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EDITED BY S. S. WILKS, 1938-1949

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Manuscripts submitted for publication in the *Annals of Mathematical Statistics* should be sent to the Editor at the following address: Ingram Olkin, Editor, *Annals of Mathematical Statistics*, Educational Testing Service, Princeton, New Jersey 08540. After June 1972, to Ingram Olkin, Department of Statistics, Stanford University, Stanford, California 94305. Manuscripts submitted for publication in the *Annals of Probability* should be sent to the Editor at the following address: Ronald Pyke, Editor, *Annals of Probability*, Department of Mathematics, University of Washington, Seattle, Washington 98105.

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The Council of the Institute of Mathematical Statistics at the annual meeting of the Institute recommended to the membership that as of January 1973,

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be superseded by the two journals:

THE ANNALS OF STATISTICS

and

THE ANNALS OF PROBABILITY

under the Editorship of Ingram Olkin and Ronald Pyke, respectively. Both journals will become, subject to the vote of the membership, official journals of the Institute of Mathematical Statistics and will appear in alternate months. Further details about this change will be forthcoming from the office of the Executive Secretary.

From January, 1972, papers submitted for publication in each journal should be sent to the appropriate Editor. In cases of doubt, a paper may be sent to either Editor; because some papers might naturally be in either journal, some members of the Editorial Board will be common to both journals.

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