

THE ANNALS of MATHEMATICAL STATISTICS

FOUNDED AND EDITED BY H. C. CARVER, 1930-1938

EDITED BY S. S. WILKS, 1938-1949

THE OFFICIAL JOURNAL OF THE INSTITUTE
OF MATHEMATICAL STATISTICS

Contents

	PAGE
Unbiased coin tossing with a biased coin	GORDON SIMONS AND WASSILY Hoeffding 341
On a class of infinite games related to Liar Dice	THOMAS S. FERGUSON 353
Approximation of age dependent, multitype branching processes . .	THOMAS G. KURTZ 363
A randomized procedure of saturated main effect fractional replicates W. T. FEDERER AND U. B. PAIK	369
Construction of β -content tolerance regions at confidence level γ for large samples from the k -variate normal distribution	IRWIN GUTTMAN 376
Cauchy's equation and sufficient statistics on arcwise connected spaces . .	J. L. DENNY 401
Some structure theorems for the symmetric stable laws	MICHAEL SCHILDER 412
A duality between autoregressive and moving average processes concerning their least squares parameter estimates	DAVID A. PIERCE 422
The asymptotic distribution of the measure of random sets with application to the classical occupancy problem and suggestions for curve fitting . .	GEDALIA AILAM 427
On an asymptotic representation of the distribution of the characteristic roots of $S_1 S_2^{-1}$	TSENG C. CHANG 440
Sufficient conditions for the admissibility under squared error loss of formal Bayes estimators	JAMES V. ZIDEK 446
Maximum likelihood estimation of a unimodal density function . .	EDWARD J. WEGMAN 457
Functions of processes with Markovian states—III . .	MARTIN FOX AND HERMAN RUBIN 472
Integral functionals of birth and death processes and related limiting distributions DONALD R. McNEIL	480
A note on a characterization of the multivariate normal distribution . .	P. R. FISK 486
A correspondence between Bayesian estimation on stochastic processes and smoothing by splines	GEORGE S. KIMELDORF AND GRACE WAHBA 495
Another look at Doob's theorem	R. K. GETOOR AND MURALI RAO 503
Estimation for distributions with monotone failure rate	B. L. S. PRAKASA RAO 507
On excess over the boundary	GARY LORDEN 520

Continued on back cover

Vol. 41, No. 2—April 1970

THE ANNALS OF MATHEMATICAL STATISTICS

Subscription Rates. Current issues are \$30 per volume (six issues of one calendar year). Single issues are \$5.75. Back numbers for all issues up to and including 1956 (Volume 27) are \$16.50 per volume, \$4.50 per issue; issues from 1957 through 1964 are \$22.00 per volume, \$5.75 per issue; from 1965 onward, \$33.00 per volume, \$5.75 per issue. Complete sets (Volumes 1 through 27) are \$275.00. If additional volumes are ordered at the same time complete sets are ordered, the price is \$16.50 per volume for Volumes 28 through 35, and \$22.00 per volume for Volumes 36 through 40. Complete sets (Volumes 1 through 40) are \$500.00.

Rates to members of the Institute of Mathematical Statistics are lower (see inside back cover).

Communications concerning subscriptions, back numbers, payment of dues, membership, changes of address, etc., should be addressed to Professor George J. Resnikoff, Treasurer, Department of Statistics, California State College, Hayward, California 94542.

Papers to be submitted for publication should be sent to the Editor, Z. W. Birnbaum, Department of Mathematics, University of Washington, Seattle, Washington 98105. Abstracts and corrected galley proofs should be sent to the Managing Editor, K. J. C. Smith, Department of Statistics, University of North Carolina, Chapel Hill, North Carolina 27514.

Instructions for Authors. Each manuscript is required to contain an abstract and to have a title descriptive of its contents.

The abstract will be printed immediately after the title, clearly separated from the rest of the paper. Its main purpose is to inform the reader quickly of the nature and results of the paper; it may also be used as an aid in retrieving information. The length of an abstract will clearly depend on the length and difficulty of the paper, but in general it should not exceed 150 words. It should be typed on a separate page, under the heading "Abstract" followed by the title of the paper. Formulas should be used as sparingly as possible. The abstract should not refer to the theorems or formulas in the body of the paper—it ought to be intelligible by itself. Occasionally the abstract will make an introduction unnecessary but more often an introductory section in the paper will still be needed.

The title should be descriptive but concise, i.e. it should indicate the topic of the paper as clearly as possible but every word in it ought to be pertinent.

In the near future the *Annals* will introduce a requirement for authors to list key words and phrases and classifying numbers, as now used in publications of the American Mathematical Society. Authors may want to anticipate this requirement by including such data now.

Manuscripts should be typewritten, entirely double-spaced including references, with wide margins at sides, top and bottom. The original should be submitted with two additional copies on paper that will take corrections. Dittoed or mimeographed papers are acceptable only if completely legible. Footnotes should be reduced to a minimum, and where possible should be replaced by remarks in the text or in the references; formulas in footnotes should be avoided. References should follow the style: [5] Wilks, S. S. (1963). Multivariate statistical outliers. *Sankhyā Ser. A* 25 407-426.

Figures, charts, and diagrams should be professionally drawn on plain white paper or tracing cloth in black India ink, twice the size they are to be printed.

(Continued on inside back cover)

Mail to the *Annals of Mathematical Statistics* should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should not be addressed to Waverly Press.

COMPOSED AND PRINTED AT THE
WAVERLY PRESS, INC., BALTIMORE, MARYLAND 21202 U.S.A.

Second-class postage paid at Baltimore, Maryland 21202 and at additional mailing offices.

Copyright © 1970 by the Institute of Mathematical Statistics

EDITORIAL STAFF

EDITOR

Z. W. BIRNBAUM

ASSOCIATE EDITORS

R. R. BAHADUR
PETER J. BICKEL
A. P. DEMPSTER
SYLVAIN EHRENFIELD
THOMAS S. FERGUSON
RONALD K. GETTOOR

FRANKLIN A. GRAYBILL
W. J. HALL
JAMES F. HANNAN
DAVID L. HANSON
SAMUEL KARLIN
EUGENE LUKACS

INGRAM OLKIN
FRANK PROSCHAN
RONALD PYKE
JEROME SACKS
FRANK L. SPITZER
HENRY TEICHER

MANAGING EDITOR

K. J. C. SMITH

PAST EDITORS OF THE ANNALS

H. C. CARVER, 1930-1938	T. E. HARRIS, 1955-1958
S. S. WILKS, 1938-1949	WILLIAM KRUSKAL, 1958-1961
T. W. ANDERSON, 1950-1952	J. L. HODGES JR., 1961-1964
E. L. LEHMANN, 1953-1955	D. L. BURKHOLDER, 1964-1967

Published bimonthly by the Institute of Mathematical Statistics in February,
April, June, August, October and December.

EDITORIAL POLICY

The main aim of the *Annals of Mathematical Statistics* is to publish original contributions to theoretical statistics and to probability theory. The emphasis is on quality, not just on formal novelty and correctness of the contents of a paper. Expository papers are also welcome, especially if they provide authoritative surveys of areas that have been in vigorous development.

IMS INSTITUTIONAL MEMBERS

AEROSPACE CORPORATION
El Segundo, California

ARIZONA STATE UNIVERSITY
Tempe, Arizona

ARTHUR D. LITTLE, INC.
Cambridge, Massachusetts

BELL TELEPHONE LABORATORIES, TECHNICAL LIBRARY
New York, New York

BOEING SCIENTIFIC RESEARCH LABORATORIES
Seattle, Washington

CASE WESTERN RESERVE UNIVERSITY, DEPARTMENT OF MATHEMATICS
Cleveland, Ohio

CATHOLIC UNIVERSITY OF AMERICA, STATISTICAL LABORATORY
Washington, D.C.

CHEVRON RESEARCH COMPANY
Richmond, California

CIBA PHARMACEUTICAL COMPANY
Summit, New Jersey

COLORADO STATE UNIVERSITY, STATISTICAL LABORATORY
Fort Collins, Colorado

CORNELL UNIVERSITY, DEPARTMENT OF MATHEMATICS
Ithaca, New York

FLORIDA STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Tallahassee, Florida

FORD MOTOR COMPANY, SCIENTIFIC LABORATORIES
Dearborn, Michigan

GENERAL MOTORS CORPORATION, RESEARCH LABORATORIES
Warren, Michigan

GEORGE WASHINGTON UNIVERSITY, DEPARTMENT OF STATISTICS
Washington, D.C.

INDIANA UNIVERSITY, THE LIBRARY
Bloomington, Indiana

INTERNATIONAL BUSINESS MACHINES CORPORATION
Armonk, New York

IOWA STATE UNIVERSITY, STATISTICAL LABORATORY
Ames, Iowa

JOHNS HOPKINS UNIVERSITY, DEPARTMENT OF BIostatISTICS, DEPARTMENT OF STATISTICS
Baltimore, Maryland

KANSAS STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Manhattan, Kansas

KENT STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS
Kent, Ohio

KNOLLS ATOMIC POWER LABORATORY
Schenectady, New York

LOCKHEED-CALIFORNIA COMPANY, SCIENTIFIC AND TECHNICAL INFORMATION CENTER
Burbank, California

MICHIGAN STATE UNIVERSITY, DEPARTMENT OF STATISTICS
East Lansing, Michigan

MINNESOTA MINING AND MANUFACTURING COMPANY, APPLIED MATHEMATICS AND STATISTICS
Saint Paul, Minnesota

NATIONAL SECURITY AGENCY
Fort George G. Meade, Maryland

NEW MEXICO STATE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES
Las Cruces, New Mexico

NORTHWESTERN UNIVERSITY, DEPARTMENT OF MATHEMATICS
Evanston, Illinois

OHIO STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS, STATISTICAL LABORATORY
Columbus, Ohio

OKLAHOMA STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS AND STATISTICS
Stillwater, Oklahoma

OREGON STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Corvallis, Oregon

PENNSYLVANIA STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS LIBRARY
University Park, Pennsylvania

PRINCETON UNIVERSITY, DEPARTMENT OF STATISTICS
Princeton, New Jersey

PROCTER & GAMBLE COMPANY, TECHNICAL LIBRARY
Cincinnati, Ohio

PURDUE UNIVERSITY LIBRARIES
Lafayette, Indiana

THE RAND CORPORATION
Santa Monica, California

THE ROCKEFELLER UNIVERSITY
New York, New York

ROCKLAND STATE HOSPITAL, RESEARCH CENTER
Orangeburg, New York

SANDIA CORPORATION, SANDIA BASE
Albuquerque, New Mexico

SIMON FRASER UNIVERSITY, DEPARTMENT OF MATHEMATICS
Burnaby, British Columbia, Canada

SOUTHERN METHODIST UNIVERSITY, DEPARTMENT OF STATISTICS
Dallas, Texas

STANFORD UNIVERSITY, GIRSCHICK MEMORIAL LIBRARY
Stanford, California

STATE UNIVERSITY OF NEW YORK, BUFFALO, DEPARTMENT OF STATISTICS
Amherst, New York

SYSTEM DEVELOPMENT CORPORATION
Santa Monica, California

TEXAS A & M UNIVERSITY, INSTITUTE OF
 STATISTICS
 College Station, Texas
 T R W SYSTEMS GROUP
 Redondo Beach, California
 UNION OIL COMPANY OF CALIFORNIA, UNION
 RESEARCH CENTER
 Brea, California
 UNITED STATES ARMY RESEARCH AND DE-
 VELOPMENT CENTER
 Aberdeen Proving Ground, Maryland
 UNITED STATES STEEL CORPORATION, AP-
 PLIED RESEARCH LABORATORY
 Monroeville, Pennsylvania
 UNIVERSITY OF ALBERTA, DEPARTMENT OF
 MATHEMATICS
 Edmonton, Alberta, Canada
 UNIVERSITY OF ARIZONA, DEPARTMENT OF
 MATHEMATICS
 Tucson, Arizona
 UNIVERSITY OF BRITISH COLUMBIA, DE-
 PARTMENT OF MATHEMATICS
 Vancouver, B.C., Canada
 UNIVERSITY OF CALIFORNIA, STATISTICAL
 LABORATORY
 Berkeley, California
 UNIVERSITY OF CALIFORNIA, DEPARTMENT
 OF MATHEMATICS
 Davis, California
 UNIVERSITY OF CALIFORNIA, DEPARTMENT
 OF MATHEMATICS
 Irvine, California
 UNIVERSITY OF CALIFORNIA, SCHOOL OF
 PUBLIC HEALTH
 Los Angeles, California
 UNIVERSITY OF CALIFORNIA, DEPARTMENT
 OF MATHEMATICS
 Riverside, California
 UNIVERSITY OF CALIFORNIA, DEPARTMENT
 OF MATHEMATICS
 Santa Barbara, California
 UNIVERSITY OF GEORGIA, DEPARTMENT OF
 STATISTICS
 Athens, Georgia
 UNIVERSITY OF GUELPH, LIBRARY, SERIALS
 DEPARTMENT
 Guelph, Ontario, Canada
 UNIVERSITY OF ILLINOIS AT CHICAGO
 CIRCLE, DEPARTMENT OF MATHEMATICS
 Chicago, Illinois
 UNIVERSITY OF ILLINOIS, LIBRARY, SERIALS
 DEPARTMENT
 Urbana, Illinois
 UNIVERSITY OF IOWA, DIVISION OF MATHE-
 MATICAL SCIENCES
 Iowa City, Iowa
 UNIVERSITY OF MANITOBA, DEPARTMENT
 OF STATISTICS
 Winnipeg-19, Manitoba, Canada
 UNIVERSITY OF MARYLAND, DEPARTMENT
 OF MATHEMATICS
 College Park, Maryland
 UNIVERSITY OF MASSACHUSETTS, DEPART-
 MENT OF MATHEMATICS & STATISTICS
 Amherst, Massachusetts
 UNIVERSITY OF MICHIGAN, DEPARTMENT OF
 MATHEMATICS
 Ann Arbor, Michigan
 UNIVERSITY OF MINNESOTA, DEPARTMENT
 OF STATISTICS
 Minneapolis, Minnesota
 UNIVERSITY OF MISSOURI, DEPARTMENT OF
 STATISTICS
 Columbia, Missouri
 UNIVERSITY OF MISSOURI AT ROLLA, DE-
 PARTMENT OF MATHEMATICS
 Rolla, Missouri
 UNIVERSITY OF MONTREAL, DEPARTMENT
 OF MATHEMATICS
 Montreal, Quebec, Canada
 UNIVERSITY OF NEW MEXICO, DEPARTMENT
 OF MATHEMATICS & STATISTICS
 Albuquerque, New Mexico
 UNIVERSITY OF NORTH CAROLINA, DEPART-
 MENT OF STATISTICS
 Chapel Hill, North Carolina
 UNIVERSITY OF PENNSYLVANIA, THE
 LIBRARY
 Philadelphia, Pennsylvania
 UNIVERSITY OF PUERTO RICO, DEPART-
 MENT OF MATHEMATICS
 Mayaguez, Puerto Rico
 UNIVERSITY OF ROCHESTER
 Rochester, New York
 UNIVERSITY OF WASHINGTON, LABORATORY
 OF STATISTICAL RESEARCH
 Seattle, Washington
 UNIVERSITY OF WISCONSIN, MADISON, DE-
 PARTMENT OF STATISTICS
 Madison, Wisconsin
 UNIVERSITY OF WISCONSIN, MILWAUKEE,
 DEPARTMENT OF MATHEMATICS
 Milwaukee, Wisconsin
 WAYNE STATE UNIVERSITY, DEPARTMENT
 OF MATHEMATICS
 Detroit, Michigan
 WEST CHESTER STATE COLLEGE
 West Chester, Pennsylvania
 WESTAT RESEARCH, INC.
 Bethesda, Maryland
 WESTINGHOUSE ELECTRIC CORPORATION,
 RESEARCH LABORATORIES
 Pittsburgh, Pennsylvania

MEETINGS OF THE INSTITUTE

NOTE: This calendar lists all of the meetings which have been approved by the IMS Council to date. Meeting dates which fall rather far in the future are subject to change.

Meet- ing No.	Place	Date	Abstract Deadline*	Joint With
124	Dallas, Texas (Central Regional)	April 8-10, 1970	February 17, 1970	ASA
125	Chapel Hill, N.C. (Eastern Regional)	May 5-7, 1970	March 16, 1970	ASA, ENAR
126	Laramie, Wyoming (Annual Meeting)	August 25-28, 1970	July 6, 1970	AMS
127	Hanover, West Ger- many (European Regional)	August 19-26, 1970	June 30, 1970	
	University Park, Penn. (Eastern Regional)	April 21-23, 1971		ASA, ENAR
	Columbia, Missouri (Central Regional)	Spring, 1971	
	Ames, Iowa (Central Regional)	Spring, 1972		ASA, ENAR

* Abstracts should be submitted in duplicate to the Managing Editor, K. J. C. Smith, Department of Statistics, University of North Carolina, Chapel Hill, North Carolina 27514, on abstract blanks which may be obtained from him. Abstracts of papers to be presented *in person* at the meetings must be received on or before the corresponding deadline. Only one contributed paper may be given in person at any one meeting. Abstracts contributed by title may be submitted at any time and will be printed in the next issue to go to press. Abstracts should not exceed 200 words and should avoid displayed expressions and complicated formulae. They can be accepted from non-members of the IMS only if introduced by members. Abstracts must follow the stylistic requests on the abstract blank or they may be returned.