let AB be a straight line. Numbers give the ordination of positions on it. The length between any two positions is a physical notion. If p is a material rod the intervals AC and BD are said to be equal if the rod p occupies at one time the interval AC , at another time BD without observable distortion in the transference. Taking a two dimensional number system we have besides the system ABC another system of positions $AA'A''$, $BB'B''$ and so on. The