

# THE ANNALS of MATHEMATICAL STATISTICS

FOUNDED AND EDITED BY H. C. CARVER, 1930-1938

EDITED BY S. S. WILKS, 1938-1949

THE OFFICIAL JOURNAL OF THE INSTITUTE  
OF MATHEMATICAL STATISTICS

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Vol. 42, No. 3—June 1971

# THE ANNALS OF MATHEMATICAL STATISTICS

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COMPOSED AND PRINTED AT THE  
WAVERLY PRESS, INC., BALTIMORE, MARYLAND 21202 U.S.A.

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Published bimonthly by the Institute of Mathematical Statistics in February,  
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Meet- ing No.	Place	Date	Abstract Deadline*	Joint With
131	Fort Collins, Colorado (Annual)	August 23-26, 1971	June 24, 1971	ASA
	Blacksburg, Virginia (Eastern Regional)	Spring 1972		ASA
	Ames, Iowa (Central Regional)	Spring 1972		ASA, ENAR
	Seattle, Washington (Western Regional)	Spring 1972		
	Ithaca, New York (Eastern Regional)	Spring 1973		ASA, ENAR
	Albuquerque, New Mexico (Western Regional)	Spring 1973		

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